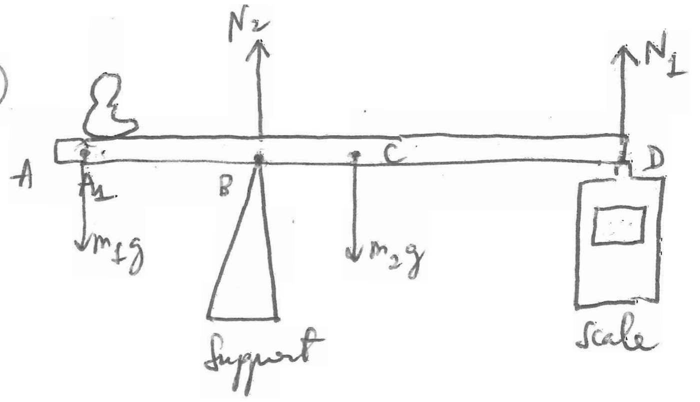
Ch12 : Static Equilibrium

Basic principles already discussed:

- $\hookrightarrow$ no motion : neither linear nor rotational

1) $\sum_i \vec{F}_i = 0$ (Net force on system is 0)
2) $\sum_i \vec{\tau}_i = 0$ (Net torque on system is 0)

12.22

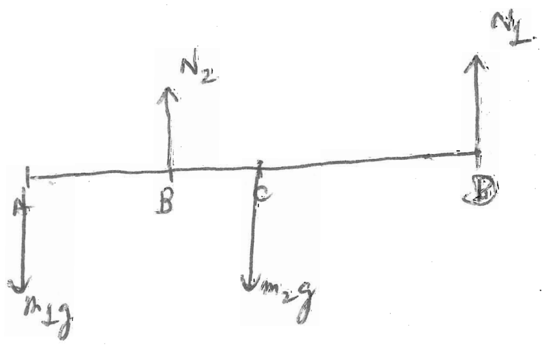


A : left edge of beam.
A_1 : child sits
B : pivot or center of rotation $\rightarrow$ essential to define torques.
D : scale
C : CM of beam (half way b/w A & D)

Child : $\{ m_1 = 40 kg$

Beam : $\begin{cases} m_2 = 60 kg \\ L = 2.4 m = AD \\ AB = 0.8 m \rightarrow \begin{cases} BD = 1.6 m ✓ \\ BC = 0.4 m ✓ \end{cases} \end{cases}$

Y, x



1) $\boxed{\sum_i \vec{F}_i = 0}$ $\rightarrow$ Forces applying on beam $\begin{cases} m_1 g \\ N_2 \\ m_2 g \\ N_1 \end{cases}$

$\hookrightarrow$ all in y $\rightarrow$ Each add arithmetically :

$$N_2 + N_1 - m_1 g - m_2 g = 0 \quad \text{(Net force is 0)}$$

2) $\boxed{\sum_i \vec{\tau}_i = 0}$ $\boxed{\text{wrt. center of rotation : } B}$ $\rightarrow$ Torques wrt B $\begin{cases} \vec{\tau}_1 \text{ (by } m_1 g) \\ \vec{\tau}_2 \text{ (by } m_2 g) \\ \vec{\tau}_{N_1} \text{ (by } N_1) \end{cases}$

Note : * N_2 applies no torque wrt B since $\vec{r}_{N_2} = 0$ (force application point is same as pivot point B)

* Common technique in static equilibrium analysis is to place the pivot point at the force application point of the force we have least information about.

Let's name pole length = L $\rightarrow r_{CA} = r_{CB} = \frac{L}{2}$

$\rightarrow \sum_i \vec{\tau}_i = 0 \rightarrow$ $\boxed{\frac{\cancel{L}}{2} T\cos\theta - \frac{\cancel{L}}{2}\mu_s N = 0}$ 2)

Net torque on pole wrt C is O

Need to solve for μ_s → Need expressions for $\left\{ \begin{matrix} T \\ \& \\ N \end{matrix} \right.$ > use equations for $\sum_i \vec{F}_i = 0$

1b): $-T\sin\theta - Mg\cos\theta + N = 0$

2) $\mu_s N = T\cos\theta \rightarrow \boxed{T = \frac{\mu_s N}{\cos\theta}}$ (2) → in 1b)

1b) → $-\underbrace{\frac{\mu_s N}{\cos\theta}\sin\theta}_{\tan\theta} - Mg\cos\theta + N = 0 \rightarrow \boxed{-\mu_s N\tan\theta - Mg\cos\theta + N = 0}$ 1b)

1a) $-T\cos\theta + Mg\sin\theta - \mu_s N = 0$ (Net force in x-direction is 0)

2) use 2): $-\mu_s N + Mg\sin\theta - \mu_s N = 0$

$\boxed{+2\mu_s N = +Mg\sin\theta}$ 1a)

$\rightarrow \boxed{Mg = \frac{2\mu_s N}{\sin\theta}}$ 1a)

1b) (use new 1a) $\boxed{\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}}$ $-\mu_s \cancel{N}\tan\theta - \frac{2\mu_s \cancel{N}}{\tan\theta} + \cancel{N} = 0$

$-\mu_s\left(\tan\theta + \frac{2}{\tan\theta}\right) + 1 = 0$

$\mu_s = \frac{1}{\tan\theta + \frac{2}{\tan\theta}}$

$\mu_s = \frac{\tan\theta}{2 + \tan^2\theta}$

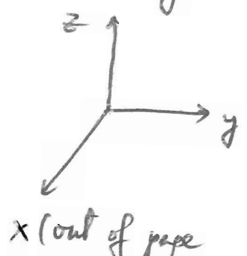
critical value for pole to be in equilibrium

*We have utilized all available equations! 1a) 1b) 2)

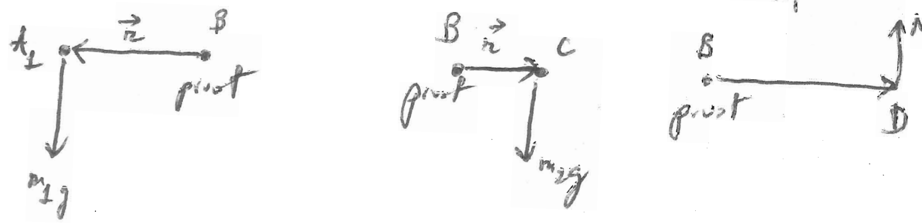
$\rightarrow \boxed{\mu_s \geq \frac{\tan\theta}{2 + \tan^2\theta}}$

→ minimum for μ_s, solely based on angle of incline!

Center of rotation : B : $\sum_i \vec{\tau}_i = 0$



$\rightarrow \quad \vec{\tau}_1 \quad + \quad \vec{\tau}_2 \quad + \quad \vec{\tau}_{N_1} = 0$



$$r_{BA_1} m_1 g \,\hat{i} \quad + \quad r_{BC} m_2 g (-\hat{i}) \quad + \quad r_{BD} N_1 (\hat{i}) = 0$$

a) $N_1 = 100\,N \rightarrow A_1 ? \rightarrow$ provide BA_1 or provide AA_1

From $\sum_i \vec{\tau}_i = 0 \rightarrow$

$$r_{BA_1} = \frac{r_{BC} m_2 g - r_{BD} N_1}{m_1 g} = \frac{0.4 \cdot 60 \cdot 9.81 - 1.6 \cdot 100}{40 \cdot 9.81}\, m = 0.19\, m$$

$AA_1 = 0.8 - 0.19 = 0.61\,m$ (child location from left edge A when $N_1 = 100\,N$)

b) $N_2 = 300\,N$ (child will need to sit further away from B & to its right)

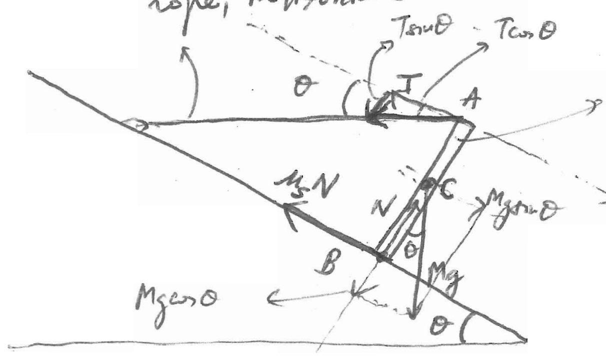
$$r_{BA_1} = \frac{0.4 \cdot 60 \cdot 9.81 - 1.6 \cdot 300}{40 \cdot 9.81} = -0.62\,m$$

child sits at 0.62m to the right of center of rotation B

$\rightarrow AA_1 = AB + BA_1 = 0.8 + 0.62 = 1.42\,m$ from left edge A of beam.

12.57 Static equilibrium for a pole on an incline of angle θ with rope, horizontal friction.

$\boxed{\mu_s \text{ min for pole in static equilibrium.}}$

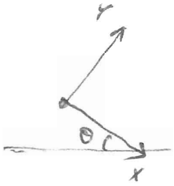


Forces on pole:

$T, Mg, N, \mu_s N$
@A @C @B @B

A: top of pole
C: center of pole
B: contact with incline

$$\begin{cases} 1) \sum_i \vec{F}_i = 0 \\ 2) \sum_i \vec{\tau}_i = 0 \end{cases}$$



T & Mg will have x & y components in this coord. syst.

$\vec{T} = -T\cos\theta\,\hat{i} - T\sin\theta\,\hat{j}$; $M\vec{g} = Mg\sin\theta\,\hat{i} - Mg\cos\theta\,\hat{j}$

$\vec{N} = N\hat{j}$; $\vec{F}_s = -\mu_s N\,\hat{i}$

1) $\sum_i \vec{F}_i = 0$
$\downarrow$
$\vec{T} + M\vec{g} + \vec{N} + \vec{F}_s = 0$

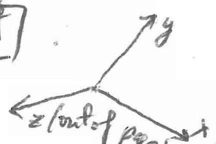
(1a) x: $-T\cos\theta + Mg\sin\theta - \mu_s N = 0$

(1b) y: $\boxed{-T\sin\theta - Mg\cos\theta + N = 0}$

Net force is 0 on pole

2) $\sum_i \vec{\tau}_i = 0$ what is the center of rotation? → Select $\boxed{C}$.

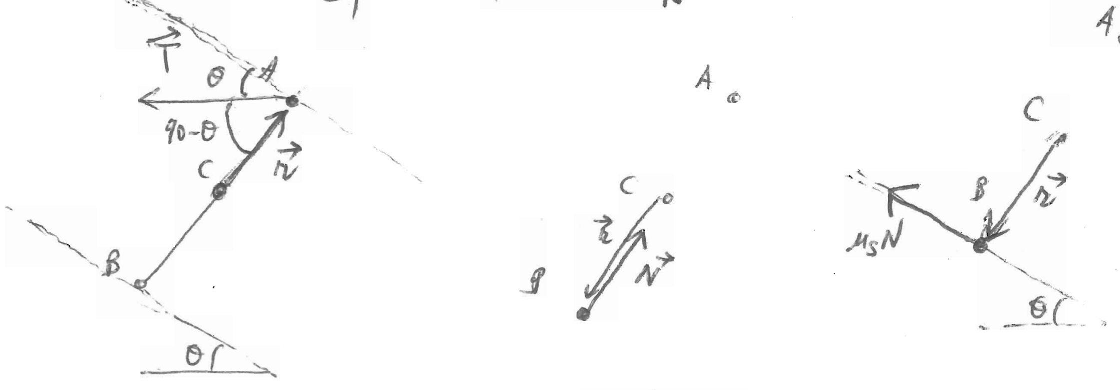
possible pivots:
A
~~B~~: then friction is not part of $\sum \vec{\tau}_i = 0$ → can't solve for μ_s.
C: M is not given → $\boxed{\text{pivot}}$.



by the RHR: when you close RH fingers from x to y thumb is z (out of page)

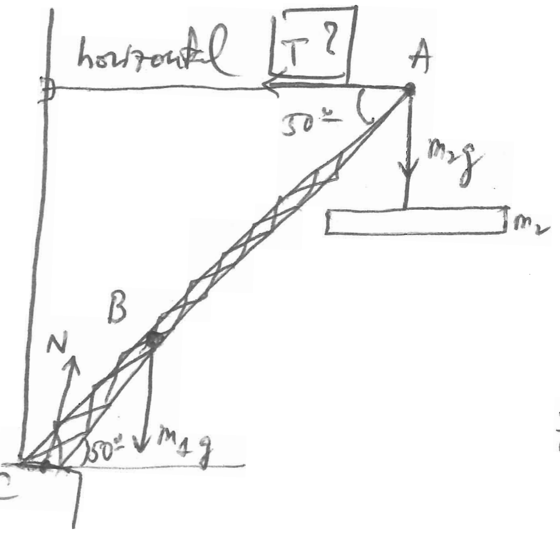
→ Three torques on pole wrt C:

$$\vec{\tau}_T + \vec{\tau}_N + \vec{\tau}_{F_s} = 0$$



$r_{CA}\, T \underbrace{\sin(90-\theta)}_{\cos\theta}\,\hat{k}$ + No torque since $\theta = 180°$ & $\sin 180° = 0$ + $r_{CB}\, \mu_s N\,(-\hat{k}) = 0$

(12-40) Crane holding 2500 kg marble slab in static equilibrium.



marble: $m_2 = 2500\,kg$ @ A

boom: $m_1 = 830\,kg$ @ B

$l = 15\,m \rightarrow CB = 5\,m$ (location of boom's CM)

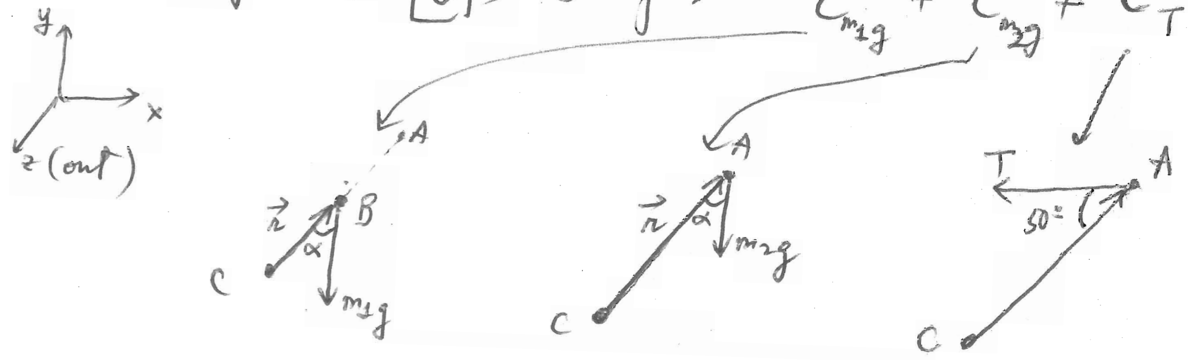
Forces on boom: $\begin{cases} m_2 g \text{ @ } A \\ m_1 g \text{ @ } B \\ \vec{N} \text{ @ } C \\ T \text{ @ } A \end{cases}$ (direction not given: depending on shape of rock crane is mounted on!)

1) $\sum_i \vec{F}_i = 0$

2) $\sum_i \vec{\tau}_i = 0$ wrt pivot point $\begin{cases} A \\ B \\ C \end{cases}$: to avoid the extra unknown which the angle of $\vec{N}$ wrt horizontal direction.

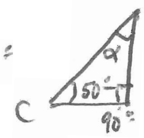
$\rightarrow \vec{\tau}_N = 0$

wrt center of rotation $\boxed{C}$ → 3 torques: $\vec{\tau}_{m_1 g} + \vec{\tau}_{m_2 g} + \vec{\tau}_T = 0$



$$\underbrace{r_{CB}}_{5m} m_1 g \sin\alpha\,(-\hat{k}) + \underbrace{r_{CA}}_{15m} m_2 g \sin\alpha\,(-\hat{k}) + \underbrace{r_{CA}}_{15} T \sin 50^\circ\,\hat{k} = 0$$

1) Find α from geometry:



$\rightarrow \alpha + 50^\circ + 90^\circ = 180^\circ \rightarrow \alpha + 50^\circ = 90^\circ \rightarrow \boxed{\alpha = 40^\circ}$

$\sin\alpha = \cos(90 - \alpha) \rightarrow \boxed{\sin 40^\circ = \cos 50^\circ}$

Finalize $\sum_i \vec{\tau}_i = 0$ & solve for T:

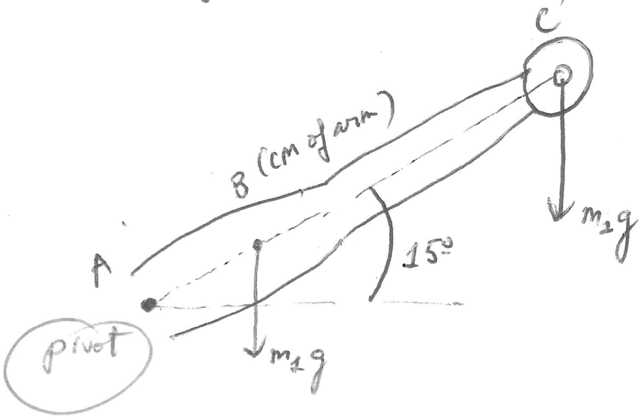
$$-\underbrace{r_{CB}}_{5} m_1 g \cos 50^\circ - \underbrace{r_{CA}}_{15} m_2 g \cos 50^\circ + \underbrace{r_{CA}}_{15} T \sin 50^\circ = 0$$

$$T = \frac{5 \cdot 830 \cdot 9.81 \cdot \cos 50^\circ + 15 \cdot 2500 \cdot 9.81 \cos 50^\circ}{15 \sin 50^\circ} N$$

$$\boxed{T = 22900 N}$$

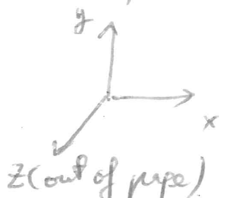
(12.28) Arm holding a weight in equilibrium:

AC = 0.56m
AB = 0.21m
$m_1 = 4.2$ kg
$m_2 = 6$ kg

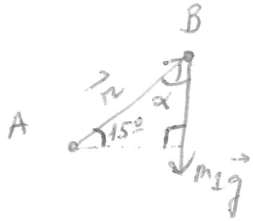


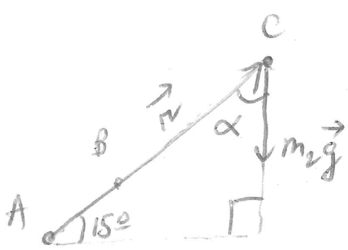
A: shoulder and center of rotation
B: CM of arm
C: where weight $m_2 g$ is applied

a) Torque about A (shoulder) by $m_1 g$ & $m_2 g$: $\vec{\tau}_{m_1} + \vec{\tau}_{m_2}$



$\vec{\tau}_{m_1}$ + $\vec{\tau}_{m_2}$





$\alpha = 75°$ $\qquad$ $\alpha = 75°$

$$\vec{\tau}_{m_1 \& m_2} = r_{AB}\, m_1 g \sin 75° (-\hat{k}) + r_{AC}\, m_2 g \sin 75° (-\hat{k}) \text{ Nm}$$

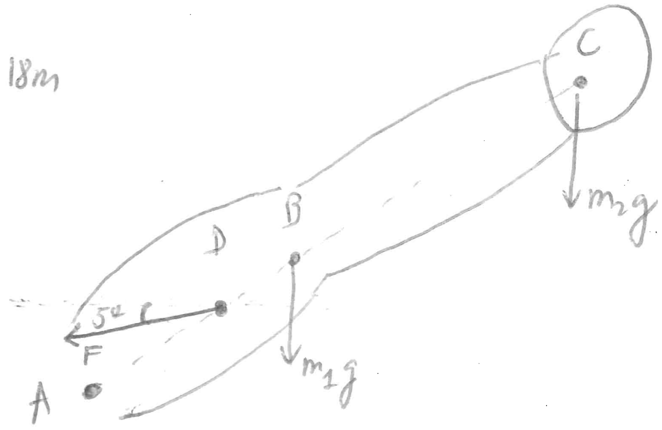
$$= (0.21 \cdot 4.2 + 0.56 \cdot 6) \cdot 9.81 \sin 75° \, (-\hat{k})$$

$$= 40.2\,(-\hat{k}) \text{ Nm} \quad \text{(w.r.t. pivot A)}$$

b) Since $\vec{\tau}_{m_1 \& m_2} \neq 0$ There needs to be a third torque (in $+\hat{k}$) that holds the arm + weight in static equilibrium.

Torque by deltoid muscle:

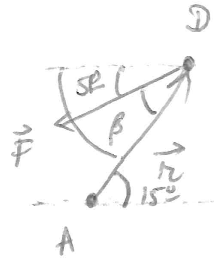
AD = 0.18m



D: deltoid muscle force is applied

F?

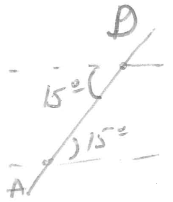
$$\vec{\tau}_F = r_{AD}\, F \sin 10^\circ\, (+\hat{k})$$



$15^\circ = 5^\circ + \beta$

$\beta = 10^\circ$

RHR: torque in $+\hat{k}$ as expected!



Since static equilibrium requires: $\sum_i \vec{\tau}_i = 0$:

$$\vec{\tau}_F + \vec{\tau}_{m_1} + \vec{\tau}_{m_2} = 0 \quad \text{or} \quad \vec{\tau}_F = -\vec{\tau}_{m_1} - \vec{\tau}_{m_2}$$
$$= 40.2\,\hat{k}\ (\text{Nm})$$

$$\rightarrow \vec{\tau}_F = 0.18 \cdot F \cdot \sin 10^\circ\, \hat{k} = 40.2\,\hat{k}$$

$$F = \frac{40.2}{0.18 \sin 10^\circ} = 1280\,N = 1.28\,kN$$

Ch 13 Oscillatory Motion

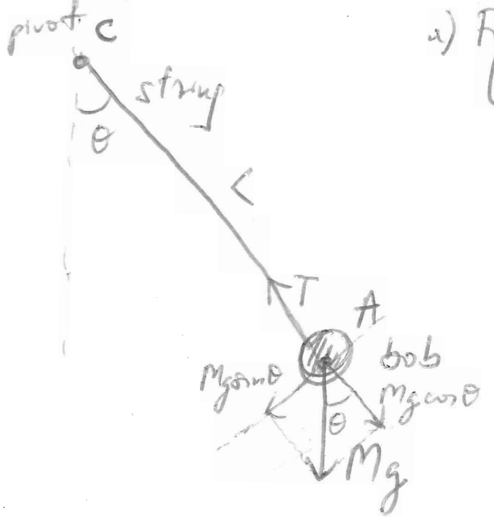
Another type of motions besides linear & rotational motion.

Examples:

1) **Pendulum**: system consists of a bob attached to a string (with a negligible mass compared to the bob). The other end of the string is attached to a fixed point: which is the center of rotation or pivot.

C = pivot
A: bob.

CA = L



a) First method: Using 2nd Newton's law $\boxed{F_{net} = Ma}$ on linear motion:

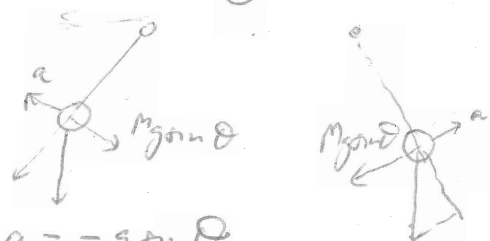
$$T - Mg\cos\theta = 0$$
$$Mg\sin\theta = Ma$$

a = tangential acceleration

Observation: is $Mg\sin\theta$ always in the same direction of motion of the bob?

If yes: bob would go in circular motion faster & and faster.

→ Not: actually:



$$\rightarrow \; -Mg\sin\theta = Ma \rightarrow a = -g\sin\theta$$

$$\alpha = \frac{a}{R} = \frac{-g\sin\theta}{L} \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

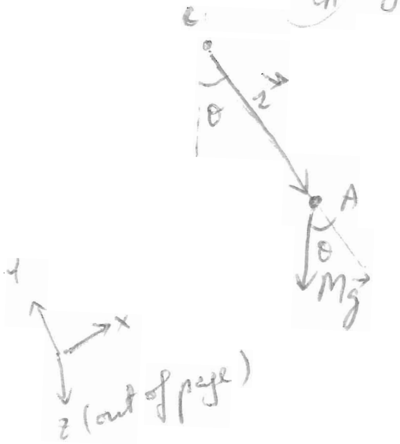
b) 2^{nd} Method: Analog of 2^{nd} Newton's law for rotational motion:

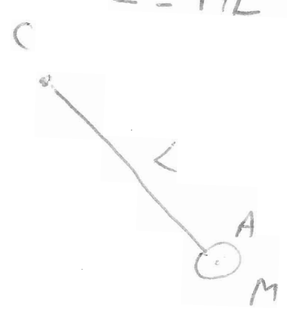
$$\boxed{\tau_{net} = I\alpha} \rightarrow \alpha = \frac{\tau}{I}$$

τ → torque on bob.

I → moment of inertia of bob. wrt pivot of pendulum.

$$\vec{\tau} = \overset{L}{r_{CA}}\, Mg \sin\theta\,(-\hat{k}) \quad ; \quad I = ML^2 \rightarrow \vec{\alpha} = \frac{\vec{\tau}}{I} = -\frac{\cancel{L}\cancel{M}g\sin\theta}{\cancel{M}L^{\cancel{2}}}(\hat{k})$$





$$\boxed{\alpha = -\frac{g\sin\theta}{L}}$$

$$\alpha = \boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

Exact eq. of motion for a pendulum.

→ Differential equation of order 2 and non-linear.

→ No simple analytical solution.

→ Use <u>small angle approximation</u> θ small

$\boxed{\sin\theta \approx \theta}$ → linear 2^{nd} order D.E!

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$ → simple analytical solution:

$$\boxed{\theta(t) = \theta_M \cos(\omega t) = \theta_M \cos\omega t}$$

θ_M : amplitude of oscillation

ω : angular frequency (# of angular oscillations per second) → how fast pendulum is swinging.

what is ω in term of L?

substitute $\theta = \theta_m \cos\omega t$ into $\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\theta$

$$\frac{d\theta}{dt} = -\theta_m \omega \sin\omega t$$

$$\frac{d^2\theta}{dt^2} = \boxed{\cancel{-}\theta_m \omega^2 \cos\omega t = \cancel{-}\frac{g}{L}\theta_m \cos\omega t}$$

$$\omega^2 = \frac{g}{L} \quad \text{or} \quad \boxed{\omega = \sqrt{\frac{g}{L}}}$$

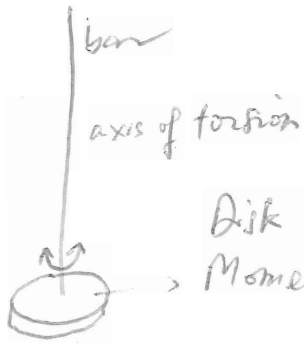
↳ Shorter pendulum swings faster as from your observation!

2) <u>Torsional Pendulum:</u> twisting motion: $\boxed{\tau = -\kappa\theta}$

Kappa: torsional constant: by material & dimension of the bar.

$\tau = I\alpha$

$\cancel{+}\kappa\theta = I \cdot \dfrac{d^2\theta}{dt^2}$



Disk undergoes rotations about its center axis

Moment of inertia: I

$$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$$

Similar to Hooke's Law in spring $F = -k\Delta x$

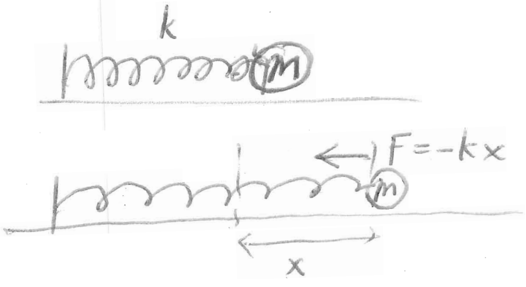
2nd order linear D.E. similar to that of the small angle approx. for pendulum $\left(\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\theta\right)$

$\theta(t) = \theta_m \cos\omega t$

θ_m max amplitude of torsion

ω: angular frequency of torsion $\quad \boxed{\omega = \sqrt{\dfrac{\kappa}{I}}}$

3) <u>Spring & bob:</u> spring constant k
mass of bob is m



→ 2nd Newton's Law

$$F_{net} = m \cdot a$$

$$-kx = m \cdot \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x$$

2nd order linear D.E !
similar to that of the small angle approx. of the pendulum & replacing θ by x

$$\rightarrow x(t) = \underbrace{x_m}_{\text{amplitude of linear oscillation}} \cos \underbrace{\omega}_{\text{angular freq. of linear osc.}} t$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

This oscillatory motion is also called SHM (simple harmonic motion)

SHM: $\frac{d^2x}{dt^2} = -\frac{k}{m}x \rightarrow x(t) = x_m \cos\omega t, \quad \omega = \sqrt{\frac{k}{m}}$

Damped SHM: $\frac{d^2x}{dt^2} = -\frac{k}{m}x \underbrace{- \frac{b}{m}\frac{dx}{dt}}_{\text{damping term}} \rightarrow x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega t + \phi)$

phase

SHM: total energy stays constant: $E = \frac{1}{2} k x_M^2$ =
(string & bob system)

↓

Total energy of bob & spring system $E = \underbrace{\frac{1}{2} m v^2}_{\text{bob}} + \underbrace{U_{elastic}}_{\text{spring}} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

eq. of motion for SHM

$$\left\{\begin{array}{l} x(t) = x_M \cos\omega t \\ v(t) = \frac{dx}{dt} = -x_M \omega \sin\omega t \\ \omega = \sqrt{\frac{k}{m}} \end{array}\right\} \quad E = \frac{1}{2} m x_M^2 \omega^2 \sin^2\omega t + \frac{1}{2} k x_M^2 \cos^2\omega t$$

$$= \frac{1}{2} k x_M^2 \underbrace{(\sin^2\omega t + \cos^2\omega t)}_{1}$$

$\omega^2 = \frac{k}{m} \rightarrow \omega^2 m = k$

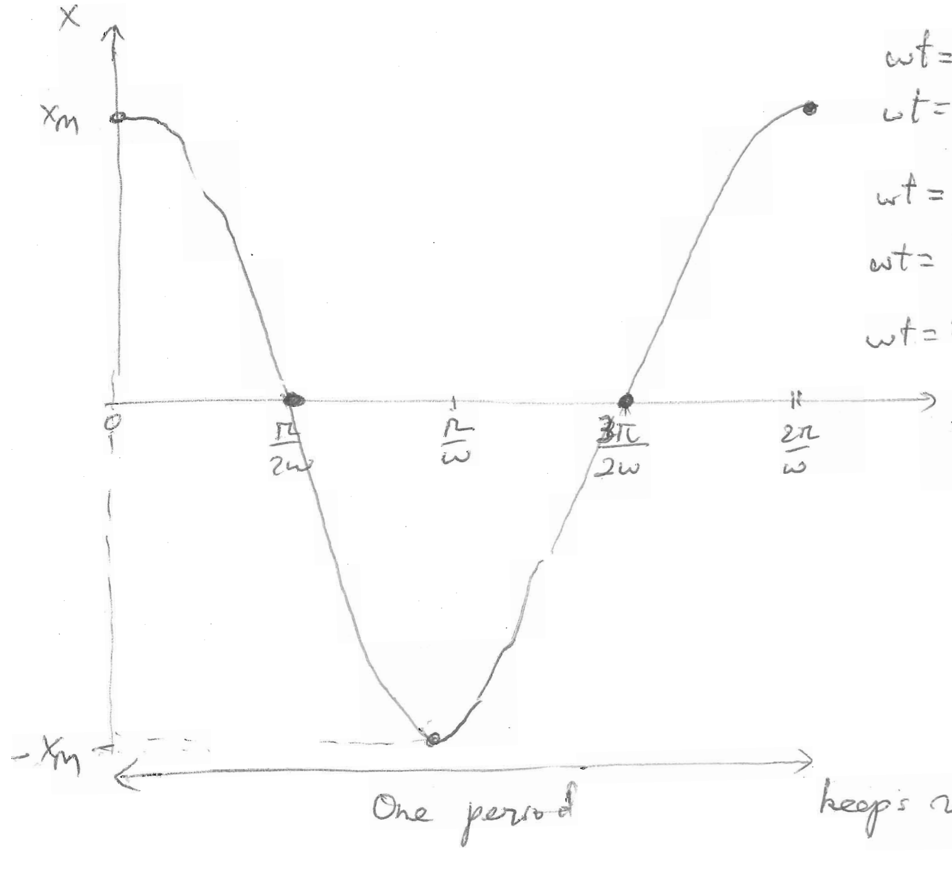
↓ Time dependence is gone!

→ Total energy of bob & spring is constant! bob ↔ spring

bob ↓ fastest & slowest

spring ↓ no elastic potential & high elastic potential

SHM position vs time: $x(t) = x_M \cos\omega t$



$\omega t = 0 \rightarrow x(0) = x_M$

$\omega t = \frac{\pi}{2} \rightarrow x(\frac{\pi}{2\omega}) = 0$

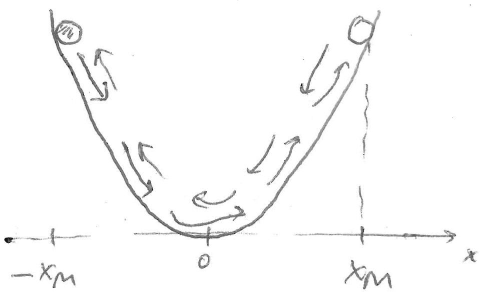
$\omega t = \pi \rightarrow x(\frac{\pi}{\omega}) = -x_M$

$\omega t = \frac{3\pi}{2} \rightarrow x(\frac{3\pi}{2\omega}) = 0$

$\omega t = 2\pi \rightarrow x(\frac{2\pi}{\omega}) = x_M$

keeps repeating in time

4) Particle trapped in a potential well :



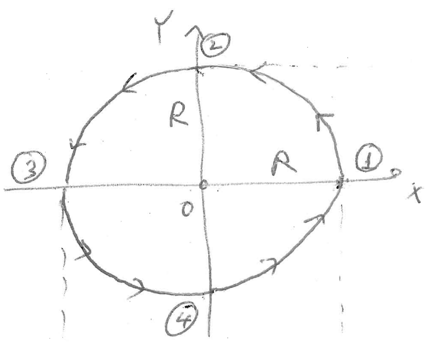
No friction : $x(t) = x_M \cos \omega t$ or SHM

With friction : damped SHM :

$$x(t) = \underbrace{x_M e^{-\frac{bt}{2m}}}_{\downarrow} \cos(\omega t + \phi)$$

exponential decay for the amplitude of displacement in x.

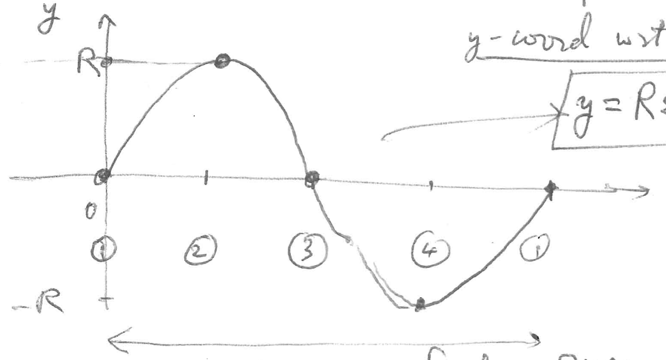
5) Coordinates x & y of a particle undergoing UCM (Uniform Circular Motion)



Particle in UCM

① → ② → ③ → ④ → ①

$t=0$ $\qquad t = \frac{2\pi}{\omega}$

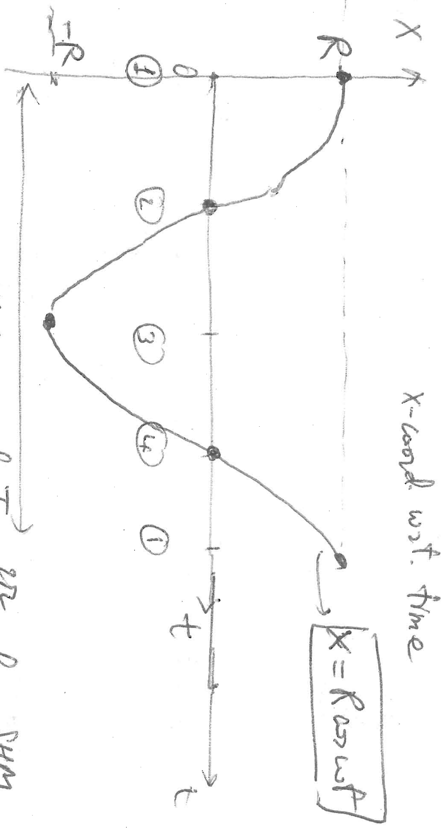


y-coord wrt time

$\boxed{y = R \sin \omega t}$

one period of a SHM

$$T = \frac{2\pi}{\omega}$$



x-coord w.r.t. time

$\boxed{x = R \cos \omega t}$

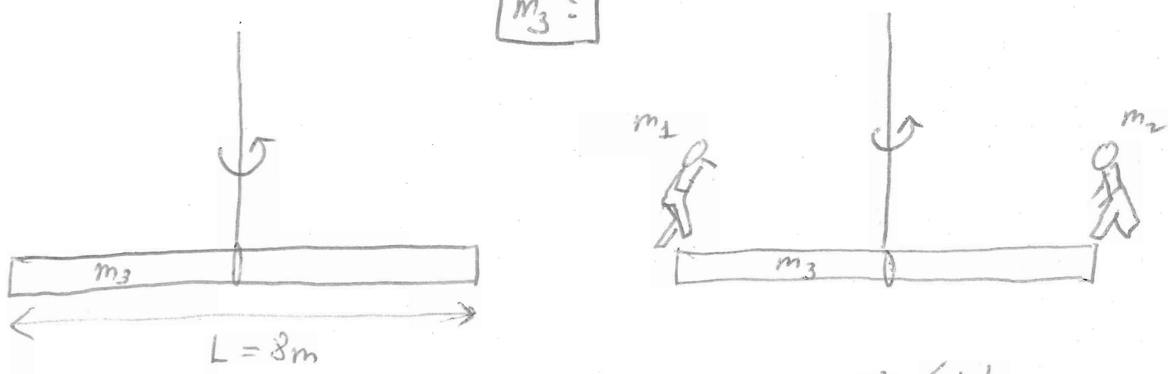
one period $T = \frac{2\pi}{\omega}$ of a SHM

$x = R \cos \omega t$
$y = R \sin \omega t$ } both are SHM's but shifted in phase by $\frac{\pi}{2}$ (or 90°)

13.50

$m_1 = m_2 = 75\,kg = m$

$\boxed{m_3\,?}$



w_i

$w_f < w_i$

(additional moment of inertia)

$w_f = 0.8\, w_i$

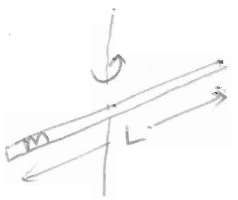
w_i & w_f are intial & final torsional frequencies

Next step: identify relevant equations for this problem. (standard names of variables used in making the sketch above are hints!)

Torsional osc. $\omega = \sqrt{\dfrac{K}{I}}$ $\begin{cases} K: \text{torsional constant} \\ I: \text{moment of inertia wrt a center of rotation or pivot.} \end{cases}$

$\downarrow$ in this problem it's the CM of the beam

Rod:



$I = \frac{1}{12} m L^2$

$$w_i = \sqrt{\frac{K}{\frac{1}{12} m_3 L^2}}$$

$$w_f = \sqrt{\frac{K}{\frac{1}{12} m_3 L^2 + 2m\left(\frac{L}{2}\right)^2}}$$

since torsions are by the rope which is the same initial & final $\rightarrow$ same torsional constant.

$\boxed{m_3\,?}$ $\quad w_f = 0.8\, w_i \rightarrow \dfrac{w_f}{w_i} = 0.8 \rightarrow \dfrac{w_f^2}{w_i^2} = 0.8^2$

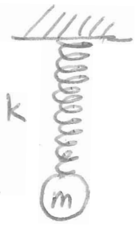
$$\rightarrow \frac{\dfrac{\cancel{K}}{\frac{1}{12}m_3 \cancel{L^2} + \frac{1}{2} m \cancel{L^2}}}{\dfrac{\cancel{K}}{\frac{1}{12} m_3 \cancel{L^2}}} = \frac{\dfrac{1}{\frac{1}{12}m_3 + \frac{1}{2}m}}{\dfrac{1}{\frac{1}{12}m_3}} = \frac{\frac{1}{12}m_3}{\frac{1}{12}m_3 + \frac{1}{2}m}$$

$$\rightarrow \frac{m_3}{m_3 + 6m} = 0.8^2 \quad \rightarrow \quad m_3 = 0.8^2(m_3 + 6 \cdot m)$$

$$m_3 = \frac{0.8^2 \cdot 6 \cdot m}{1 - 0.8^2}$$

$$= 6.75 \cdot \frac{0.8^2}{1 - 0.8^2} = \boxed{800\,kg}$$

13.64



$m = 0.49\,kg$

$k = 74\frac{N}{m}$

a) Amplitude of oscillation ?

b) Period of resulting motion ?

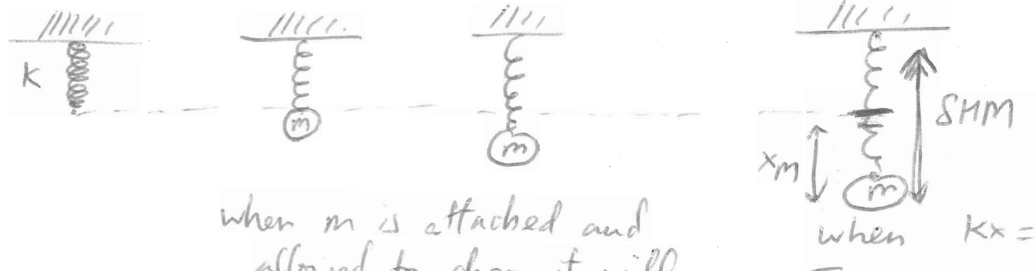
Look at bob

(when mass is attached, the spring is stretched, it pulls up with a force kx given by Hooke's law.)

$$F_{net} = 0 \rightarrow kx - mg = 0$$
$$kx = mg$$

oscillation for spring & bob is a SHM : $x(t) = x_m \cos(\omega t)$

$\boxed{x_m\ ?}$



x_m is max stretching.

$$\boxed{x_m = \frac{mg}{k}}$$

when m is attached and allowed to drop it will stretch the spring
kx will increase as mg stretch the spring

when $kx = mg$ we have $F_{net} = 0 \rightarrow$ it will stop stretching and goes back up in SHM

a) $x_M = \frac{mg}{k} = \frac{0.49 \cdot 9.81}{74} = 0.0649\ m$

b) Period of oscillation T: time seperation b/w two consecutive max = time to complete one full cycle.

$\omega = \sqrt{\frac{k}{m}}$: angular frequency = angular oscillations per second
(for SHM of a spring & bob system)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.49}{74}} = 0.511\,s.$$

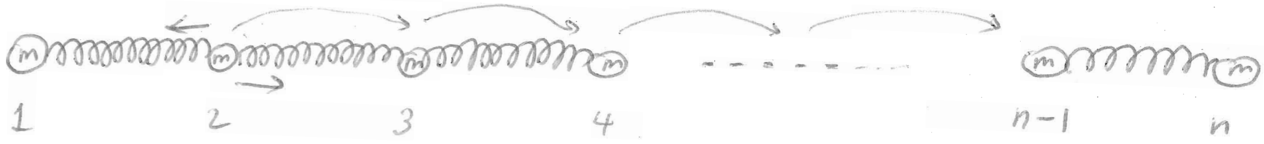
Ch. 14 Wave Motion:

Oscillatory motion: time repeating variation of position or angle
periodic variation

Wave motion: a step beyond: the oscillation/$\boxed{\text{time}}$ periodic variation of position or angle/or perturbation is propagated in $\boxed{\text{space}}$ → (time & space)

1) Propagation

An example of a longitudinal wave: system of identical bobs connected by identical springs



n = 1000

If I give bob #2 a displacement in the horizontal direction:

1) #2 will undergo SHM. At that time the perturbation is still local: bob # 800 and others are not affected

2) #3 will undergo SHM, #4, #5, etc..

Perturbation is then propagated out:

a) all in the horizontal direction = propagation is in the same direction as that of the oscillation → longitudinal wave

b) when propagation goes to #3, #4, #5, ... #800
#2: stays around its original position!
What is propagated is not matter (bob)! but the oscillation or perturbation!

Propagation: of a perturbation is such that the objects involved (for example: springs & bobs) stay local while the perturbation reaches as far as possible.

→ Sound wave:
- Air molecules stay local
- Perturbation: change of air pressure is propagated as far as possible.

→ Light wave:
- No matter
- Perturbation: oscillation of the electric & magnetic fields

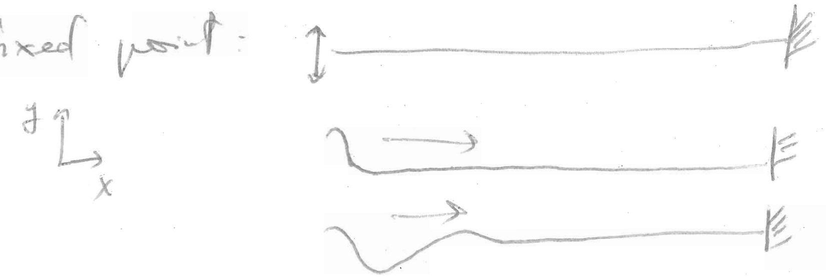
2) **Waves:** involve both time & space variations.

Transverse wave: $y(x, t) = A \sin(kx - \omega t)$

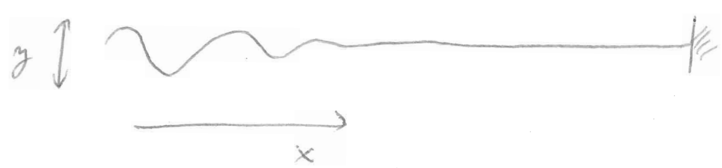
This wave propagates along x, perturbation is along y. It's the math expression for a perturbation along y-direction that is propagated in the $+x$ direction

3) **Types:**
- Transverse waves: the perturbation is perpendicular to direction of propagation
- Longitudinal waves: perturbation is parallel to direction of propagation

String attached to fixed point:



4) Math description of transverse wave:



$y(x,t) = A \sin(kx - \omega t)$ { Perturbation along y; Propagation is along +x

A: wave amplitude (amplitude of perturbation or oscillation)

$k =$ wave number (# wavelength in 2π) $= \dfrac{2\pi}{\lambda}$ (m^{-1})

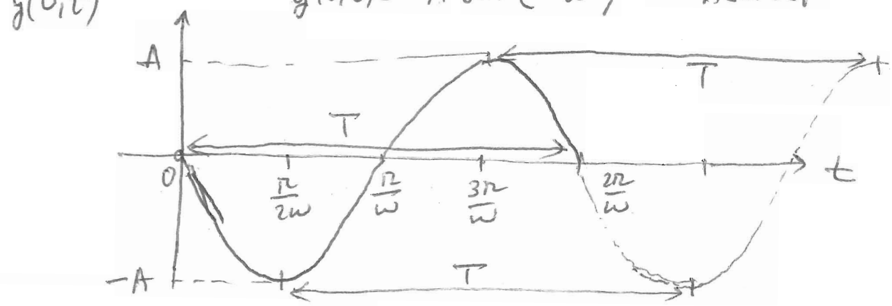
Lambda

$\lambda =$ wavelength : space separation b/w 2 consecutive peaks (m)

$\omega =$ angular frequency : $T = \dfrac{2\pi}{\omega}$ (T = period or time separation b/w 2 consecutive peaks)

$y(0,t) = A \sin(-\omega t) = -A \sin \omega t$

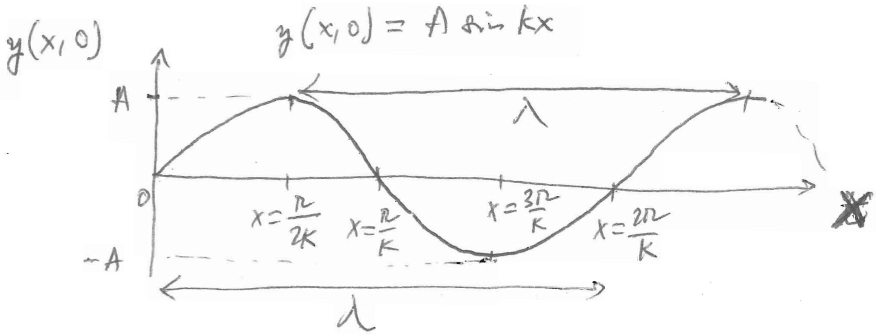
$x=0$



T = period

$y(x,0) = A \sin kx$

$t=0$



$\lambda =$ wavelength.

14.20

Earthquake waves: $\left\{ \begin{array}{l} 1200\text{ km in } 300\text{ s} \\ \text{seismograph}: f = 3.1\text{ Hz}. \end{array} \right\} \lambda$? wavelength

ω ($\frac{\text{rad}}{s}$ or s^{-1}) : angular frequency
f (Hz) : linear frequency : # oscillations per second $\Big\} \boxed{\omega = 2\pi \cdot f}$
Hertz

waves: fn of $(\underbrace{kx - \omega t}_{\text{phase of the wave.}})$

If we travel with the wave (e.g. when you surf water waves)
→ phase is constant: $\phi = kx - \omega t = \text{constant}$

$$\rightarrow \frac{d\phi}{dt} = 0 \rightarrow k \underbrace{\frac{dx}{dt}}_{\substack{\text{phase velocity} \\ \text{wave velocity} = v}} - \omega = 0 \rightarrow kv = \omega \rightarrow \boxed{k = \frac{\omega}{v}}$$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{v} = \frac{2\pi f}{v} \rightarrow \frac{\cancel{2\pi}}{\lambda} = \frac{\cancel{2\pi} f}{v} \rightarrow \boxed{\lambda = \frac{v}{f}}$$

wave number

$$\lambda = \frac{\frac{1.2 \times 10^6 m}{300 s}}{3.1} = 1.29 \times 10^3 \ m$$

14.54

Transverse wave in a wire: $\left\{ \begin{array}{l} \text{oscillation in } y \text{ direction} \\ \text{propagation in } \oplus x \end{array} \right.$

$$y(x,t) = 1.5 \sin(0.1x - 560 \cdot t)$$

Tension in wire is 28N

cm, cm, s

λ?
A?, k?, T?, v?, P?

1) Amplitude $A = 1.5$ cm
2) Wave number $k = 0.1\ cm^{-1}$
$y(x,t) = A \sin(kx - \omega t)$
$k = \frac{2\pi}{\lambda}$ → wavelength $\rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.1 cm^{-1}} = 20\pi\ cm = 62.8\ cm$

3) Period T? $\omega = 560\ s^{-1} \rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{560 s^{-1}} = 11.2 \times 10^{-3}$ s

$\left(T = \frac{1}{f}\right)$ $= 11.2$ ms

milli-seconds

10^{-3}

4) Wave speed $v =$

$\lambda = \frac{v}{f} \rightarrow v = \lambda \cdot f = \frac{\lambda}{T} = \frac{62.8 \times 10^{-2}\,m}{11.2 \times 10^{-3}\,s} = 56\ \frac{m}{s}$

(transverse wave in a wire)

average car speed: high way $= \frac{100\,km}{h} \cdot \frac{1h}{3600 s} \cdot \frac{1000 m}{1 km} = \frac{100}{3.6} = 27.8\ \frac{m}{s}$

5) Power carried by wave: $\overline{P} = \frac{1}{2} \mu \omega^2 A^2 v$

μ: linear density of wire

ω: angular frequency of wave

A: amplitude of wave

v: speed of wave

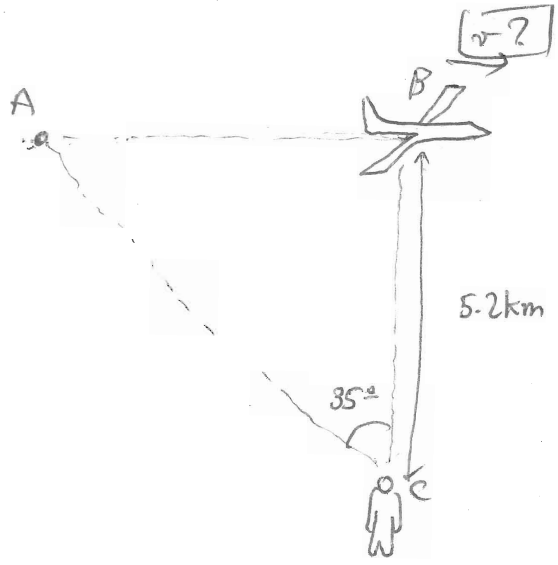
Need μ for this wire: $T = 28N$, $v = \sqrt{\frac{T}{\mu}} \rightarrow v^2 = \frac{T}{\mu} \rightarrow \boxed{\mu = \frac{T}{v^2}}$

Tension!

$\mu = \frac{28}{56^2}\ \frac{kg}{m}$

$\overline{P} = \frac{1}{2} \cdot \frac{28}{\cancel{56^2}} \cdot 560^2\ 0.015^2 \cdot \cancel{56}\ W = 17.4\ W$

14.61



See a plane right above @ 5.2km @ B. But sound comes from a previous point A of the plane's trajectory : since light travels a lot faster than sound: Sound from A reaches your ears at the same time as light from B reaches your eyes! Sound from B is still travelling toward your ears!

Sound wave speed of $330 \frac{m}{s}$

(light wave speed $= c = 300\ 000\ 000 \frac{m}{s}$)
↳ (fiber optics)

What is the plane's speed v?

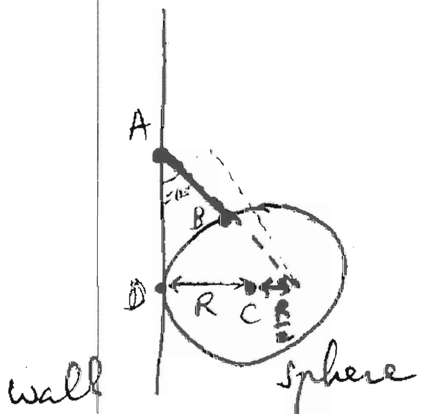
$$v_{airplane} = \frac{d_{AB}}{t_{sound\,A\to C}} = \frac{d_{AB}}{\frac{d_{AC}}{v_{sound}}} = v_{sound} \underbrace{\frac{d_{AB}}{d_{AC}}}_{\sin 35°}$$

$$= 330 \cdot \underbrace{\sin 35°}_{<1} = 189 \frac{m}{s}$$

↳ $189 \cdot 3.6 = \boxed{680 \frac{km}{h}}$

12.29

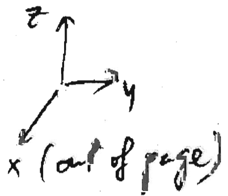
→ Smallest coef of friction $\boxed{\mu_s}$ b/w sphere & wall

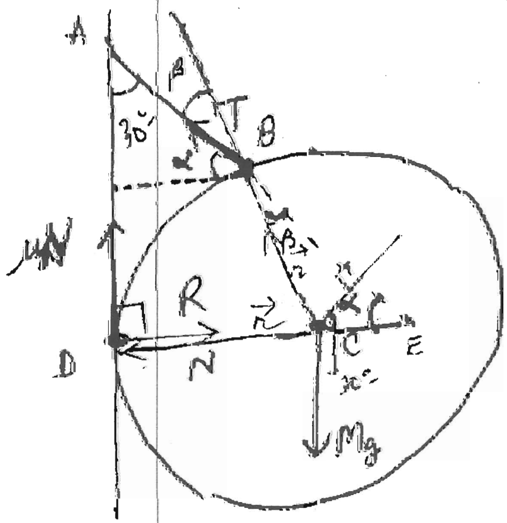


Static Equilibrium

$$\sum_i \vec{F}_i = 0 \begin{cases} y: & \boxed{N - T\cos 60^\circ = 0 \quad (1)} \\ z: & \boxed{T\sin 60^\circ + \mu N - Mg = 0 \quad (2)} \end{cases}$$

$$\sum_i \vec{\tau}_i = 0 \rightarrow \underline{\text{Pivot}}: \ C \ (\text{no information on } M!)$$





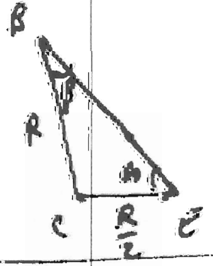
$$\underset{\substack{\| \\ 0}}{\vec{\tau}_N} + \vec{\tau}_{\mu N} + \vec{\tau}_T = 0$$

$$\cancel{R}\,\mu N(-\hat{i}) + \cancel{R}\,T\sin 25.6^\circ(\hat{i}) = 0$$

$$\boxed{-\mu N + T\sin 25.6^\circ = 0 \quad (3)}$$

$\alpha = 90 - 30^\circ = 60^\circ$

Law of sines:



$$\frac{\sin 60}{R} = \frac{\sin\beta}{\frac{R}{2}} \rightarrow 2\sin\beta = \sin 60 \rightarrow \beta = \sin^{-1}\left(\frac{\sin 60}{2}\right) = 25.6^\circ$$

Use (1) in (3) : (1) → $N = T\cos 60^\circ$

→ (3) $-\mu \cancel{T}\cos 60^\circ + \cancel{T}\sin 25.6^\circ = 0$

$$\boxed{\begin{aligned} \mu &\geq \frac{\sin 25.6^\circ}{\cos 60^\circ} \\ \mu &\geq 0.86 \end{aligned}}$$