Ch 1 Doing Physics

Dimensional Analysis:

Dimension of speed: $[v] = \dfrac{[\Delta s]}{[\Delta t]} = \dfrac{L}{T}$

$[\Delta s]$: dimension of the change of position $= L$

$[\Delta t]$: dimension of $\underbrace{\text{the change}}_{(\Delta \to \text{"delta"})}$ of time $=$ dimension of time $= T$

Dimension of time: T

Dimension of length/position/space: L

→ Units of speed: $\underbrace{\dfrac{m}{s}}_{\substack{\text{S.I. units} \\ \text{(International units)}}}$, $\dfrac{km}{h}$, $\dfrac{mi}{h}$, etc...

Dimension of acceleration: $[a] = \dfrac{[\Delta v]}{[\Delta t]} = \dfrac{\frac{L}{T}}{T} = \dfrac{L}{T^2}$

Acceleration: change of velocity over change of time

→ Units of acceleration: $\dfrac{m}{s^2}$ (SI. unit), $\dfrac{km}{h^2}$, $\dfrac{m}{h^2}$, $\dfrac{ft}{s^2}$, $\dfrac{cm}{s^2}$, etc...

Application of dimensional analysis:

$$v = \frac{1}{2} g h^2 \rightarrow \left[\tfrac{1}{2} g h^2\right] = \left[\tfrac{1}{2}\right] \cdot [g] \cdot [h]^2 = 1 \cdot \frac{L}{T^2} \cdot L^2 = \frac{L^3}{T^2}$$

$$\boxed{v = \sqrt{gh}} \rightarrow [\sqrt{gh}] = \sqrt{[g \cdot h]} = \{[g] \cdot [h]\}^{\frac{1}{2}} = \left\{\frac{L}{T^2} \cdot L\right\}^{\frac{1}{2}} = \left\{\frac{L^2}{T^2}\right\}^{\frac{1}{2}} = \frac{L}{T}$$

g : acceleration of gravity (SI: $g = 9.81\ m/s^2$) $\rightarrow [g] = \dfrac{L}{T^2}$

h : height $\longrightarrow [h] = L$

<u>Dimensional analysis loop hole:</u> numeric constants (with dimension = 1)

$v = \frac{1}{2} g h^2$

$v = \frac{2}{5}\sqrt{gh} \rightarrow \left[\frac{2}{5}\sqrt{gh}\right] = \frac{L}{T}$ ✓

<u>Dimension ↔ units:</u> S.I. or international system

L : m (meter)

T : s (second)

M (mass) : kg (kilogram : kilo = 10^3)

Area : m^2 (meter squared)

Volume : m^3

Energy (kinetic) : $kg.\frac{m^2}{s^2} \equiv J$ (Joules)

$\frac{1}{2}mv^2$

<u>Conversions</u> :

1 ft = 0.3048 m

1 lb = 0.454 kg

1 cal = 4.184 J

1 h = 3600 s

1 day = 86400 s

1 km = 1000 m ; 1 km^2 = 10^6 m^2

1 cm = 10^{-2} m ; 1 cm^2 = 10^{-4} m^2

nano ↑ 1 nm	micron ↑ 1 μm	1 mm	1 cm	1 km	1 mi	1 in	1 light-year
10^{-9} m	10^{-6} m	10^{-3} m	10^{-2} m	10^{3} m	1609 m	2.54 cm	9.46×10^{15} m

Results: numbers + $\boxed{\text{units}}$

↓

Scientific notation: using powers of 10 with a coefficient b/w 0 & 9

$$\Delta s = 3\,105\,000 \text{ m} = \underbrace{3.105}_{\substack{\text{coefficient} \\ <10}} \times \underbrace{10^6}_{\substack{\text{power} \\ \text{of } 10}}$$

in calculator: 3.105 E+6

Accuracy: number of decimal digits

$\pi = 3.1416$ is more accurate than 3.14

→ **Addition & subtraction**: $\pi - 2.14 = 3.1416 - 2.14$ } Calculator: 1.0016; You: 1.00

→ keep lowest accuracy

→ limits of measuring instruments.

Significant figures s.f.: $\underbrace{11\,275\,000}_{\text{s.f.} = 5}$ $\qquad$ $\underbrace{11\,275\,002}_{\text{s.f.} = 8}$

(zeroes at the end are not significant)

→ **Multiplication & division**: keep smallest number of s.f.'s (except for numeric constant like π, e, etc. which are not measured)

Earth's circumference: $2\pi R_E = 2 \times \underbrace{3.1416}_{\text{s.f}=5} \times \underbrace{6.37 \times 10^6 \text{ m}}_{\text{s.f}=3}$

$= \underbrace{4.002398\text{E}7}_{\text{s.f}=7} \rightarrow \cancel{4.00 \times 10^7}$

$= 4.00 \times 10^7 \text{ m}$

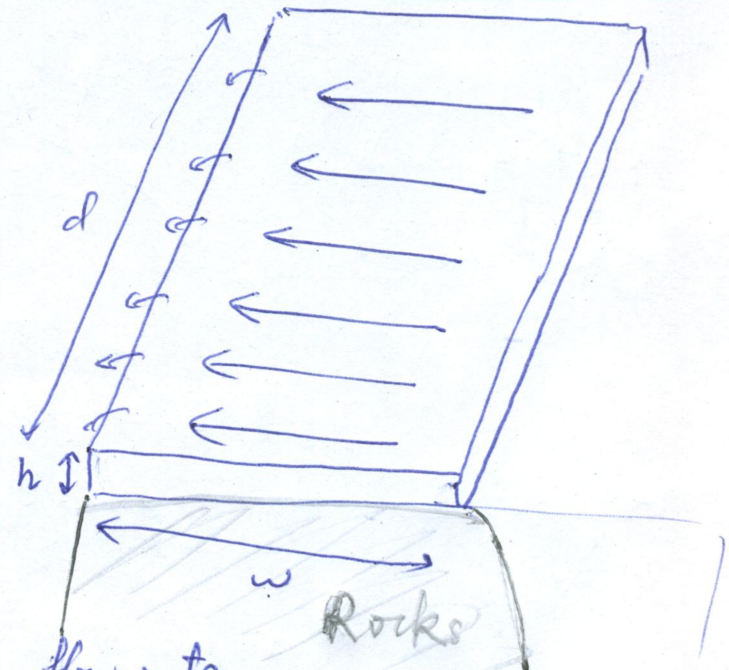
1.41 a) Estimate volume of water over Niagara Falls per second.

Guess: $5\times10^6\ \frac{m^3}{s}$; $1.2\times10^4\ \frac{m^3}{s}$; $8\times10^5\ \frac{m^3}{s}$

Estimation: approximate shape of water using simple geometries: rectangular slab:

Top of falls:

Volume: hwd



Volume per unit time: or flow rate

$$\frac{h\cdot w\cdot d}{t} = \begin{cases} 1)\ \frac{h}{t}\cdot w\cdot d \\ 2)\ h\cdot \frac{w}{t}\cdot d\ \checkmark \\ 3)\ h\cdot w\cdot \frac{d}{t} \end{cases}$$

✓: Use this to proceed with estimation since $\frac{w}{t}$ = speed of water (motion in the other two directions are negligible)

→ Flow rate, need estimates:

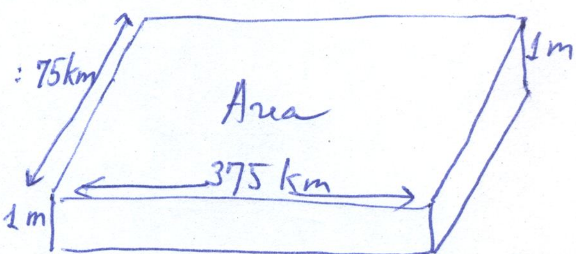
h: thickness of water slab (m)	1m ✓	10m	100m	
$\frac{w}{t}$ = speed of water ($\frac{m}{s}$)	$1\frac{m}{s}$	$2\frac{m}{s}$ (1)	$5\frac{m}{s}$ (9) ✓	$10\frac{m}{s}$ (10)
d = depth of Falls (m)	100m	1000m ✓	10 000m	

Flow rate: $\sim 1m \times 5\frac{m}{s} \times 1000m = 5000\ \frac{m^3}{s}$

b)

Normally Niagara Falls serve as outlet for Lake Erie. If Falls are shutoff <u>how long</u> for water in Lake Erie to rise 1 m:

→ We need area of Lake Erie :



↳ Volume of water: $1\,m \times 75\times10^3\,m \times 375\times10^3\,m$

→ How long? → $t = \dfrac{\text{volume}}{\text{Flow rate}} = \dfrac{75\times375\times10^6\,m^3}{5\times10^3\,\frac{m^3}{s}} = 5.6\times10^6\,s$

(≈ 1D : $\dfrac{\text{distance}}{\text{speed}}$)

$5.6\times10^6\,\cancel{s}\ \dfrac{1\,\text{day}}{84600\,\cancel{s}} = 66$ days $\simeq 2$ months.

Ch 2 Motion in a Straight Line (1D)

Average motion:

↳ $\text{speed} = \dfrac{\text{distance}}{\text{time}}$; $\text{velocity} = \dfrac{\text{displacement}}{\text{time}}$

Example: W —— 10 mi ——→ E $t = 2\text{hrs}.$

←— 2 mi

W→E 10 mi
then returns 2 mi
in 2 hrs.

↳ $\text{speed} = \dfrac{12\text{mi}}{2\text{hrs}} = 6\text{mph}$; $\text{velocity} = \dfrac{8\text{mi}}{2\text{hrs}} = 4\text{mph}$

direction of motion is included

Average velocity: $\bar{v} = \dfrac{\Delta x}{\Delta t}$ $\begin{cases} \Delta x: \text{change of position or displacement} \\ \Delta t = \text{change of time or time increment} \end{cases}$

Instantaneous velocity: $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ (time derivative of the position)

Example: $x = a_1 t^4$ → instantaneous vel.

x: position; a_1: constant; t: time

$v = \dfrac{dx}{dt} = \dfrac{d(a_1 t^4)}{dt} = a_1 4 t^3$

$v = 4a_1 t^3$

unit for a_1: $\dfrac{m}{s^4}$

$\boxed{\text{Review: } \dfrac{d\,t^n}{dt} = n \cdot t^{n-1}}$

Velocity can change over time $\rightarrow$ acceleration

<u>Average</u> acceleration $= \bar{a} = \dfrac{\Delta v}{\Delta t}$ $\left\{ \begin{array}{l} \Delta v: \text{change of velocity} \\ \Delta t: \text{change of time} \end{array} \right.$

<u>Instantaneous</u> acceleration $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$ (time derivative of velocity)

<u>Example</u>: $a = \dfrac{dv}{dt} = \dfrac{d(4a_1 t^3)}{dt} = 12 a_1 t^2$

More common type of motion on our planet:

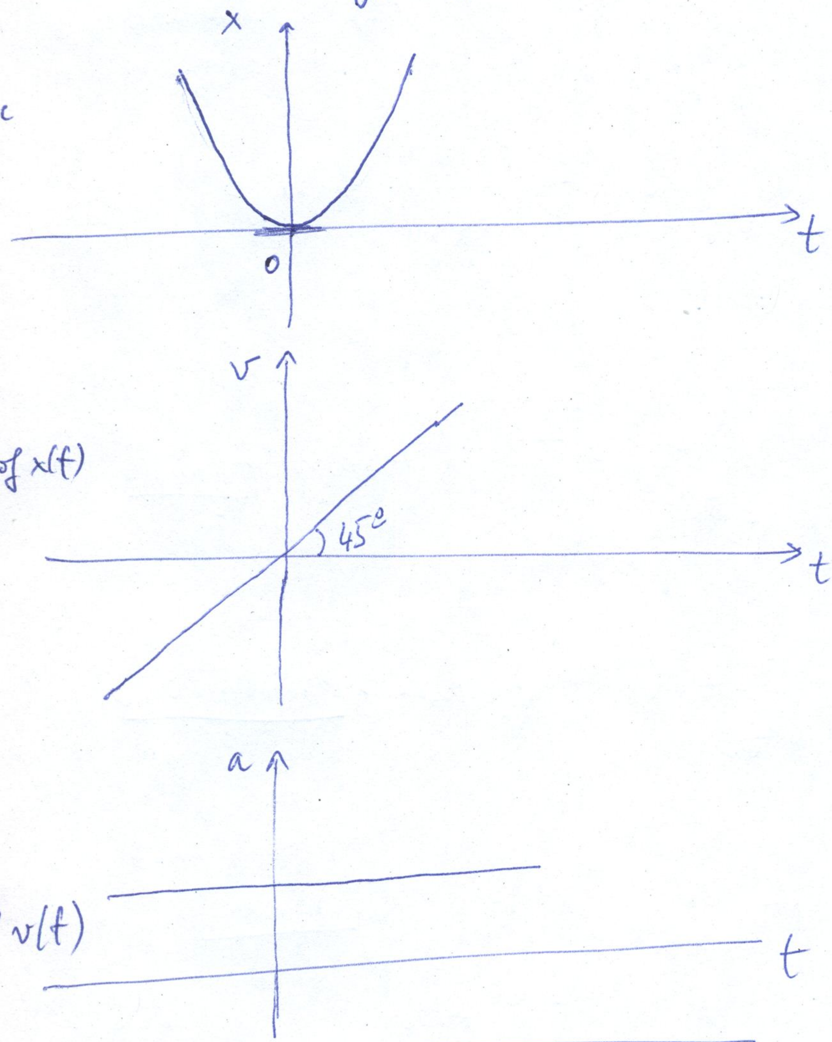
x(t) parabolic

$x = \frac{1}{2} a_1 t^2$

$v = \dfrac{dx}{dt}$ slope of x(t)

$v = a_1 t$

$a = \dfrac{dv}{dt}$ slope of v(t)



parabolic $\rightarrow$ constant acceleration (gravity)

$a = a_1$

Derive kinematic equations (3) for constant acceleration motion:

Constant acceleration $\Rightarrow$ $\bar{a} = a$ (average = instantaneous)

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$$

v = final velocity
v_0 = initial velocity
t = final time
0 = initial time

$$a \cdot t = v - v_0 \Rightarrow \boxed{v = v_0 + a \cdot t} \quad (1)$$

1st Kinematic Eq. for constant acceleration

2nd Kinematic eq:

Average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \Rightarrow x = x_0 + \bar{v} \cdot t \quad (A)$$

Mathematically: $\bar{v} = \frac{1}{t-0}\int_0^t v \cdot dt \overset{(1)}{=} \frac{1}{t}\int_0^t (v_0 + a \cdot t) \cdot dt$

$$= \frac{v_0}{t} \underbrace{\int_0^t dt}_{[t]_0^t = t} + \frac{a}{t} \underbrace{\int_0^t t\,dt}_{\left[\frac{1}{2}t^2\right]_0^t = \frac{1}{2}t^2}$$

$$= v_0 + \frac{1}{2} a \cdot t$$

$$= \frac{1}{2} v_0 + \frac{1}{2} a \cdot 0 + \frac{1}{2} v_0 + \frac{1}{2} a \cdot t$$

$$= \frac{1}{2}\left[(v_0 + a \cdot 0) + (v_0 + a \cdot t)\right]$$

(B) $\bar{v} = \frac{1}{2}\left[v_0 + v\right]$ ($\bar{v}$ is $(v_0 + v)$ divided by 2)

Plug (B) into (A) : $x = x_0 + \bar{v}\cdot t$

$$= x_0 + \underbrace{\frac{1}{2}(v_0 + v)}_{\bar{v}\ (B)}\cdot t$$

$$\overset{(1)}{=} x_0 + \frac{1}{2}(v_0 + v_0 + a\cdot t)\cdot t$$

$$\Rightarrow \boxed{x = x_0 + v_0\cdot t + \frac{1}{2}a\cdot t^2} \quad (2)$$

2nd Kinematic eq. for constant acceleration

Comes from { Definitions of $\bar{v}$, $\bar{a}$,
Constant acceleration $\bar{a} = a$
Math expression for $\bar{v}$ & some algebra.

<u>Constant acceleration</u> motion in 1D:

$$v = v_0 + a\cdot t \quad (1)$$

$$x = x_0 + v_0\cdot t + \frac{1}{2}at^2 \quad (2)$$

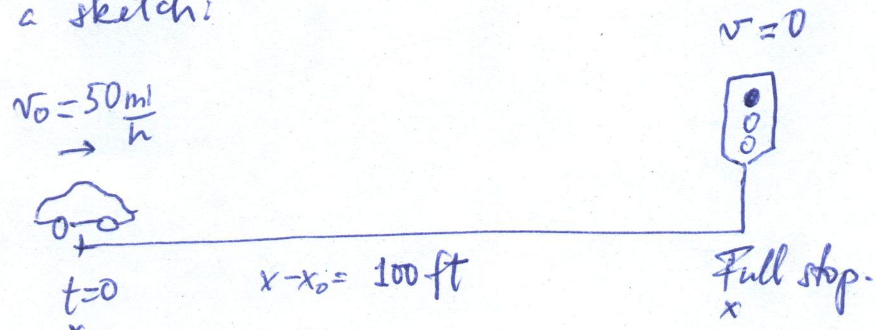
can derive from (1) & (2) : $\dfrac{v^2 - v_0^2}{x - x_0} = 2\cdot a \quad (3)$

<u>Note:</u> (i) There is no time in (3)
(when time is not given)

(ii) Notation: final : x, v
Initial : x_0, v_0

2.33

Step 1) Reword the problem identifying all information and make a sketch:



starts slowing down at <u>constant</u> <u>rate</u> $\longrightarrow$ <u>can apply the kinematic eqs for constant acceleration</u>

<u>Alternative #1</u>: Start with eq. 3) (time is not given)

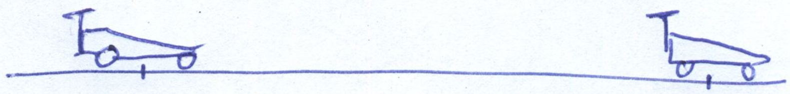
$$\frac{0 - 22.35^2}{30.48} = 2 \cdot a \rightarrow \boxed{a = \frac{1}{2}\,\frac{-(22.35^2)}{30.48} = -8.192\,\frac{m}{s^2}}$$

<u>Conversion</u>: $$50\,\frac{\cancel{mi}}{\cancel{h}} \cdot \frac{1609\,m}{\cancel{mi}} \cdot \frac{1\,h}{3600\,s} = 22.35\,\frac{m}{s}$$

$$x - x_0 = 100\,\cancel{ft} \cdot \frac{0.3048\,m}{1\,\cancel{ft}} = 30.48\,m$$

2.59 Statement: Racing car under (horizontal) constant acceleration $\rightarrow$ Kinematic eqs. for constant accel. in 1D.

Step 1) Sketch with given information



$v_0\,?$ $\qquad\longleftrightarrow\qquad$ $v = 53\,\frac{m}{s}$

$$x - x_0 = 140\,m$$
$$t - t_0 = 3.6\,s$$

Step 2)

a)

$$\begin{cases} \text{Eq 2:} & \overset{\checkmark}{x - x_0} = \underset{?}{v_0}\overset{\checkmark}{t} + \frac{1}{2}\underset{?}{a}t^2 \\ \text{Eq. 1:} & \overset{\checkmark}{v} = \underset{?}{v_0} + \underset{?}{a}\cdot\overset{\checkmark}{t} \end{cases}$$

one eq. & 2 unknowns $\rightarrow$ need Eq 1.

$\rightarrow$ Math solution:

(i) solve for a $\rightarrow$ $a = \dfrac{v - v_0}{t}$ (Eq. 1) (a is now a function of v_0)

(ii) solve for v_0 then for a $\rightarrow$ $\overset{\checkmark}{x - x_0} = \underset{?}{v_0}\overset{\checkmark}{t} + \frac{1}{2}\cdot\left(\frac{v - \overset{?}{v_0}}{\cancel{t}}\right)\cdot t^{\cancel{2}}$

$$x - x_0 = \frac{1}{2}v_0 t + \frac{1}{2}v t$$

Now solve for v_0 $\rightarrow$ $v_0 = \dfrac{2}{t}\left[(x - x_0) - \dfrac{1}{2}v\cdot t\right]$

$$v_0 = \frac{2(x - x_0)}{t} - v$$

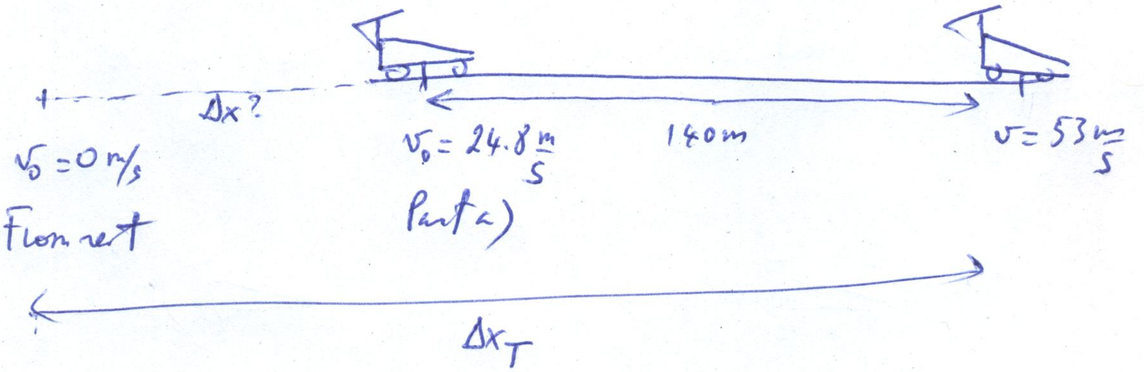
$$= \frac{2\cdot 140}{3.6} - 53 = \boxed{24.8\,\frac{m}{s}}$$

Check: $v_0 < v$ ✓

$a = \dfrac{53 - 24.8}{3.6} = 7.83\,m/s^2$ also: Eq: $\dfrac{v^2 - v_0^2}{x - x_0} = 2\cdot a$

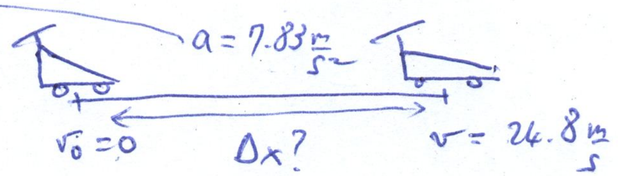
$$a = \frac{1}{2}\,\frac{53^2 - 24.8^2}{140} = 7.83\,\frac{m}{s^2}$$

b)



Alternatives
$\begin{cases} \text{(i) Find } \Delta x \text{ then } \Delta x_T = \Delta x + 140\,m \\ \text{(ii) } \Delta x_T \end{cases}$

(i) Find Δx:



Same acceleration as in the second part of trajectory (Part a)

↑

Constant acceleration!

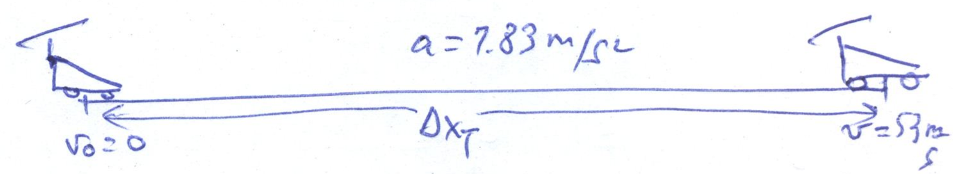
→ No time ⇒ Eq. 3: $\frac{v^2 - v_0^2}{x - x_0} = 2a$

$x - x_0$? → Δx

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{24.8^2 - 0}{2 \times 7.83} = 39.37\,m$$

$$\Delta x_T = 39.37 + 140 = 179.37\,m \quad ✓$$

(ii) Find Δx_T:



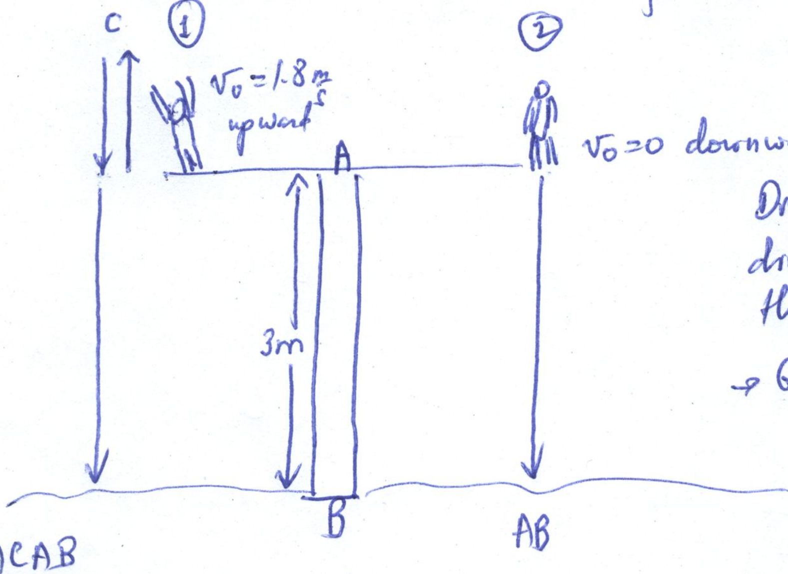
$$\Delta x_T = \frac{v^2 - v_0^2}{2a} = \frac{53^2}{2 \times 7.83} = 179.37\,m \quad ✓$$

2.69 **Statement:** Divers under constant acceleration (vertical, due to gravity)

↳ Kinematic eqs for constant acceleration 1D

Step 1: Sketch

a)



Diver #2 steps off when diver #1 passes platform on the way down.

→ Goal: compare motions of the two divers as they hit water.

Trajectory: ACAB

Step 2: use eqs to find final speeds @ B for both divers

Anticipation: (before you start calculations)

a) Diver #1 will hit water first: when he pass platform on the way down (at A) he starts with $v_0 = 1.8\frac{m}{s}$ downward (while diver #2 starts with $0\,m/s$ downward)

b) Also diver #1 will hit water @ higher speed.

Calculations: Eq 3: $\frac{v^2 - v_0^2}{(x - x_0)} = 2 \cdot a \Leftrightarrow \frac{v_B^2 - v_A^2}{x - x_0} = 2 \cdot g$

This problem $a = g = 9.81\,m/s^2$

Diver #1

$v_{B_1}^2 = 2 \cdot g \cdot (x - x_0) + v_{A_1}^2$

$v_{B_1} = \sqrt{2 \times 9.81 \times 3 + 1.8^2}$

$= 7.88\,m/s$

Diver #2

$v_{B_2} = \sqrt{2 \times 9.81 \times 3 + 0}$

$= 7.67\,\frac{m}{s}$

b) Which diver hits water first & by how much?

→ Calulate time for each to cover AB

↳ { Eq 1 ✓ $v_B = v_A + \underset{\uparrow g}{a}.t$ → solve for t
Eq 2

Diver #1

$$t_1 = \frac{v_{B_1} - v_{A_1}}{g} = \frac{7.88 - 1.8}{9.81} = 0.62\,s$$

Diver #2

$$t_2 = \frac{v_{B_2} - v_{A_2}}{g} = \frac{7.67 - 0}{9.81} = 0.78\,s$$

Diver #1 hits water 1st, 0.16s before Diver #2

Other alternatives to calculate t_1:

(i) Include whole trajectory for diver #1 $\underbrace{A \to C \to A}_{\text{up \& down}} \to B$

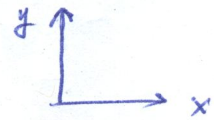
Total time: t_{ACAB} ⇒ $t_{AB} = t_{ACAB} - t_{ACA}$

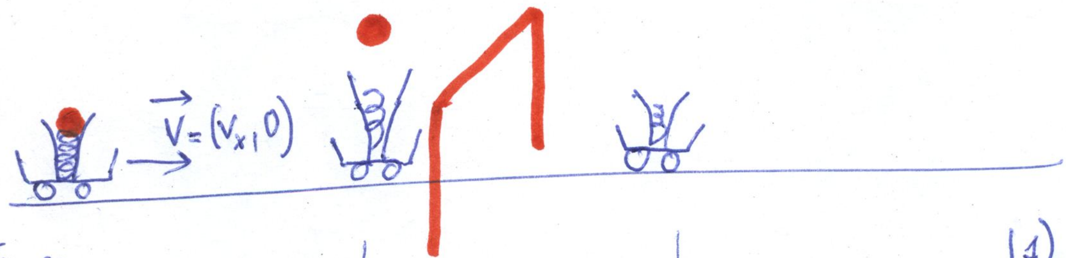
↳ sign is important ($a = -g$ for AC ; $a = +g$ for CAB)

(ii) Include trajectory from highest point to water: $C \to A \to B$

ch3 Motion in two & three dimensions:

Visual experiment #1: Reference





Horizontal track

→ Toy car to roll on track @ $\vec{V}$
→ Has funnel with spring.
→ Velocity of ball: is $\vec{V} = (v_x, 0)$

→ spring released ball is launched up ward
→ Velocity of ball is $\vec{V} = (V_x, \underbrace{V_y}_{\text{due to spring release}})$

car continues at same speed through the gate.

Assumptions.
1) (no friction) b/w car wheels & track
2) air drag is negligible

Big question: will ball fall back into funnel on car? (or in front of it or behind it?)

Statements.

1) acceleration for ball: Yes in y-direction due to gravity; None in x-direction
acceleration for car: none in either direction.

2) ✓ If motion in y affects motion in x then ball will miss funnel
✓ If motion in perpendicular directions are independent → ball will fall back into funnel