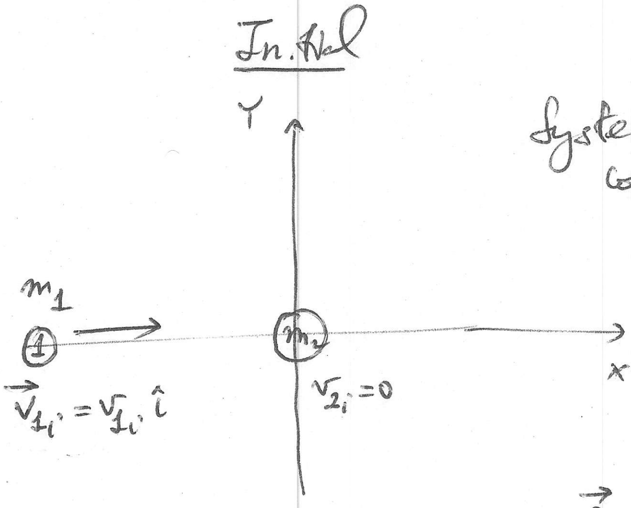
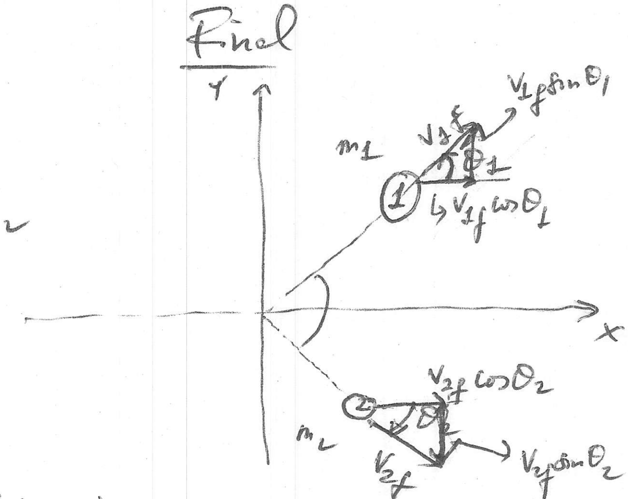
Elastic Collision in 2D:

We will prove that the final directions of two colliding objects (elastic collisions: no deformation, hard-ball collisions) of equal masses form 90° angle!



System of 2 components m_1 & m_2



- Elastic collisions: $\begin{cases} \vec{P}_i = \vec{P}_f & (2\text{ eqs}) \\ KE_i = KE_f & (1\text{ eq.}) \end{cases}$

- Known variables: $m_1,\ m_2,\ \vec{v}_{1i},\ \theta_1$ (final direction for m_1)

  $\downarrow$ can only solve for 3 unknowns ($v_{1f},\ v_{2f},\ \theta_2$)

Write the conservation laws in terms of the 2 components of the system.

$$\text{(1)}\quad m_1 v_{1i} = m_1 v_{1f}\cos\theta_1 + m_2 v_{2f}\cos\theta_2$$

$$\text{(2)}\quad 0 = m_1 v_{1f}\sin\theta_1 + m_2 v_{2f}\sin\theta_2$$

↳ Const of θ_2 is negative as shown

Standard convention for angle: $\begin{cases} + \text{ CCW from x-axis} \\ - \text{ CW from x-axis} \end{cases}$

CW from x

Eq ② is also: $0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin|\theta_2|$

③ $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

Now use math manipulation (algebra) to solve for v_{1f}, v_{2f}, θ_2

$$①^2: \quad \cancel{m_1^2} v_{1i}^2 = \cancel{m_1^2} \left(v_{1f} \cos\theta_1 + \frac{m_2}{m_1} v_{2f} \cos\theta_2 \right)^2$$

$$v_{1i}^2 = \boxed{v_{1f}^2 \cos^2\theta_1} + \frac{m_2^2}{m_1^2} v_{2f}^2 \cos^2\theta_2 + 2 \frac{m_2}{m_1} v_{1f} v_{2f} \cos\theta_1 \cdot \cos\theta_2$$

$$②^2 \quad 0 = m_1^2 \left(v_{1f} \sin\theta_1 + \frac{m_2}{m_1} v_{2f} \sin\theta_2 \right)^2$$

$$0 = \boxed{v_{1f}^2 \sin^2\theta_1} + \frac{m_2^2}{m_1^2} v_{2f}^2 \sin^2\theta_2 + 2 \frac{m_2}{m_1} v_{1f} v_{2f} \sin\theta_1 \cdot \sin\theta_2$$

$\cos^2\alpha + \sin^2\alpha = 1$

$$①^2 + ②^2: \quad v_{1i}^2 = v_{1f}^2 + \frac{m_2^2}{m_1^2} v_{2f}^2 + 2 \frac{m_2}{m_1} v_{1f} v_{2f} \underbrace{\left[\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \right]}_{\cos(\theta_1 - \theta_2) \; = \; \cos(\theta_2 - \theta_1)}$$

$$\boxed{\boxed{v_{1i}^2} = \boxed{v_{1f}^2} + \frac{m_2^2}{m_1^2} v_{2f}^2 + 2 \frac{m_2}{m_1} v_{1f} v_{2f} \cos(\theta_1 - \theta_2)} \quad ⓐ$$

Direct consequence of $\vec{P}_i = \vec{P}_f$!

Divide both side of eq ③ by $\frac{1}{2} m_1$:

$$\boxed{\boxed{v_{1i}^2} = \boxed{v_{1f}^2} + \frac{m_2}{m_1} v_{2f}^2} \quad ⓑ$$

$KE_i = KE_f$

Similar terms in ⓐ & ⓑ $\longrightarrow$ ⓐ − ⓑ !

ⓐ − ⓑ :
$$0 = \underbrace{\left(\frac{m_2^2}{m_1^2} - \frac{m_2}{m_1}\right)}_{\boxed{\frac{m_2}{m_1}}\left(\frac{m_2}{m_1}-1\right)} \boxed{V_{2f}^2} + 2\boxed{\frac{m_2}{m_1}} V_{1f} \boxed{V_{2f}} \cos(\theta_1 - \theta_2)$$

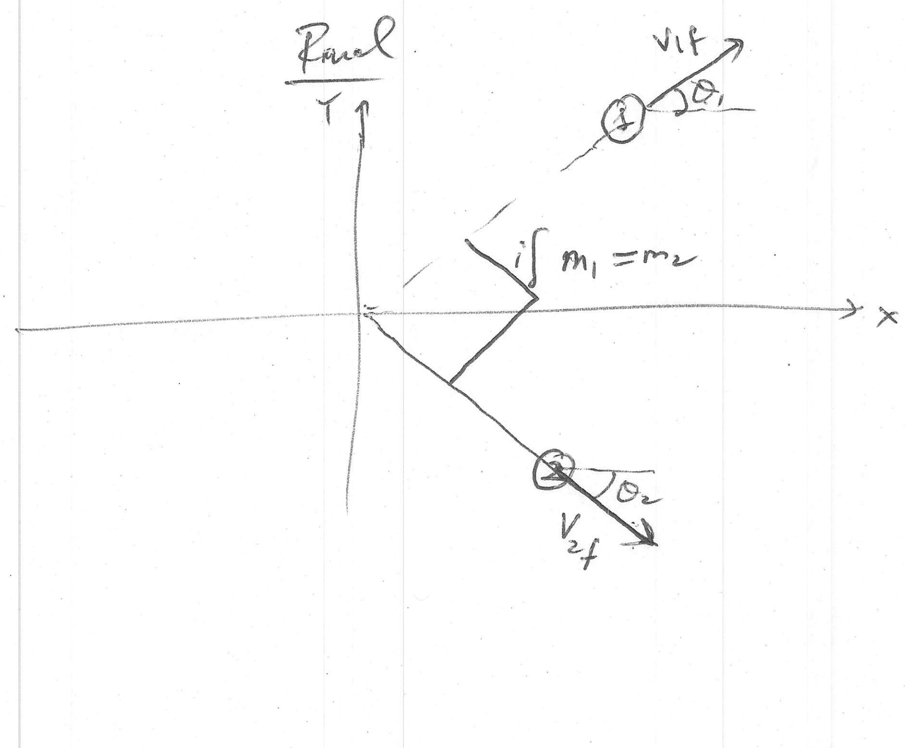
$$\frac{ⓐ - ⓑ}{\frac{m_2}{m_1} V_{2f}} : \quad \boxed{0 = \left(\frac{m_2}{m_1} - 1\right) V_{2f} + 2 V_{1f} \cos(\theta_1 - \theta_2)}$$

2D Elastic Collision $\begin{cases} \vec{P_i} = \vec{P_f} \\ KE_i = KE_f \end{cases}$

If $m_2 = m_1$ $\rightarrow$ $2 V_{1f} \cos(\theta_1 - \theta_2) = 0 \rightarrow \cos(\theta_1 - \theta_2) = 0$

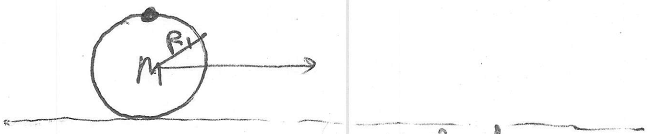
$\Rightarrow \theta_1 - \theta_2 = 90°$

Direction of the objects after collision form 90° angle if $m_1 = m_2$!



Ch. 10 Rotational Motion:

Translation



No friction

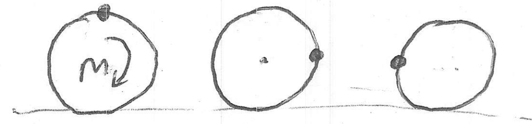
No friction

→ Sliding balls of equal masses M but different radii $R_2 < R_1$;

→ Same translational motion: since their motions (translations) can be described as that of a point-like particle of mass M located @ their center of masses (CM)

→ Sliding or translation: top point (as shown) always stay @ top.

Rotation

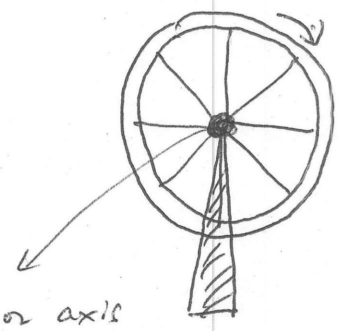


→ More realistically, because of friction we see rolling balls:

Rolling motion involves both rotation & translation:

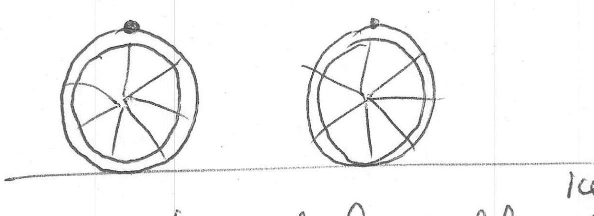
→ ① When we look at rotation size does matter.

→ ② ~~Are translation~~
Are rotations always associated with translations? No!



→ A bike wheel on a support rotates without any translation!

Center or axis of rotation or Pivot point

→ 

Ice surface

Same bike wheel would only translate on frictionless surface.

Other situations:

1) Car wheels under normal ~~situation~~ road conditions:

→ Rolling motion : both translation & rotation

2) Car wheels stuck in sand :

→ Only rotation

3) Car wheels while brake pedal is applied (no ABS) : only translation

" " " " " " (ABS) : both rotation & translation.

ABS : anti-blocking braking system : advantage :

w/o ABS : can only rely on friction to loose final KE to come to a stop.

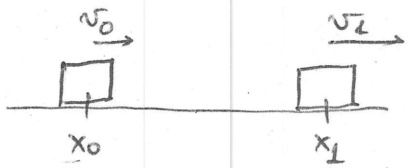
w ABS : in addition to friction can loose some of the final KE into rotational motion of wheels

→ stopping distance is shorter.

→ more vehicle control at curves.

Translational Motion

(Change of position)



$$\bar{v} = \frac{v_0 + v}{2}$$

$$\bar{v} = \frac{\Delta x}{\Delta t}$$ (average vel.)

Change of position over change of time $\left(\frac{m}{s}\right)$

$$v = \frac{dx}{dt}$$ (instantaneous vel.)

$$v = v_0 + a \cdot t \quad (1)$$

a: linear acceleration $\left(\frac{m}{s^2}\right)$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (2)$$

$$\frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a \quad (3)$$

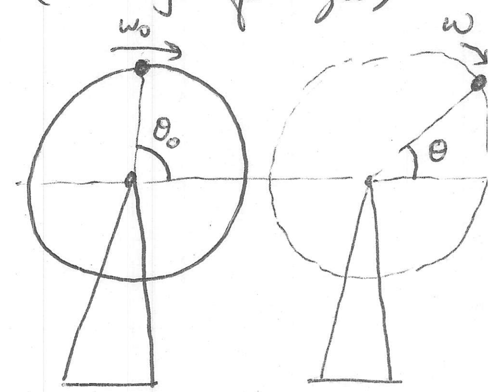
$$F_{net} = m \cdot a$$

↓ inertia

(constant masse)

Rotational Motion

(change of angle)



Same system (it did not translate!)

$$\bar{\omega} = \frac{\omega_0 + \omega}{2}$$

ω: angular velocity

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$ (change of angle over change of time)

$\frac{radian}{s}$ or $\frac{rad}{s}$

(2π radians in 360°)

$$\omega = \frac{d\theta}{dt}$$ (instantaneous ang. velocity).

$$\omega = \omega_0 + \alpha \cdot t \quad (1)$$

α: angular acceleration $\left(\frac{rad}{s^2}\right)$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \quad (2)$$

$$\frac{\omega^2 - \omega_0^2}{\theta - \theta_0} = 2\alpha \quad (3)$$

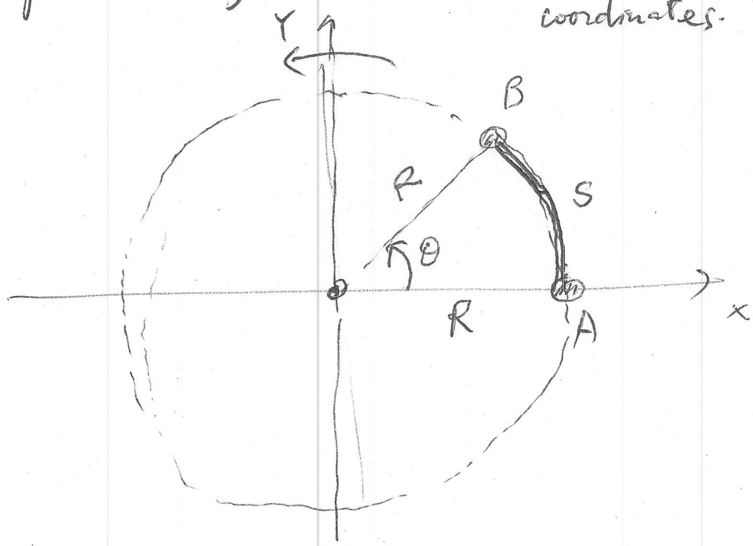
$$\tau_{net} = I \cdot \alpha$$

τ_{net} = "tau": net torque } size of object does matter
I = moment of inertia

- Angular velocity (ω) and acceleration (α)
- Net torque & moment of inertia (size matters)

1) Any connection b/w angular velocity & linear velocity of a point on object?

Point A on a rotating disk wrt origin of coordinates.



a) From A to B angle went from 0 to θ (CCW)

b) A has changed position along a circular trajectory of radius R, an arc length S

b) $\theta = \frac{S}{R}$ — θ → angular rotation; S → linear motion on circular trajectory

$$\frac{d}{dt}\left[\ \theta = \frac{S}{R}\ \right] \rightarrow \frac{d\theta}{dt} = \frac{1}{R}\frac{dS}{dt} \rightarrow \boxed{\omega = \frac{v}{R}}$$

For a point on the rotating disk.

$$\omega = \underbrace{\frac{v}{R} \qquad \frac{m/s}{m}}_{\frac{1}{s}}$$

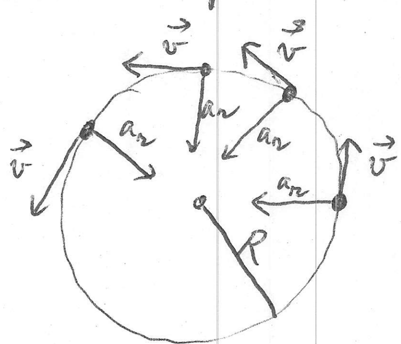
ω → $\frac{rad}{s}$ → rpm (rev per minute)

→ correct since angle is dimensionless ($[\theta] = 1$)

2) <u>Angular acceleration</u>: α ("alpha")

$$\bar{\alpha} = \frac{\Delta \omega}{\Delta t}, \quad \alpha = \frac{d\omega}{dt} \qquad \left(\frac{\text{rad}}{s^2} = \frac{1}{s^2} = s^{-2}\right)$$

→ UCM (Uniform circular motion) { - Constant speed (tangential)
- Constant radial acceleration a_r (to change direction of motion)
- Tangential acceleration is 0



$$\begin{cases} a_r = \dfrac{v^2}{R} \\ a_t = 0 \end{cases}$$

(UCM: v = constant, R constant)

$$\Rightarrow \boxed{a_r = \frac{\omega^2 R^{\cancel{2}}}{R} = \omega^2 R}$$

$$\omega = \frac{v}{R}$$

What is α? $\quad \alpha = \dfrac{d\omega}{dt} = \dfrac{1}{R}\underbrace{\dfrac{dv}{dt}}_{a_t} = \dfrac{1}{R} a_t = 0$

Makes sense: in UCM = constant v → constant ω $\left(\omega = \frac{v}{R}\right)$

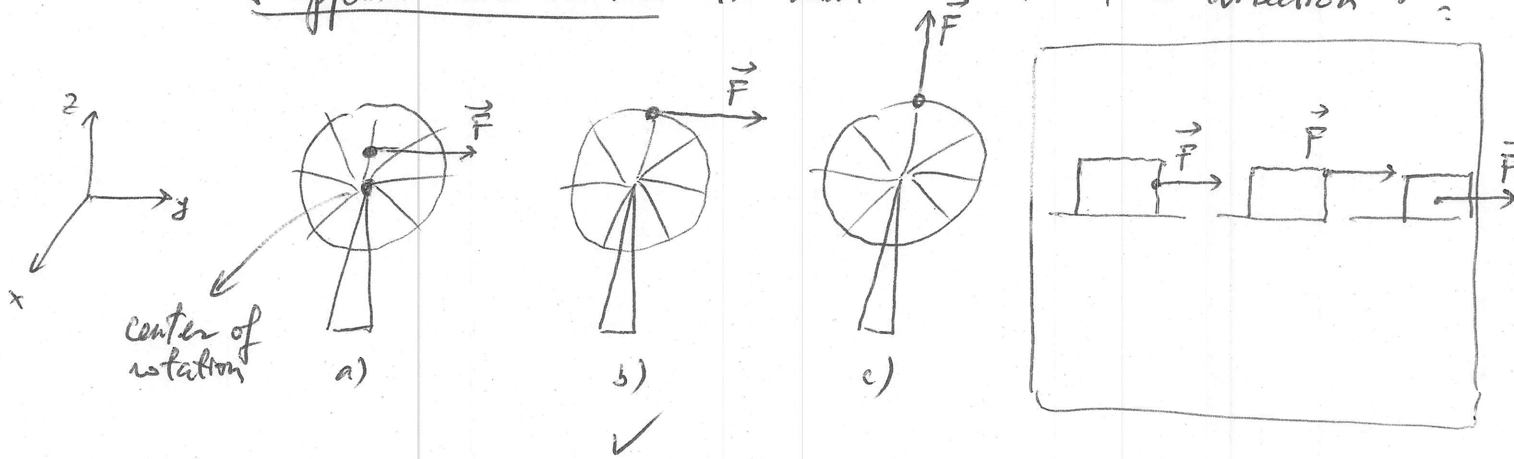
→ $\alpha = 0$

→ Non-UCM or non-uniform circular motion:

$$\begin{cases} \boxed{a_r = \text{still there (whenever there is a circular motion!)} = R\omega^2} \\ \boxed{a_t = \dfrac{dv}{dt} = \dfrac{d(\omega R)}{dt} = R\dfrac{d\omega}{dt} = R\alpha} \end{cases}$$

3) Torque $\vec{\tau}$ (tau) :

Is needed in rotations b/c size matters: → <u>where a force is applied does matter</u> in addition to the force direction ! :



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\vec{r}$: position vector of the force application point w.r.t pivot or center of rotation.

$\times$: "cross product": product of two vectors that is another vector that is perpendicular to both

$$= r F \sin\theta \, \hat{\tau}$$

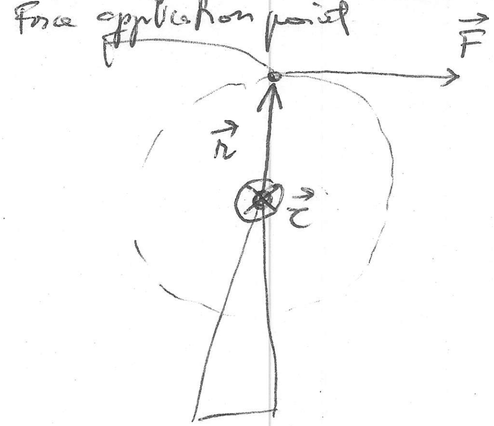
θ : angle b/w $\vec{r}$ & $\vec{F}$

$\hat{\tau}$: unit vector that is perpendicular to the plane formed by $\vec{r}$ & $\vec{F}$ direction given by <u>Right Hand Rule</u> (RHR)

Unit: Nm

Torque is a vector whose magnitude is the product of the magnitudes of the position vector & force vector times the sine of the angle b/w them. Direction of the torque is perpendicular to both $\vec{r}$ & $\vec{F}$ and given by RHR.

Cross-product & RHR:

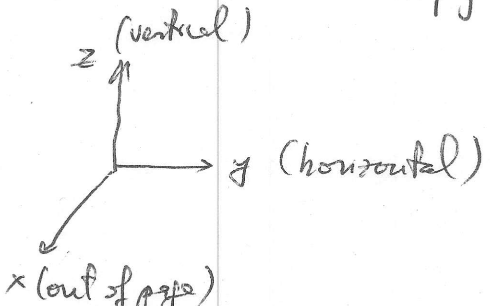


$$\vec{\tau} = \vec{r} \times \vec{F}$$

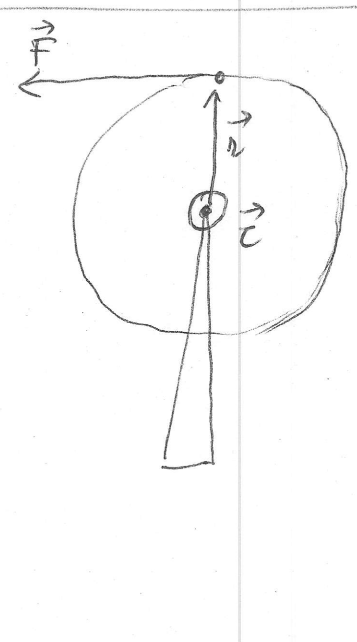
As shown: $\vec{r}$ & $\vec{F}$ are in plane of this page. → $\vec{\tau}$ is perpendicular to the page.

RHR: align RH ~~thumb~~ fingers along 1st vector ($\vec{r}$), as you ~~turn~~ close RH fingers toward direction of 2nd vector ($\vec{F}$), RH thumb points in direction of torque:

As shown: $\vec{\tau}$ = into page $\otimes$



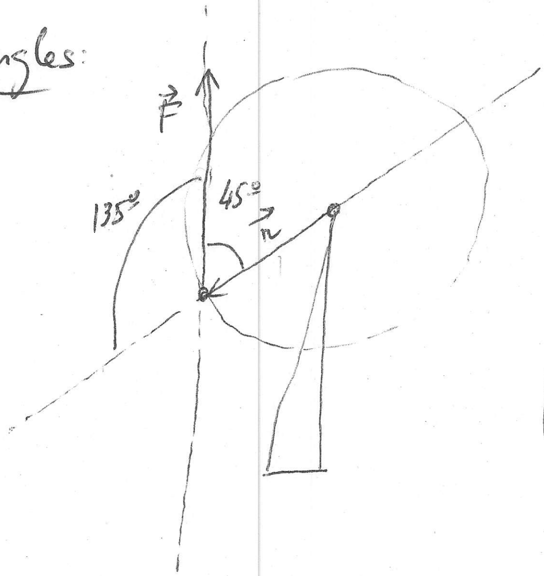
$$\vec{\tau} = rF \underbrace{\sin 90°}_{1} (-\hat{i}) = -rF\hat{i}$$



RHR: $\vec{\tau}$ = out of page $\odot$

$$\vec{\tau} = +rF\hat{i}$$

Angles:

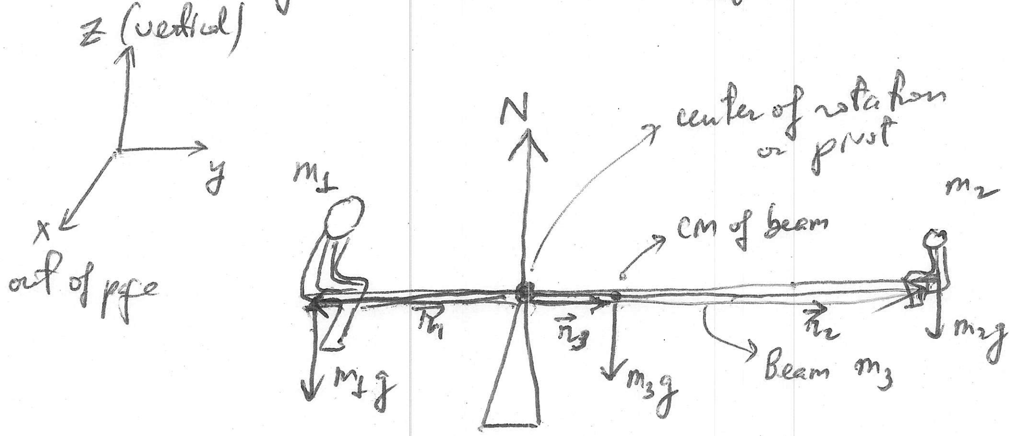


$\tau = rF\sin\theta$ (Magnitude: +)

$\sin 45° = -\sin 135°$

$\tau = rF|\sin\theta|$

Balancing a beam: $\vec{\tau}_{net} = 0 = I \cdot \alpha \quad (\alpha = 0)$



Forces applying on beam: its weight $m_3 g$; $m_1 g$; $m_2 g$; N: normal force from pivot point.

Torques apply on beam: (wrt to center of rotation or pivot)

$$\begin{cases} \vec{\tau}_1 = r_1 m_1 g \underbrace{\sin 90}_{1} \ \hat{i} \\ \vec{\tau}_2 = r_2 m_2 g \underbrace{\sin 90}_{1} (-\hat{i}) \\ \vec{\tau}_3 = r_3 m_3 g \cdot 1 \cdot (-\hat{i}) \\ \text{No torque by } \vec{N} \ (\vec{r} = 0) \end{cases}$$

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = (r_1 m_1 g - r_2 m_2 g - r_3 m_3 g)\, \hat{i} = 0$$

Example:
- beam Length $L = 3m$
- pivot at 0.75m from left edge
- $m_1 = 75$ kg ; $m_2 = 25$ kg

m_3?

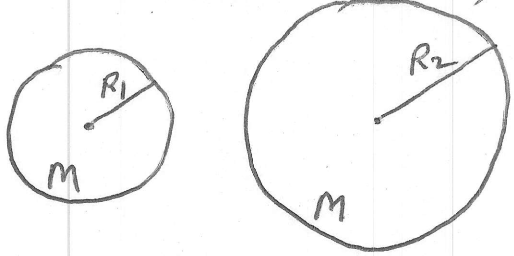
$$m_3 = \frac{(r_1 m_1 - r_2 m_2)\cancel{g}}{r_3 \cancel{g}} = \frac{0.75 \cdot 75 - 2.25 \cdot 25}{0.75} = 25\left(\frac{3}{2} - \frac{3}{2}\right) = 0 \text{ kg}.$$

$r_1 = 0.75m$; $r_2 = 2.25$ m ; $r_3 = 0.75m$

4) Moment of Inertia (I) $\quad (\check{\tau}_{net} = I \cdot \check{\alpha})$

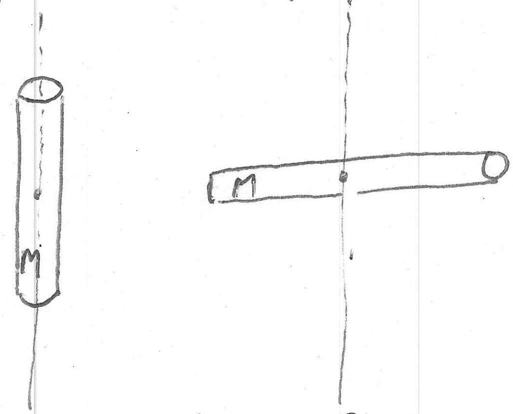
(size of mass distribution matters!)

↳



$I_1 \quad < \quad I_2$

location of the axis of rotation also matters:



$I_1 \quad < \quad I_2$

$$I = \begin{cases} \text{Discrete system}: & \sum_i m_i r_i^2 \\ \text{Continuous system}: & \int dm\, r^2 \end{cases}$$

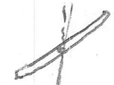
Moment of inertia for round & symmetrical objects:

disks, rods, spheres, etc...

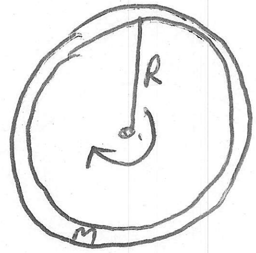
$I = \alpha \quad MR^2$ $\begin{cases} M: \text{total mass of object} \\ R: \text{radius of mass distribution wrt center of rotation} \end{cases}$

↳
1) Sphere wrt center axis  $\rightarrow \alpha = \frac{2}{5}$ (proved by $\int dm\, r^2$)

2) Cylinder wrt center axis  $\rightarrow \alpha = \frac{1}{2}$

3) Thin rod of length L wrt middle axis  $\alpha = \frac{1}{12}$; $R = L$ or $I = \frac{1}{12} ML^2$

4) Ring of mass M & radius R wrt center axis.

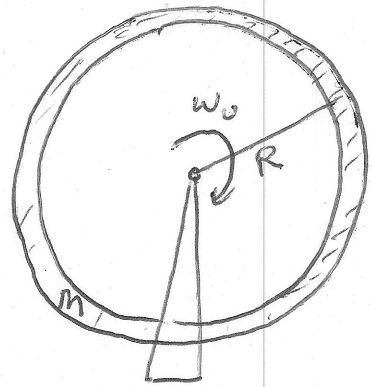


$\alpha = 1$

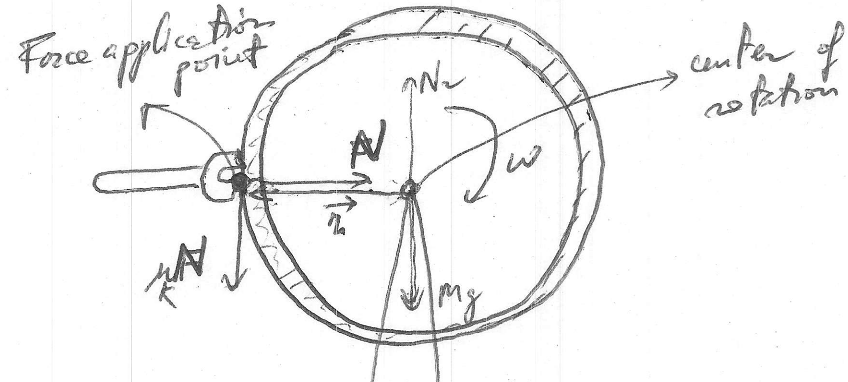
or $I = MR^2$

Example: 10.58

Spinning bike wheel: $R = 0.33m$, $M = 1.9kg \rightarrow I = MR^2$



$\omega_0 = 230$ rpm



gets slowed down with a wrench at normal direction $N = 2.7N$ during $\Delta t = 3.1s$

$\mu_K = 0.46$

(b/w wrench & tire)

Solve for final angular speed ω (after $N = 2.7N$ is applied during $\Delta t = 3.1s$):

$$\omega = \omega_0 - \alpha \Delta t$$

(deceleration happens while wrench is applied)

ω_0 ✓ α ? Δt ✓

As 2nd Newton's law was center piece to solve linear motion, its analog $\tau_{net} = I\alpha$ is the center piece to solve rotational motion.

$$\tau_{net} = I \cdot \alpha \rightarrow \alpha = \frac{\tau_{net}}{I} = \frac{\mu_K N R}{MR^2}$$

τ_{net} = sum of torques on bike wheel:

→ There are 4 forces on bike wheel: N, $\mu_K N$, Mg (weight), N_2 (Normal @ pivot)

→ There is 1 torque (wrt center rotation): $\tau_{net} = R \mu_K N \underbrace{\sin 90}_{1}$

N_2 & Mg apply no torque since center of rotation is also force application point ($\vec{r} = 0$!)

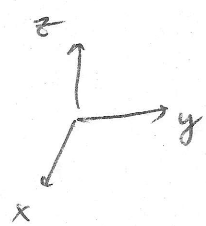
N applies no torque: although center of rotation is R from force application point $\theta = 180 \Rightarrow \sin\theta = 0$

$$\omega = \omega_0 - \frac{\tau_{net}}{I} \cdot \Delta t = \omega_0 - \underbrace{\frac{\mu_k N R}{M R^2} \Delta t}_{\Delta\omega}$$

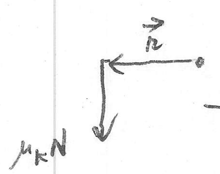
$$\Delta\omega = \underbrace{\frac{0.46 \cdot 2.7 \cdot \cancel{0.33}}{1.9 \cdot 0.33^{\cancel{2}}} \, 3.1}_{\text{SI units.}} = 6.14 \frac{\cancel{rad}}{\cancel{s}} \cdot \frac{1\,rev}{2\pi\,\cancel{rad}} \cdot \frac{60\cancel{s}}{1\,min}$$

$$= 58.6 \frac{rev}{min} = 58.6 \text{ rpm}$$

$$\rightarrow \omega = \omega_0 - \Delta\omega = 230 - 58.6 = 171 \text{ rpm.}$$



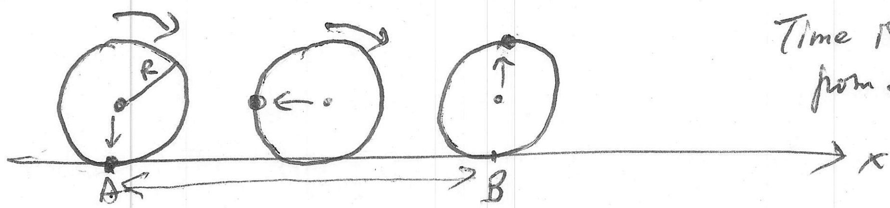
$$\vec{\tau}_{net} = \mu_k N R \, \hat{i}$$



$\rightarrow$ RHR: $\vec{\tau}$ is out of page.

Rolling Motion:

↳ Non-skidding



Time it takes to go from A to B is Δt.

Disk rolling on horizontal surface, non-skidding

1) Center of Mass of disk has traveled from A to B
2) Disk in contact with surface at all time (non-skidding)
   ↳ half of its circumference has been in contact with the surface : $\Delta x_{AB} = \pi R$

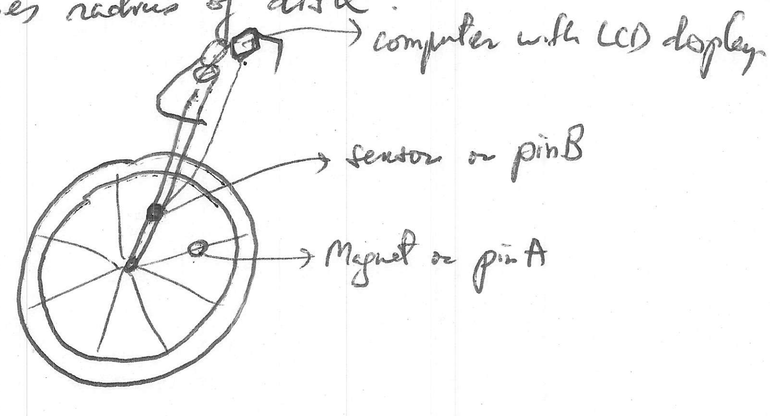
$$v_{CM} = \frac{\Delta x_{AB}}{\Delta t} = \frac{\pi R}{\Delta t} = \omega R \rightarrow \boxed{v_{CM} = \omega R}$$

angular speed of disk = $\omega = \dfrac{\Delta \theta}{\Delta t} = \dfrac{\pi}{\Delta t}$

In rolling motion : linear speed of CM is angular speed times radius of disk.

Apply this to a ring:

↳ "Bike speedometer"

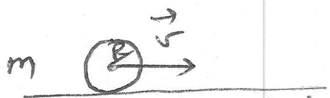


Every time pin A passes pin B : computer assumes wheel has completed one revolution :
→ calculate ω in rpm, then use rolling motion formula $v_{CM} = \omega . R$ to display linear speed.

Kinetic energy in rolling motion:

1. Linear motion

$\frac{1}{2}mv^2$



Non-friction

Disk skidding w/o rotation on a frictionless surface

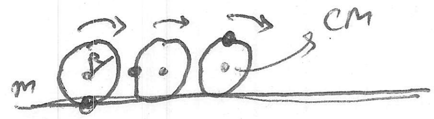
2. Rotational Motion

$\frac{1}{2}I\omega^2$

Disk is rotating freely on a support (no translational motion)

3. Rolling motion

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$



Disk rolling on surface with friction: translation of CM + rotation wrt CM

KE for rolling motion: $\boxed{v_{CM} = \omega . R} \rightarrow \omega = \frac{v_{CM}}{R}$

$$KE = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{2}\underbrace{\left(m + \frac{I}{R^2}\right)}\, v_{cm}^2$$

For a disk $= I = \frac{1}{2}mR^2$

$$m + \frac{I}{R^2} = m + \frac{\frac{1}{2}m\cancel{R^2}}{\cancel{R^2}} = m + \frac{m}{2} = \frac{3}{2}m$$

Disk: wrt center axis

Sliding

$$KE = \frac{1}{2}mv_{cm}^2$$

Rolling

$$KE = \frac{1}{2}\left(\frac{3}{2}m\right)v_{cm}^2$$

In term of KE rolling increases mass ~~inter~~ inertia a factor of 1.5 (150%)

ABS : anti-blocking brakes

w/o ABS : inertia mass is m

w/ ABS : inertia mass is 1.5m per wheel

→ shorter stopping distance!

Ch 11 Rotational Vectors & Angular Momentum:

Linear Motion

$\vec{p}$ = linear momentum

$\vec{p} \equiv m\vec{v}$

2nd Newton's Law:

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$

$\vec{F}_{net} = 0 \rightarrow \vec{P}_i = \vec{P}_f$

Total linear momentum is conserved if $\vec{F}_{net} = 0$

↓

Collisions!

Rotational Motion

$\vec{L}$ = angular momentum

$\vec{L} = \vec{r} \times \vec{p}$

→ cross product b/w position vector (wrt center of rotation) & linear momentum $\vec{p}$

→ as with the torque, $\vec{L}$ is perpendicular to both $\vec{r}$ & $\vec{p}$ with direction given by RHR

Analog of 2nd Newton's Law:

$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

$\vec{\tau}_{net} = 0 \rightarrow \vec{L}_i = \vec{L}_f$

Total angular momentum is conserved if $\vec{\tau}_{net} = 0$

(And $\vec{\tau}_{net} = I \cdot \vec{\alpha}$

$\boxed{\vec{L} = I \cdot \vec{\omega}}$)

Two cross products: with a defined center of rotation

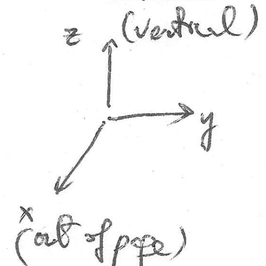
$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{r}$ = position vector, from center of rotation to force application point

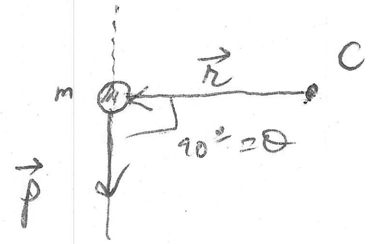
$\vec{L} = \vec{r} \times \vec{p}$

$\vec{r}$ = position vector, from center of rotation to the position of mass m that has linear momentum $\vec{p}$

Examples:

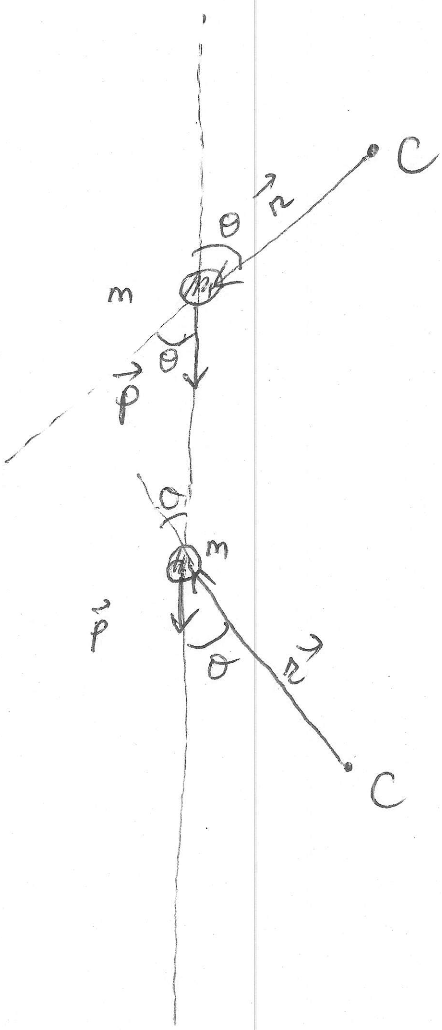


Rotation with centers C:
object of mass m moving along $-z$ $(-\hat{k})$
(an object does not need to rotate to have an angular momentum)



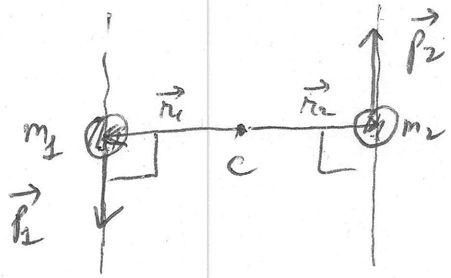
Angular momentum for this object: $\vec{L} = \vec{r} \times \vec{p} = rp\,\hat{i}$ given by RHR

What is the angular momentum now?



$\vec{L} = \vec{r} \times \vec{p} = rp \sin\theta \; \hat{i}$ by RHR

$\vec{L} = rp\sin\theta \; \hat{i}$ by RHR



System with 2 masses →

Total angular momentum $\vec{L}$ is

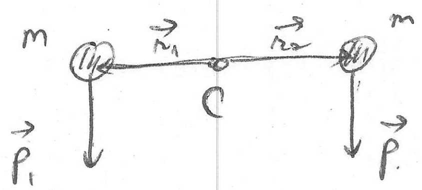
$$\vec{L} = \vec{L}_1 + \vec{L}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$
$$= r_1 p_1 \hat{i} + r_2 p_2 \hat{i}$$
$$= 2rp\,\hat{i}$$

C right b/w both masses $r_1 = r_2 = r$

both masses with same speed $\vec{p}_1 = \vec{p}_2 = \vec{p}$

In a car: this happens in the steering wheel. ~~when~~

it turns → makes sense there is a total $\vec{L}$!



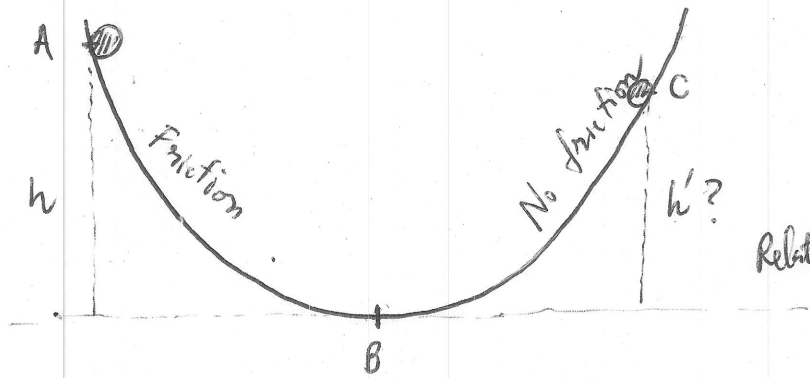
$$\vec{L} = rp\,\hat{i} - rp\,\hat{i} = 0$$

↓

makes sense!

10.64

Solid ball → sphere $I = \frac{2}{5}MR^2$
M, R



Point C : { Above A / Same height as A / $\boxed{\text{Below A}}$

- Energy lost due to friction.
- There was rotation beside translation b/w A & B.

Relate

Find h':

AB : Rolling motion
(Translation & rotation and $v_{cm} = \omega R$)

BC : Skidding motion
(only translation)

Conservation of energy:

AB: initial (A) final (B)
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \quad (1)$$

BC: initial (B) final (C)
$$\frac{1}{2}Mv_B^2 = Mgh' \quad (2)$$
$$\boxed{h' = \frac{v_B^2}{2g}}$$

Solve for v_B in (1):

a) spherical solid ball rotating wrt its center axis
$I = \frac{2}{5}MR^2$

b) Rolling motion b/w A & B : $\omega = \frac{v_B}{R}$

$$(1) \quad \cancel{M}gh = \frac{1}{2}\cancel{M}v_B^2 + \frac{1}{2}\cdot\frac{2}{5}\cancel{M}\cancel{R^2}\cdot\frac{v_B^2}{\cancel{R^2}}$$

$$gh = \frac{v_B^2}{2} + \frac{v_B^2}{5} = \frac{v_B^2}{2}\underbrace{\left(1+\frac{2}{5}\right)}_{\frac{7}{5}} \rightarrow \boxed{v_B^2 = \frac{10}{7}gh}$$

$$(2) \quad h' = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h < h$$

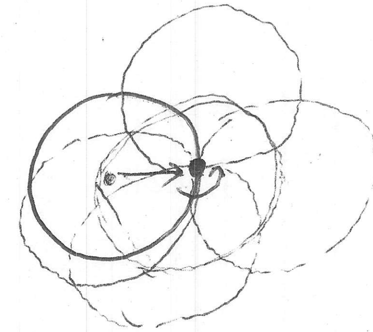
Note: this confirms our prediction that $h' < h$ since some initial gravitational potential energy was lost due to friction (which went into the rotational part of rolling motion b/w A & B)

10.65 Background information: Parallel Axis Theorem for Moments of Inertia

Rotation of a ~~sph~~ $\boxed{\text{disk}}$ as view from above:

Rotates wrt its center axis:

$$I_1 = \frac{1}{2}MR^2$$

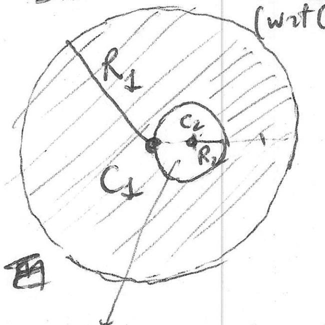


Rotates wrt an axis on the outer edge

$$I_2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

Parallel Axis Theorem: when the axis is displaced by a distance R : the moment of intertia wrt the new axis is increased by $\underbrace{M}_{\text{mass of object}} \cdot \underbrace{R^2}_{\text{displacement incurred.}}$

Disk with hole : I (wrt C_1)



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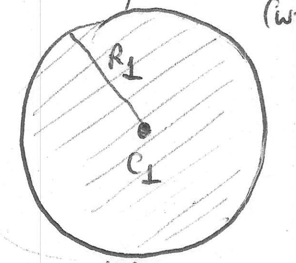
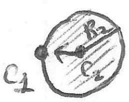
Disk w/ no hole : I_1 (wrt C_1)



−

Smaller disk : I_2 (wrt C_1)



hole of radius $R_2 = \frac{R_1}{4}$

- Mass of disk before hole was drilled is M
- C_1 : center of disk and $\boxed{\text{center of rotation}}$
- C_2 : center of the hole

$$I = I_1 - I_2$$

all three I's (wrt to same central axis C_1!)

$$= \frac{1}{2}MR_1^2 - \Big(\frac{1}{2}mR_2^2 + \underbrace{mR_2^2}_{\text{Parallel axis Theorem}} \Big)$$

Note: $\left\{ \begin{array}{l} R_2 = \frac{R_1}{4} = \frac{R}{4} \\ m = \frac{\pi R_2^2}{\pi R_1^2} M = \frac{1}{16} M \end{array} \right.$

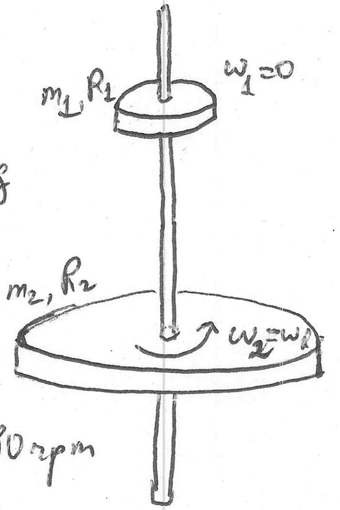
$$I = \frac{1}{2}MR_1^2 - \frac{3}{2}mR_2^2 = \frac{1}{2}MR_1^2 - \frac{3}{2}\frac{M}{16}\frac{R_1^2}{16} = \frac{1}{2}MR_1^2\underbrace{\left(1 - \frac{3}{16^2}\right)}_{253}$$

11.51

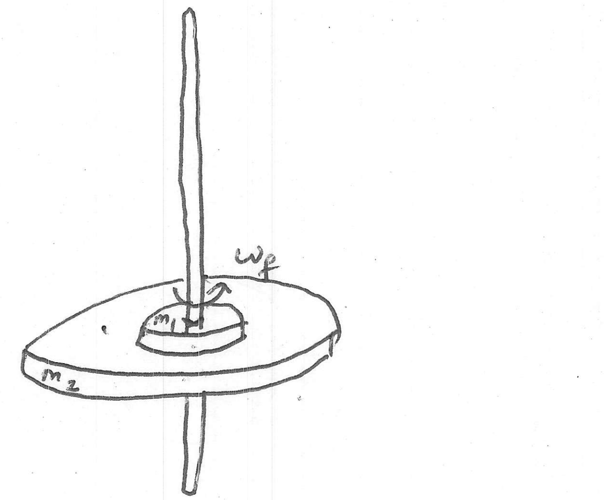
$m_2 = 0.44 kg$
$R_2 = 0.035 m$

$m_1 = 0.27 kg$
$R_1 = 0.023 m$

$\omega_2 = \omega_i = 180 rpm$



Initial
(Only m_2 is rotating)
$\omega_i = \omega_2$



Final
(both are rotating at same speed ω_f due to friction b/w the two disks)

Initial & final → conservation of angular momentum (rotations are involved)

<u>Check</u>: System of disks (shaft's mass is ignored) <u>w/o friction</u> on shaft: $\vec{\tau}_{net} = 0 \rightarrow \boxed{\vec{L}_i = \vec{L}_f}$ ↓ "m_1 drops freely"

Force applied in direction of rotation: none

a) ω_f ?

$$L_i = L_f$$

$$I_2 \omega_2 = I_1 \omega_f + I_2 \omega_f = (I_1 + I_2)\omega_f$$

$$\omega_f = \frac{I_2}{I_1 + I_2}\omega_2 = \frac{\frac{1}{2} m_2 R_2^2}{\frac{1}{2} m_1 R_1^2 + \frac{1}{2} m_2 R_2^2}\omega_2$$

$$\omega_f = \frac{0.44 \times 0.035^2}{0.44 \times 0.035^2 + 0.27 \times 0.023^2} 180 rpm$$

$$\omega_f = 142 rpm.$$

b) Fraction of energy lost due to friction:

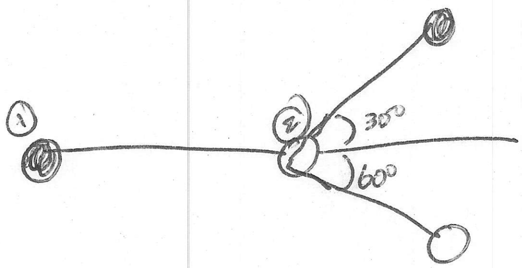
$KE_i > KE_f$ since some energy was lost to friction b/w the two disks! (for them to rotate at same final ω_f!)

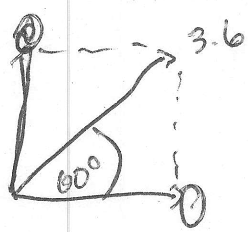
$$\text{Lost fraction} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{\frac{1}{2}(I_1 + I_2)\omega_f^2}{\frac{1}{2} I_2 \omega_2^2}$$

$$\text{lost fraction} = 1 - \frac{\cancel{\tfrac{1}{2}}\left(\tfrac{1}{2}m_1R_1^2 + \cancel{\tfrac{1}{2}}m_2R_2^2\right)\ \omega_f^2}{\cancel{\tfrac{1}{2}}\cdot\cancel{\tfrac{1}{2}}m_2R_2^2\ \omega_2^2}$$

$$= 1 - \frac{0.44\cdot 0.035^2 + 0.27\cdot 0.023^2}{0.44\cdot 0.035^2}\cdot\underbrace{\frac{142^2}{180^2}} = 0.210 \text{ or } \boxed{21\%}$$

conversions of rpm to $\frac{\text{rad}}{\text{s}}$ is a scaling factor common to both numerator & denominator and would cancel out.



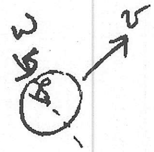


$$v_1 = \sin 60^\circ \cdot 3.6 = 3.12\ m/s$$

$$v_2 = \cos 60 \cdot 3.6 = 1.8\ m/s$$

10.37 Translational & rotation KE

$\quad\quad\quad\quad\quad\quad\quad\quad v \quad\quad\quad\quad\quad \omega$



Fraction of rotational in the total KE?

$M = 0.15\,kg$
$R = 0.037\,m$

$v = 33\,\frac{m}{s}$

$\omega = 42\,\frac{rad}{s}$

Baseball → sphere : $I = \frac{2}{5}MR^2$

$$\frac{KE_{Rot.}}{KE_{Trans} + KE_{rot}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2}$$

$$\rightarrow = \frac{\cancel{\frac{1}{2}}\frac{2}{5}\cancel{M}R^2\omega^2}{\cancel{\frac{1}{2}}\cancel{M}v^2 + \cancel{\frac{1}{2}}\frac{2}{5}\cancel{M}R^2\omega^2}$$

$$= \frac{\frac{2}{5}R^2\omega^2}{v^2 + \frac{2}{5}R^2\omega^2}$$

$$= \frac{\frac{2}{5}\,0.037^2 \cdot 42^2}{33^2 + \frac{2}{5}\,0.037^2 \cdot 42^2}$$

$$= 8.86 \cdot 10^{-4} \text{ or } 0.0886\,\%$$

(very small)

Note: $\omega R = 42 \cdot 0.037 = 1.554\,\frac{m}{s}$ → This is the linear speed of a point on the surface of baseball

$v = 33\,\frac{m}{s}$

→ It's not v (velocity of CM of ball) because this is not a rolling motion!

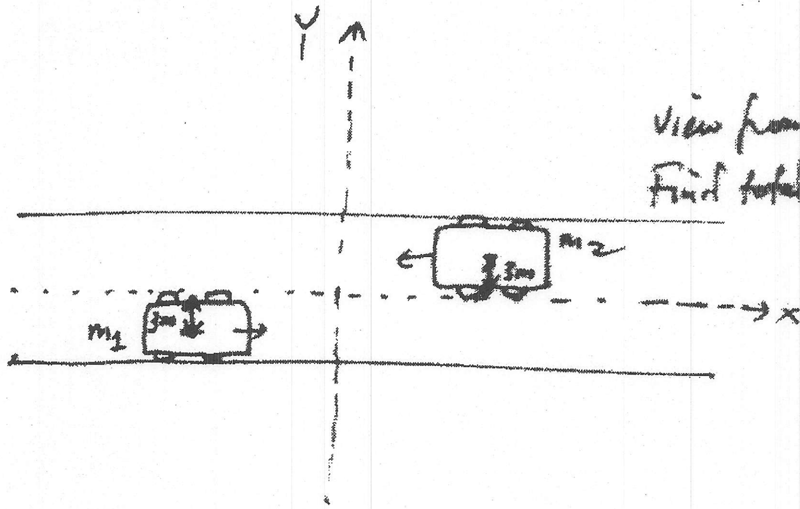
→ It's very small so most KE is in the translational motion

11-37

view from above!

Find total $\vec{L} = \vec{L}_1 + \vec{L}_2$

$= \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times$



$m_1 = m_2 = 1800\,kg$

$90 \frac{km}{h} = 25 \frac{m}{s}$ $\quad \vec{v}_1 = 25\,\hat{i}$ m/s ; $\quad \vec{r}_1 = x_1 \hat{i} - 3\hat{j}$ (m)

$\vec{v}_2 = 25(-\hat{i})$ m/s ; $\quad \vec{r}_2 = x_2 \hat{i} + 3\hat{j}$ (m)

$$\vec{L} = \vec{L}_1 + \vec{L}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

$$= (x_1 \hat{i} - 3\hat{j}) \times \underbrace{25\,\hat{i} \cdot 1800}_{45000\,\hat{i}} + (x_2 \hat{i} + 3\hat{j}) \times \underbrace{25(-\hat{i})1800}_{-45000\,\hat{i}}$$

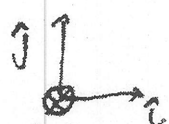
$$= 135000\,\hat{k} + 135000\,\hat{k} = 270000\,\hat{k}$$

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{i} = -\hat{k}$

$\frac{kg\,m^2}{s}$

$= J \cdot s$



$\hat{j} \times \hat{i}$

into page $\rightarrow (-\hat{k})$

11.40

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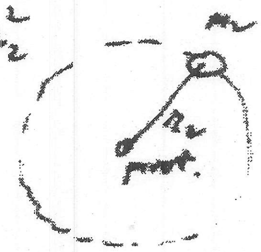
Equations:

$$L_i = L_f$$

$$I_1 \omega_i + |\vec{r}_2 \times \vec{p}_2| = (I_1 + I_2)\,\omega_f$$

$$I_2 \downarrow m_2 r_2^2 \downarrow ?$$

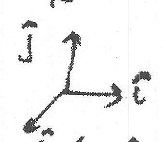


$\boxed{\text{clay}}$

$$\vec{r}_2 \times \vec{p}_2 = (x_2 \hat{i} + y_2 \hat{j}) \times v_2 \hat{j}\, m_2$$

$$= x_2 v_2 m_2 \underbrace{\hat{i} \times \hat{j}}_{\hat{k}} = \underbrace{0.15 \times 1.3}_{0.195} \times m_2\, \hat{k}$$

$$\hat{j} \times \hat{j}$$



$$\boxed{\begin{array}{l} \vec{L}_{Table} = I_1 \omega_i (-\hat{k}) \\ \vec{L}_{clay} = 0.195 m_2 (+\hat{k}) \end{array} \Big\} \text{ opposite signs} \\ \rightarrow \text{clay was opposing rotation!}}$$

$\boxed{\text{Turn table}}$

$$I_1 \vec{\omega}_i = I_1 \omega_i (-\hat{k})$$ (Direction by RHR: fingers turning as $\vec{\omega}_i$, thumb in direction of $\vec{\omega}_i$)

$$-0.021 \times 0.29 + 0.195 m_2 = -(0.021 + m_2\, 0.15^2)\, 0.085$$

$$+0.021 \times 0.29 - 0.195 m_2 = (0.021 + 0.0225)\, 0.085 \quad 0.0225$$

$$+\frac{0.021 \times 0.29}{0.085} - 0.021 = m_2 \left(0.0225 + \frac{0.195}{0.085}\right)$$

$$m_2 = \frac{\frac{0.021 \times 0.29}{0.085} - 0.021}{0.0225 + \frac{0.195}{0.085}}$$

$$\boxed{m_2 = 0.0218\ kg = 21.8\ g.}$$

→ Pay attention on the direction of angular momentum!

$$m = \frac{\pi \left(\frac{R}{4}\right)^2}{\pi R^2} M = \frac{M}{16}$$