1D

position x

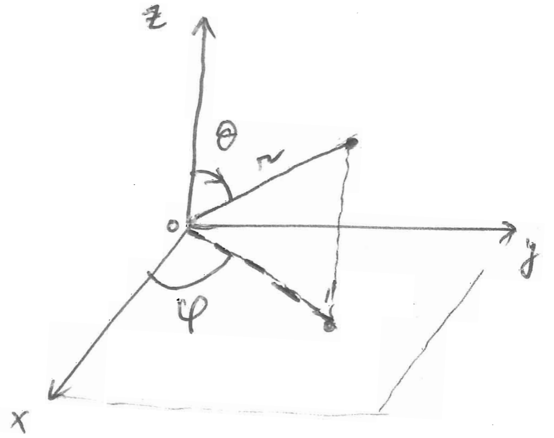
2D

position vector $\begin{cases} \text{Cartesian } (x, y) \\ \text{Polar } (r, \theta) \end{cases}$

3D

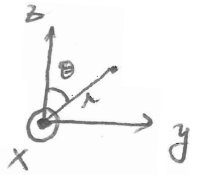
position vector $\begin{cases} \text{Cartesian } (x, y, z) \\ \text{Spherical } (r, \theta, \varphi) \end{cases}$

Spherical coordinates: r, φ, θ



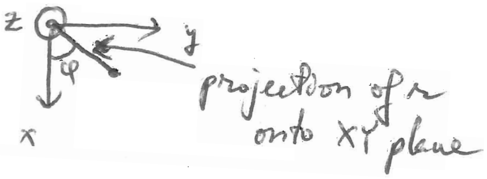
- r: length or distance b/w the position of interest and the origin of coordinates O
- φ: angle of the projection of r onto the XY plane, wrt the X-axis.
- θ: angle of r wrt Z-axis.

Front view:



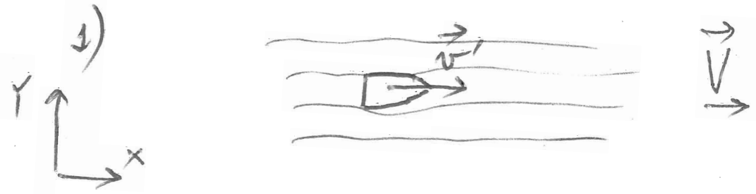
(out of page: ⊙)

Top view:



Units vectors & Relative Motion:

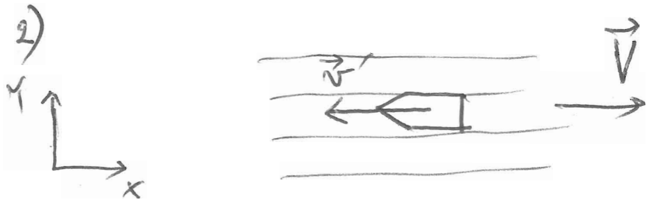
1)



Top view of a river.

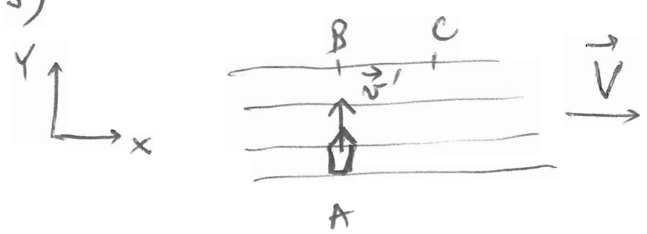
- Water's velocity: $\vec{V} = V\hat{i}$
- Boat's velocity wrt water $\vec{v}' = v'\hat{i}$ (downstream)
- Boat's velocity wrt ground $\boxed{\vec{v} = \vec{v}' + \vec{V}}$ $= (v' + V)\hat{i}$

2)



- Velocity of water $\vec{V} = V\hat{i}$
- Velocity of boat wrt water $\vec{v}' = -v'\hat{i}$ (upstream)
- Velocity of boat wrt ground $\boxed{\vec{v} = \vec{v}' + \vec{V}}$ $= (-v' + V)\hat{i}$

3)



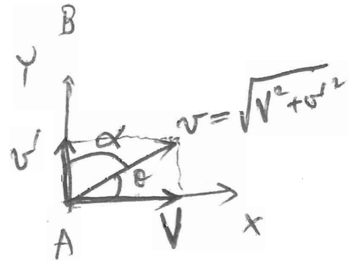
- Velocity of water $\vec{V} = V\hat{i}$
- Velocity of boat wrt water $\vec{v}' = v'\hat{j}$ (across river)
- Velocity of boat wrt ground $\boxed{\vec{v} = \vec{v}' + \vec{V}}$

$$= v'\hat{j} + V\hat{i}$$

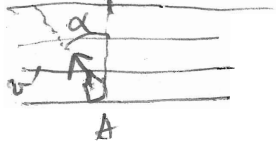
$$= V\hat{i} + v'\hat{j}$$

- $\vec{v} = V\hat{i} + v'\hat{j}$ (Cartesian) $\longrightarrow$ Polar $\begin{cases} v = \sqrt{V^2 + v'^2} \\ \theta = \tan^{-1}\left(\dfrac{v'}{V}\right) \end{cases}$

θ: angle wrt x-axis (2D)



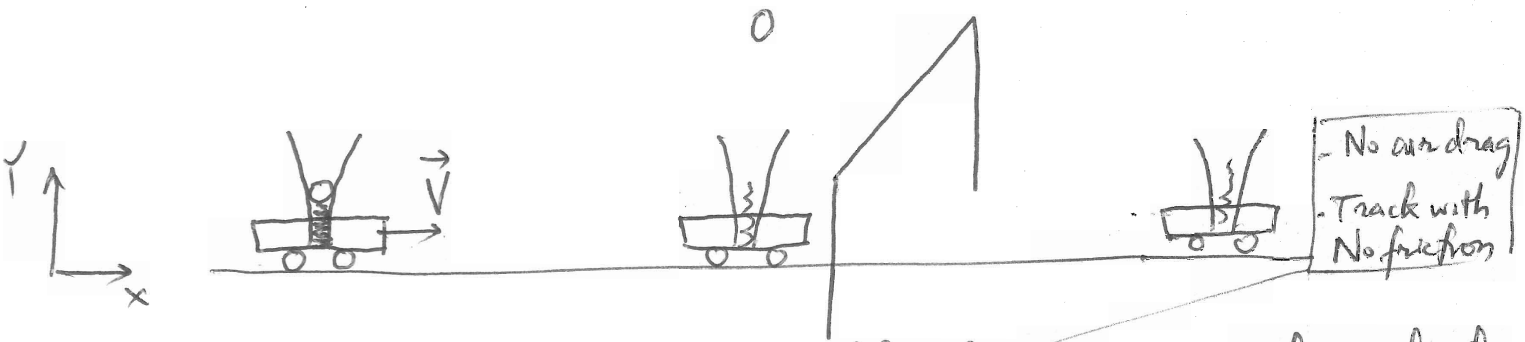
- Because the ~~tot~~ total velocity of boat wrt ground is pointing at an angle. If you started at point A and aimed at B you would end up at C.
- To end up at B you can aim your boat an angle α to the left of AB



Equations of Motion in 2D:

Important assumption for motions in more than one dimension:

Visual experiment #1: (Boston's Museum of Science)



- cart
- compressed spring
- ball
- funnel
- $\vec{V} = V\hat{i}$

- cart moved forward
- spring is released
- ball is launched upward

- cart passed gate
- does ball fall back into funnel or it would fall back behind it

Cart with uniform motion (constant speed): let's focus on motion of ball:

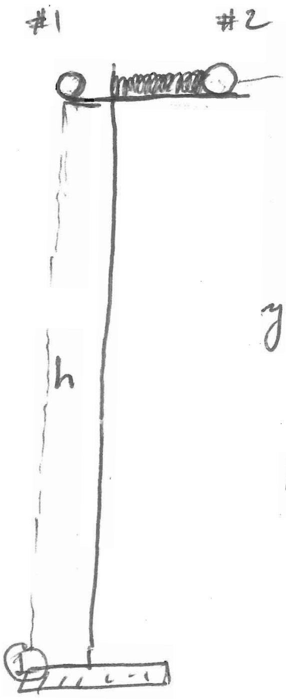
- motion along x-direction: same as that of cart: $v_x = V$
- motion along y-direction by the spring: $v_y = v_{y_0} \mp gt$ (constant acceleration due to gravity)

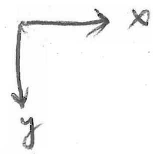
These facts along with the important assumption;

"Motions along perpendicular directions are independent"

allows us to conclude that the ball will fall back into funnel (its motion along x-direction was unchanged due to lack of drags & friction)

Visual experiment #2





Ball #1:

$$v_{x_0} = 0$$
$$v_{y_0} = 0$$
$$a_y = g$$
$$y - y_0 = h$$

$$h = y - y_0 = \cancel{v_{y_0}}^{\,0} \cdot t + \frac{1}{2} g t_1^2$$

Ball #2:

$$v_{x0} = v_0$$
$$v_{y0} = 0$$
$$a_y = g$$
$$y - y_0 = h$$

$$h = y - y_0 = \cancel{v_{y0}}^{\,0} \cdot t + \frac{1}{2} g t_2^2$$

2

Ball #1 will be let go

Ball #2 will be launched horizontally by compressed spring

→ Which will hit the ground first?

They hit ground @ same time: although they have different motions in x-direction, they have identical motion in the y-direction, which is perpendicular to the x-direction. Since motion in perpendicular directions are independent, time to hit ground in y-direction is not affected by different motions in x-direction. $(t_1 = t_2)$

Kinematic Equations for $\boxed{\text{Constant Acceleration}}$

1D

$$v = v_0 + a.t \quad (1)$$
$$x = x_0 + v_0.t + \tfrac{1}{2}at^2 \quad (2)$$

2D

$$\left.\begin{aligned} v_x &= v_{0x} + a_x.t \\ v_y &= v_{0y} + a_y.t \end{aligned}\right\} (1)$$
or $\boxed{\vec{v} = \vec{v}_0 + \vec{a}.t}$

$$\left.\begin{aligned} x &= x_0 + v_{0x}.t + \tfrac{1}{2}a_x t^2 \\ y &= y_0 + v_{0y}.t + \tfrac{1}{2}a_y t^2 \end{aligned}\right\} (2)$$
$\boxed{\vec{r} = \vec{r}_0 + \vec{v}_0.t + \tfrac{1}{2}\vec{a}t^2}$

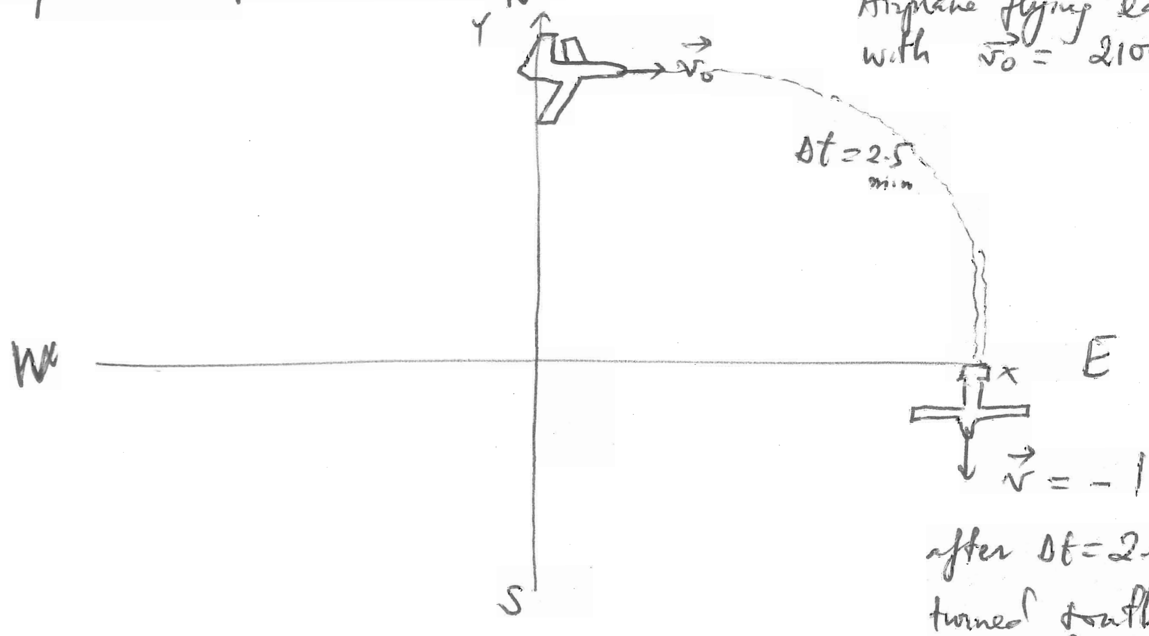
velocity vector $\vec{v} = v_x\hat{i} + v_y\hat{j} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

position vector $\vec{r} = x\hat{i} + y\hat{j}$

$\vec{a} = a_x\hat{i} + a_y\hat{j} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$

$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$

Example: airplane taking a turn (2D)



Airplane flying eastward with $\vec{v}_0 = 2100 \frac{km}{h}\hat{i}$

$\vec{v} = -1800 \frac{km}{h}\hat{j}$

after $\Delta t = 2.5$ min it turned southward at final velocity $\vec{v}$

Average acceleration in this Δt?

SI: $\frac{m}{s^2}$

Conversions:

$$2100\,\frac{km}{h}\cdot\frac{1000\,m}{km}\cdot\frac{h}{3600\,s} = \frac{2100}{3.6} = 583.3\,\frac{m}{s}$$

$$1800\,\frac{km}{h} = \frac{1800}{3.6} = 500\,\frac{m}{s}$$

$$\Delta t = 2.5\,min = 150\,s$$

Average acceleration: $\vec{\bar{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}-\vec{v}_0}{\Delta t} = \frac{\overbrace{(-500\hat{j})}^{\vec{v}} - \overbrace{(583.3\hat{i})}^{\vec{v}_0}}{150}$

$$= -3.3\hat{j} - 3.9\hat{i}\ \frac{m}{s^2}$$

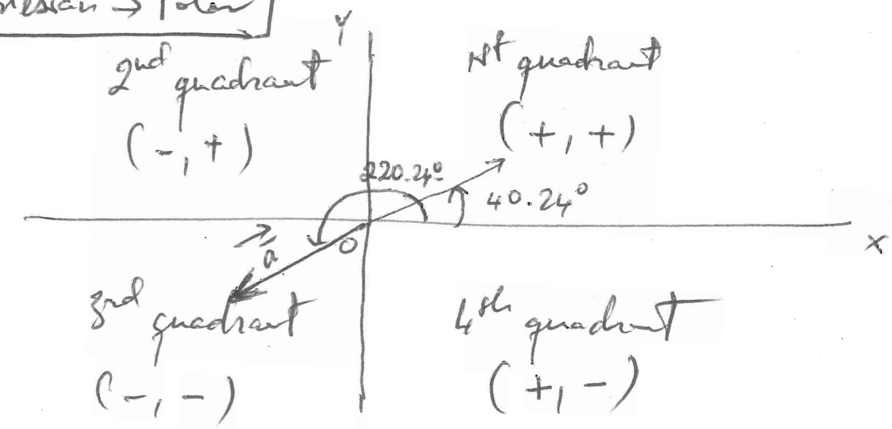
Cartesian Components: $\bar{a}_x = -3.9\,\frac{m}{s^2}$; $\bar{a}_y = -3.3\,\frac{m}{s^2}$

Polar components
$$\begin{cases} \bar{a} = \sqrt{(-3.9)^2 + (-3.3)^2} = \boxed{5.1\,\frac{m}{s^2}} \\ \theta = \tan^{-1}\frac{\bar{a}_y}{\bar{a}_x} = \tan^{-1}\left(\frac{-3.3}{-3.9}\right) = 40.24^\circ \end{cases}$$

Magnitude of the average acceleration

Calculator

Cartesian → Polar



Average acceleration should be in the 3rd quadrant!
$(\vec{a} = (-3.9, -3.3))$

$$\theta = 40.24^\circ + 180^\circ$$

$$\boxed{\theta = 220.24^\circ}$$

(180° was added)

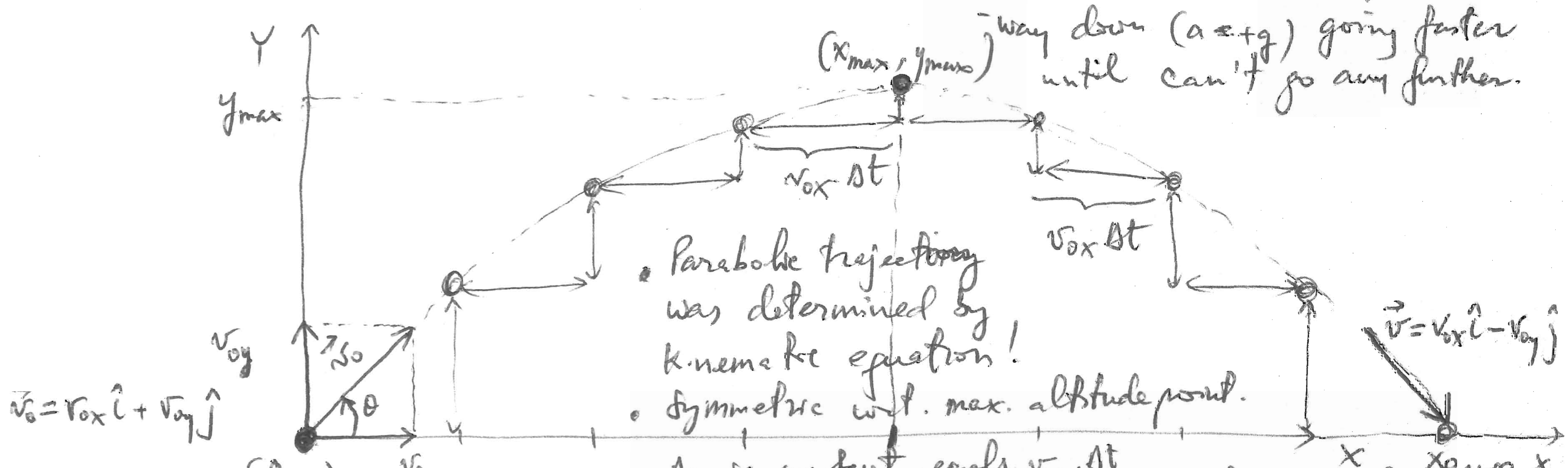
Projectile Motion :

- Constant acceleration ? Yes.
- Motion of objects under effect of gravity : constant downward acceleration.
- <u>Example:</u> baseball, water from a sprinkler, missiles, etc.
- No new physics. Just an application of the kinematic equations in 2D.

<u>Example:</u> Rolling cart under uniform with ball ejected vertically up by a spring. The ball follows a projectile motion.

<u>Parabolic trajectory</u> $\begin{cases} x = \text{uniform motion} \\ y = \text{constant acceleration} \begin{cases} a = -g \\ a = +g \end{cases} \end{cases}$

↳ way up $(a = -g)$ going slower until v_y reaches 0 @ max. altitude point
way down $(a = +g)$ going faster until can't go any further.



- Parabolic trajectory was determined by kinematic equation!
- Symmetric w.r.t. max. altitude point.
- Δx is constant, equals $v_{ox} \cdot \Delta t$
- Δy smaller and smaller on way up, bigger and bigger way down

1) when spring is released & ball starts upward motion with v_{oy}. It already had a uniform motion in x with velocity v_{ox}

2) After Δt : $\begin{cases} x = v_{ox} \cdot \Delta t \\ y = v_{oy} \cdot \Delta t - \frac{1}{2} g \, \Delta t^2 \end{cases}$

After $2\Delta t$: $\begin{cases} x = v_{ox} \cdot 2\Delta t \\ y = v_{oy} \cdot 2\Delta t - 2 g \, \Delta t^2 \end{cases}$

3) Camera snapshots of position of the ball at time intervals Δt, $2\Delta t$, $3\Delta t$, - are shown in the sketch.

Math description (consequence of 2D Kin. equations for constant acceleration!)

$\hookrightarrow \vec{a} = \begin{cases} a_x = 0 \\ a_y = \pm g \end{cases}$ ($-g$ upward; $+g$ downward)

1) $\vec{v} = \vec{v}_0 + \vec{a}.t$ $\begin{cases} v_x = v_{0x} \\ v_y = v_{0y} \mp gt \end{cases}$ ($-$ upward, $+$ downward)

vector equation ⟷ component equation

2) $\vec{r} = \vec{r}_0 + \vec{v}_0.t + \frac{1}{2}\vec{a}t^2$ $\begin{cases} x = v_{0x}.t \\ y = v_{0y}.t \mp \frac{1}{2}gt^2 \end{cases}$ ($-$ upward; $+$ downward)

($\vec{r}_0 = 0$)

θ: (aim angle) is useful in practical applications of projectile motion (trebuchet, catapult, etc.).

How do you insert θ into these equations? $\begin{cases} v_{0x} = v_0 \cdot \cos\theta \\ v_{0y} = v_0 \cdot \sin\theta \end{cases}$

2) $\begin{cases} x = (v_0 \cos\theta).t \longrightarrow t = \frac{x}{v_0 \cos\theta} \\ y = (v_0 \sin\theta).t \mp \frac{1}{2}gt^2 \end{cases}$

$y = (\cancel{v_0}\sin\theta) \cdot \frac{x}{\cancel{v_0}\cos\theta} \mp \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\theta}$

$$\boxed{y = x\tan\theta \mp \frac{g}{2}\frac{x^2}{v_0^2\cos^2\theta}}$$

If θ & v_0 are known, this equation determine a trajectory for x & y → **Trajectory Equation** (no time, only position!)

Maximum Altitude Point

$$(x_{max}, y_{max}) = \left(\frac{v_0^2 \boxed{\sin(2\theta)}}{2g} \checkmark, \frac{v_0^2 \boxed{\sin^2\theta}}{2g} \checkmark \right)$$

Proof:

Eq 1): $v_y = \underbrace{v_0 \sin\theta}_{v_{0y}} - g t$ (upward)

@ y_{max}: $0 = v_{0y} - g t_{max} \rightarrow \boxed{t_{max} = \frac{v_{0y}}{g}}$

Eq 2) $\rightarrow y_{max} = v_0 \sin\theta \underbrace{\frac{v_0 \sin\theta}{g}}_{t_{max}} - \frac{1}{2}\cancel{g} \underbrace{\frac{(v_0 \sin\theta)^2}{g^{\cancel{2}}}}_{t_{max}^2} = \frac{v_0^2 \sin^2\theta}{2g}$

$$x_{max} = v_{0x} \cdot t_{max} = v_0 \cos\theta \cdot \frac{v_0 \sin\theta}{g} = \frac{v_0^2 \cos\theta \sin\theta}{g}$$

$\rightarrow \cos\theta \sin\theta = \frac{1}{2} \sin 2\theta$ (Trigonometry) $\boxed{x_{max} = \frac{v_0^2 \sin 2\theta}{2g}}$

Range: $\boxed{x_{Range} = 2 x_{max} = \frac{v_0^2 \sin 2\theta}{g}}$

(UCM)

- Uniform Circular Motion: circular motion with constant speed (not constant velocity).

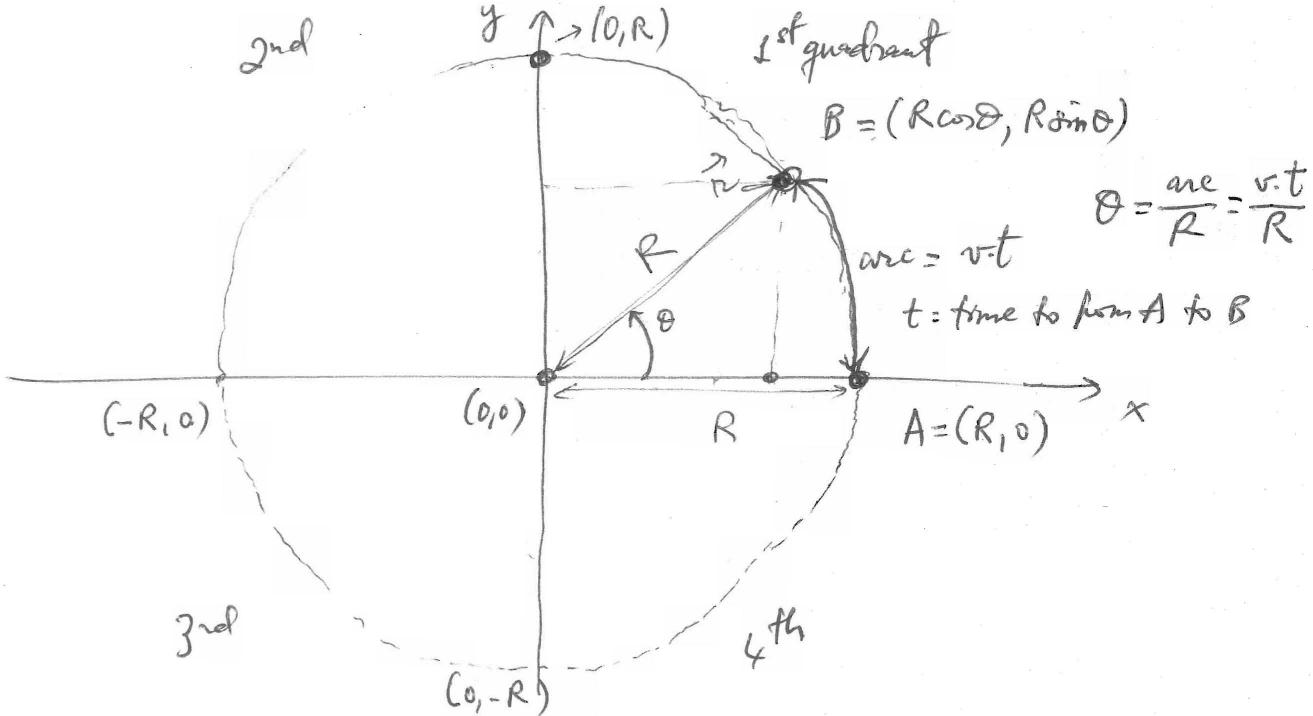
Motion in straight line

In 2D: projectile motion (parabola)
circular motion (when we take turns)

UCM: $\vec{v} = (v, \theta)$

v → speed → constant

θ → angle → changing



- Origin is the center of curvature
- look @ x: $R, R\cos\theta, 0, \cdots -R, \cdots 0, \cdots R$
  y: $0, R\sin\theta, R, \cdots 0, \cdots -R, \cdots 0,$

  } Repeating patterns. Not an accident!

$$\vec{r} = x\hat{i} + y\hat{j} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j} = R\cos\left(\frac{v.t}{R}\right)\hat{i} + R\sin\left(\frac{v.t}{R}\right)\hat{j}$$

UCM: $\vec{r} = R\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + R\sin\left(\frac{v \cdot t}{R}\right)\hat{j}$

$$|\vec{r}| = \sqrt{R^2\cos^2\left(\frac{v \cdot t}{R}\right) + R^2\sin^2\left(\frac{v \cdot t}{R}\right)} = R\underbrace{\sqrt{\cos^2\frac{vt}{R} + \sin^2\frac{vt}{R}}}_{1} = R$$

$$\vec{v} = \frac{d\vec{r}}{dt} = R\left[-\frac{v}{R}\sin\left(\frac{v \cdot t}{R}\right)\hat{i} + \frac{v}{R}\cos\left(\frac{vt}{R}\right)\hat{j}\right] = -v\left[\sin\left(\frac{v \cdot t}{R}\right)\hat{i} - \cos\left(\frac{v \cdot t}{R}\right)\hat{j}\right]$$

$\vec{v}$ is changing with time as expected.

$$\vec{a} = \frac{d\vec{v}}{dt} = -v\left[\frac{v}{R}\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \frac{v}{R}\sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$\boxed{\vec{a} = -\frac{v^2}{R}\underbrace{\left[\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]}_{\text{magnitude} = 1}}$$

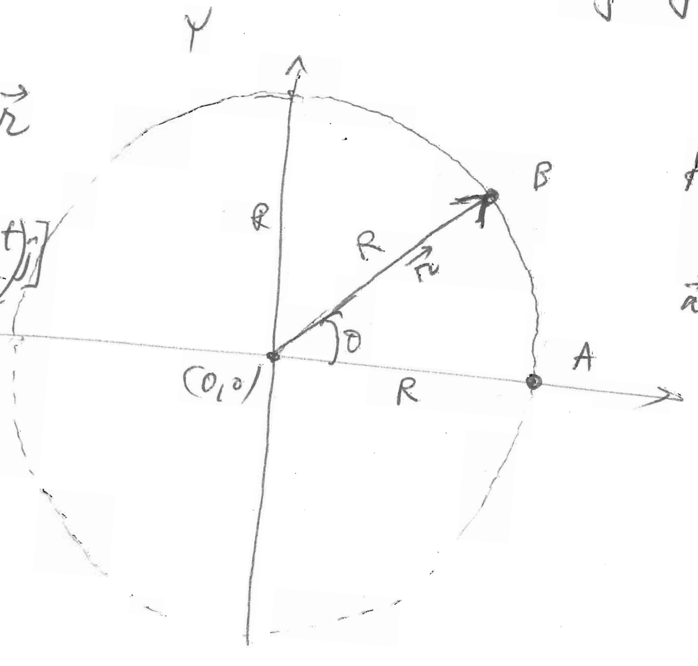
$$\boxed{|\vec{a}| = \frac{v^2}{R}} \quad \boxed{\text{UCM}}$$

Acceleration connected with circular change of direction.

Position vector is $\vec{r}$ (length R)

$\vec{r} = R\underbrace{\left[\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]}_{\text{unit vector}}$

unit vector for position: pointing away from center of curvature (location B)



Acceleration vector is $\vec{a}$ (length $\frac{v^2}{R}$)

$\vec{a} = -\frac{v^2}{R}\underbrace{\left[\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]}_{\text{unit vector}}$

unit vector acceleration:

$-\left[\cos\left(\frac{v \cdot t}{R}\right)\hat{i} + \sin\left(\frac{v \cdot t}{R}\right)\hat{j}\right]$

pointing towards center of curvature.

3.49 | Vector addition in 2D

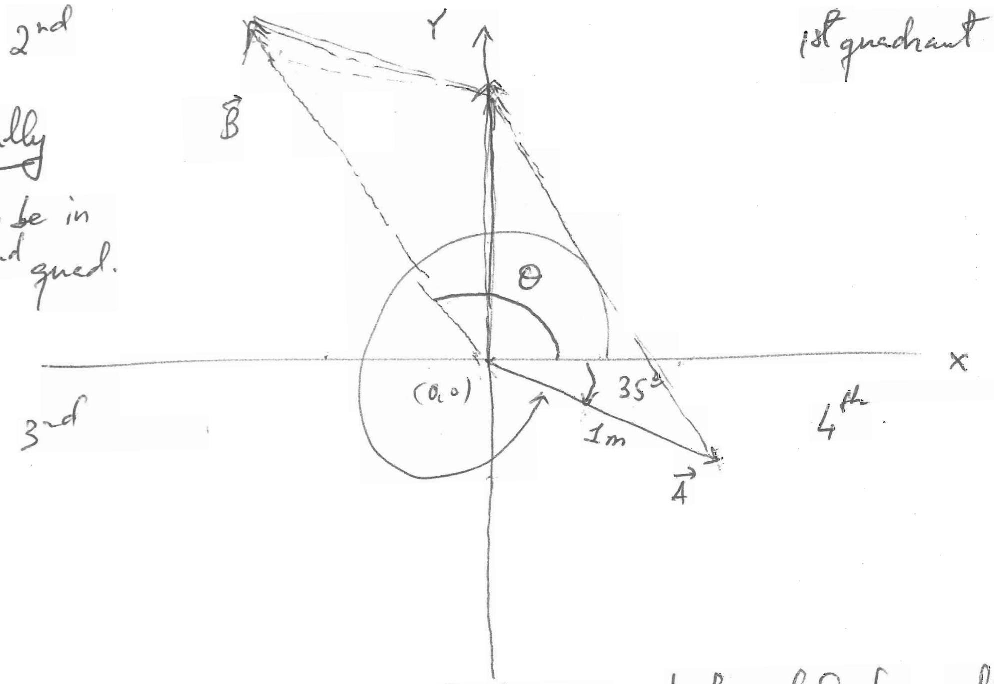
$\vec{A} = (1m, 35^\circ)$ clockwise from x-axis

$\vec{B} = (1.8m, \theta)$

θ? $\vec{A}+\vec{B}$ vertical

Graphically

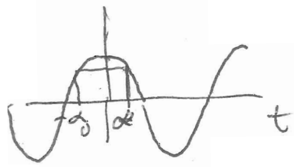
$\vec{B}$ has to be in the 2nd quad.



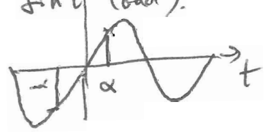
Mathematically: 2D $\left\{ \begin{array}{l} \text{Cartesian : best suited for addition \& subtraction} \\ \text{Polar : " " " multiplication \& division.} \end{array} \right.$

We need to convert ~~cartesian~~ $\rightarrow$ polar. $\rightarrow$ cartesian.

cos t (even)



sin t (odd).



$$\vec{A} = (1, -35^\circ) = (1, 325^\circ) \equiv (1\cos(-35^\circ), 1\sin(-35^\circ))$$
$$= (1 \cdot \cos 35^\circ, -1 \cdot \sin 35^\circ)$$
$$= \cos 35^\circ \hat{i} - \sin 35^\circ \hat{j}$$

$$\vec{B} = (1.8, \theta) = 1.8\cos\theta\,\hat{i} \pm 1.8\sin\theta\,\hat{j}$$

$$\vec{A}+\vec{B} = \underbrace{(\cos 35^\circ \pm 1.8\cos\theta)}_{=0}\hat{i} + (-\sin 35^\circ \pm 1.8\sin\theta)\hat{j}$$

since $\vec{A}+\vec{B}$ needs to be purely vertical.

$$\cos\theta = -\frac{\cos 35^\circ}{1.8} \rightarrow \theta = \cos^{-1}\left[-\frac{\cos 35^\circ}{1.8}\right] = \boxed{\pm 117^\circ}$$

cos is even.

2nd quadrant as from graphical method.

3.60

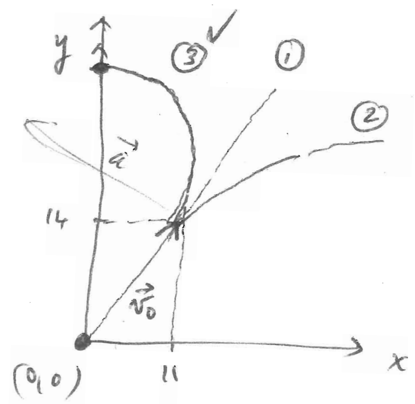
$\vec{v}_0 = 11\hat{i} + 14\hat{j}$ $\frac{m}{s}$ @ $\vec{r} = (0,0) = $ origin

$\boxed{\text{Constant acceleration}}$ $\vec{a} = -1.2\hat{i} + 0.26\hat{j}$ $\frac{m}{s^2}$

a) When does this particle cross y-axis?

t?

Qualitative check: will it cross the y-axis? Yes!



$\vec{a}$ is pointing in the 2nd quadrant $\begin{cases} a_x < 0 \\ a_y > 0 \end{cases}$

Particle trajectory:

$\begin{cases} \text{① No, since } \vec{a} \text{ not parallel to } \vec{v_0} \\ \text{② No, since } \vec{a} \text{ points to 2nd quadr.} \\ \text{③ ✓} \end{cases}$

1) $\vec{v} = \vec{v_0} + \vec{a} \cdot t$

2) $\vec{r} = \vec{r_0} + \vec{v_0} \cdot t + \frac{1}{2}\vec{a} t^2$ $\begin{cases} x: \boxed{0 = 0 + 11 \cdot t - \frac{1.2}{2} t^2} \text{ (I)} \\ y: y = 0 + 14 \cdot t + \frac{0.26}{2} t^2 \text{ (II)} \end{cases}$

$\vec{r} \downarrow$ Final position $(0, y)$; $\vec{r_0} \downarrow (0,0)$

$-0.6t^2 + 11t = 0 \rightarrow -0.6t + 11 = 0 \rightarrow \boxed{t = \frac{11}{0.6} = 18.3\,s}$

Note: by setting $x_0 = y_0 = 0$ you have set also $t=0$ a solution of this quadratic equation.

b) (II) $\rightarrow$ y coord. when it crosses the y-axis:

$$y = 14 \cdot 18.3 + 0.13 \cdot (18.3)^2 = 300\,m$$

c) How fast & in what direction when it crosses the y-axis?

$\vec{v} \rightarrow$ 1) $\begin{cases} x: v_x = 11 - 1.2 \times 18.3 = -10.96 \frac{m}{s} \\ y: v_y = 14 + 0.26 \times 18.3 = 18.8 \frac{m}{s} \end{cases}$ 2nd quadr.

$(v_x, v_y) \rightarrow (v, \theta_v) = \left(\sqrt{10.96^2 + 18.8^2}, \underbrace{\tan^{-1}\left(\frac{18.8}{-10.96}\right)}_{-60°}\right)$ Need to add 180°

$= (21.7 \frac{m}{s}, 120°)$

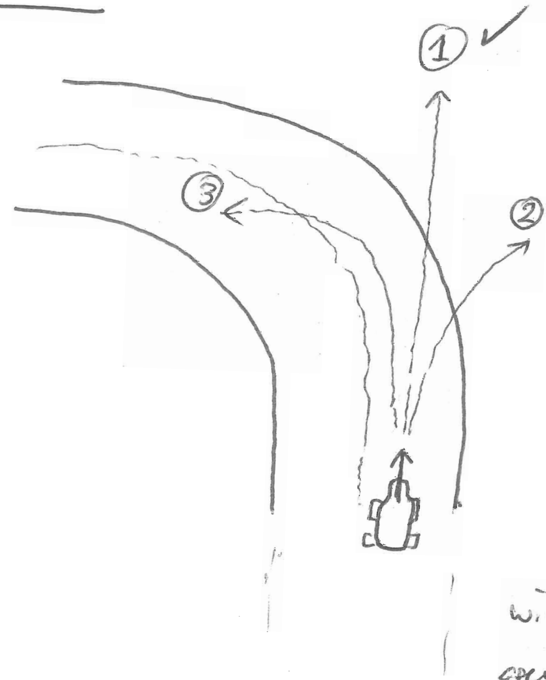
Ch.4 Force & Motion

$\vec{r}, \vec{v}, \vec{a}, t$

$\vec{F} \longleftrightarrow$

Force is the agent that causes the acceleration or the change of motion.

Visual Experiment to introduce the force.



② Driving: taking a turn downhill in icy road.

① : 2

② : 1

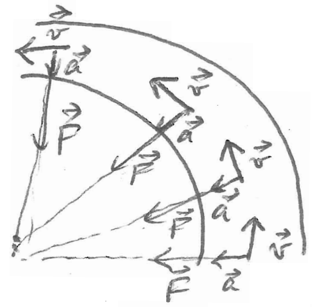
③ : 28

In this situation vehicle will follow path ①: the agent or force that would normally provide the acceleration toward center of curvature is missing: friction is absent in icy road.

Conclusion: vehicle entering a curve in forward direction will continue to do so if there is no force or agent to change its motion. A force is needed to change a motion.

Observations:

1) UCM: speed was constant but velocity was changing as its direction needs to change constantly to conform the circular path.



Force & acceleration both point towards center of curvature.

2) Force will be a vector

3) If several forces are involved, what changes motion is the net force.

1st Newton's Law: a body at rest will continue at rest, a body in uniform motion will continue in uniform motion unless there is a net force acting on the body. Law of inertia

→ Glider on air track

→

2nd Newton's Law: $\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$ $\left\{ \begin{array}{l} \vec{p}: \text{linear momentum} \\ \vec{p} \equiv m\vec{v} \end{array} \right.$

$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt} = \frac{dm}{dt}\cdot\vec{v} + m\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}$$

if $m = \text{Constant} \rightarrow \frac{dm}{dt} = 0 \longrightarrow \boxed{\vec{F}_{net} = m\vec{a}}$

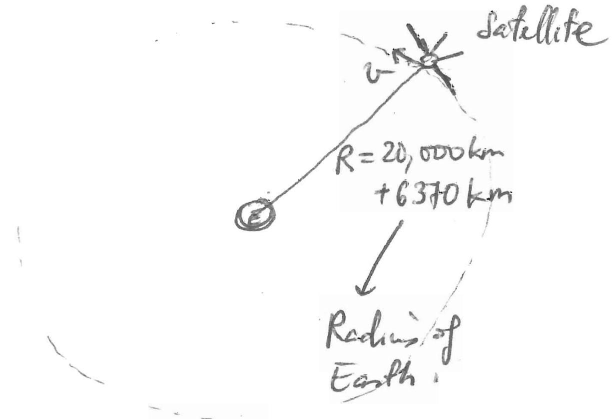
Dimension $[F] = M \cdot \frac{L}{T^2} \rightarrow$ SI units: $kg\frac{m}{s^2} \equiv N(\text{Newton})$

3rd Newton's Law: Law of action & reaction

If A exerts a force on B, B exerts an equal and opposite force on A.

(3.47) Orbital period of GPS satellite : $\begin{cases} \text{UCM: constant speed } v \\ R = 20000\text{ km} \end{cases}$ (33)

T (time to complete one circular orbit)



$$T = \frac{2\pi R}{v}$$

v : in any UCM we need an acceleration towards center of curvature, in this case that acceleration is $g = 0.058 \cdot 9.81$

$$g = \frac{v^2}{R} \rightarrow v = \sqrt{gR}$$

$$T = \frac{2\pi R}{\sqrt{gR}} = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{2.637\times10^7}{0.058\cdot 9.81}}$$

$$= 42774\,s \cdot \frac{1h}{3600s} = 11.88\text{ hrs.}$$

$T \simeq 12$ hrs.

(3.52) $\vec{r} = \underbrace{12t}\,\hat{i} + \underbrace{(15t - 5t^2)}\,\hat{j}$ m $\begin{cases} x = \text{uniform motion} \\ y = \text{constant deceleration} \end{cases}$

a) $\vec{r}(t=2s) = 24\,\hat{i} + 10\,\hat{j}$ (m)

b) Av. velocity b/w $t=0$ & $t=2s$: $\bar{\vec{v}} = \frac{\vec{v}(t=0) + \vec{v}(t=2s)}{2}$

$$= \frac{12\hat{i} + 15\hat{j} + 12\hat{i} - 5\hat{j}}{2}$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = 12\,\hat{i} + (15 - 10t)\,\hat{j}}$$

$$= \frac{24\hat{i} + 10\hat{j}}{2} = \boxed{12\hat{i} + 5\hat{j}}\ \frac{m}{s}$$

c) $\vec{v}(t=2s) = 12\hat{i} - 5\hat{j}$ $\frac{m}{s}$

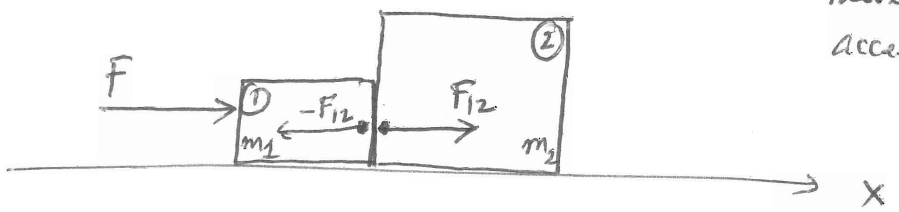
F_{12} : interaction b/w (1) & (2) is internal (F_{12} on (2), $-F_{12}$ on (1)). Externally they combine to 0

Ch4 Force & Motion (Cont.)

1) Two boxes on a horizontal surface (No friction): Force F is applied on box ① causing system to move in +x with acceleration a.

$\boxed{\text{Net force on each component of a system}}$

$\xrightarrow{a}$



a) What force is applied on box ② ?

Observation: a as related to F:

2^{nd} Newton's law $F = (m_1 + m_2) \cdot a$

→ If F is also acting on m_2: $F = m_2 \cdot a$ → Not possible

→ Box ② moves due to $\boxed{F_{12}}$ (force applied by ① on ②):

2^{nd} Newton's law $F_{12} = m_2 \cdot a$

3^{rd} Newton's law: ② applies an equal but opposite force on ①: $-F_{12}$

Focus on Box ②: $F_{net_{②}} = F_{12} = m_2 \cdot a$

Focus on Box ①: $F_{net_{①}} = F - F_{12} = m_1 \cdot a \rightarrow F_{12} = F - m_1 a$

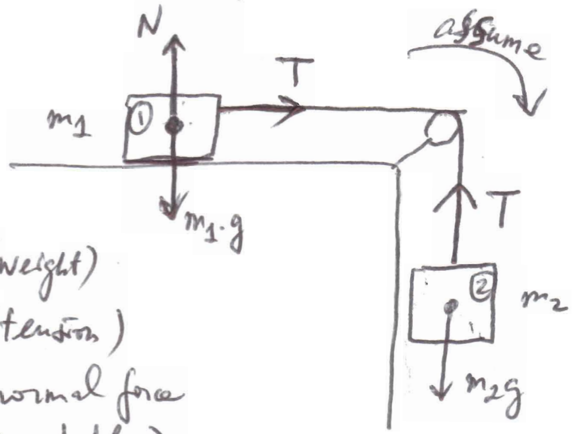
b) Observations:

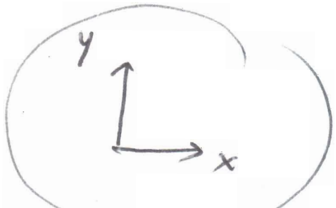
Total force on system	Net force on ①	Net force on ②
F	$F - F_{12}$	F_{12}
external to system of 2 boxes.	internal to system	internal to system.

F_{12}: interaction b/w ① & ② is internal (F_{12} on ②, $-F_{12}$ on ①). Externally they combine to 0

2) Two boxes connected via a $\boxed{\text{mass-less string}}$, no friction:

$\boxed{\text{Net force on each component}}$





On Box① : $m_1 g$ (weight)
T (tension)
N (normal force by table)

can find net force in each direction x & y → can write 2nd Newton's law → get a → get $\vec{v}$ & $\vec{r}$.

On Box② : $m_2 g$
T (tension)

Note $T_1 = T_2 = T$ (massless string)

Box①:

$$\vec{F}_{net_1} = T\hat{i} + \underbrace{(N - m_1 g)}_{0}\hat{j}$$

Box②

$$\vec{F}_{net_2} = (T - m_2 g)\hat{j}$$

2nd Newton: $\vec{F}_{net_1} = m_1 \vec{a}_1$

$T\hat{i} = m_1 a \hat{i}$

$\boxed{T = m_1 a}$ ①

2nd Newton: $\vec{F}_{net_2} = m_2 \vec{a}_2$

$(T - m_2 g)\hat{j} = m_2 (-a\hat{j})$

($a_1 = a_2 = a$ since the boxes are connected)

→ $\boxed{T - m_2 g = -m_2 a}$ ②

These two equation complete the description for these two boxes.
→ can solve for any situation.

Example: m_1 & m_2 are given → ask for a & T;

Plug ① into ② $m_1 a - m_2 g = -m_2 a \rightarrow (m_1 + m_2) a = m_2 g$

→ $\boxed{a = \frac{m_2}{m_1 + m_2} g}$ → Then $\boxed{T = m_1 a = \frac{m_1 m_2}{m_1 + m_2} g}$

$$a = \frac{m_2}{m_1 + m_2} g$$

Observations: 1) If m_2 is doubled $a' = \dfrac{2m_2}{m_1 + 2m_2} g \cancel{>} a$

2) If both masses are doubled $a' = \dfrac{\cancel{2}m_2}{\cancel{2}m_1 + \cancel{2}m_2} g = a$

$\boxed{\text{Spring force}}$: Hooke's Law:

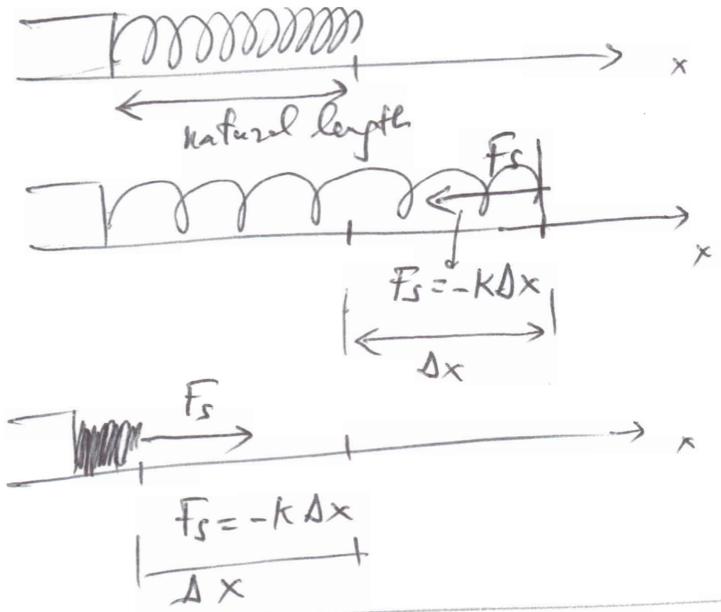
resistance to change

$$F_s = -k \Delta x$$

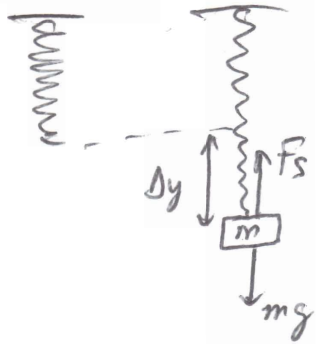
F_s → Force by spring

Δx: stretch from its natural length (or compression)

k: spring constant (SI: $\frac{N}{m}$)



Spring scale:

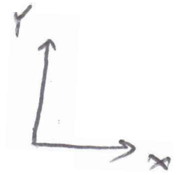


On mass m:

$$F_{net} = mg - k\Delta y = 0$$

$$\rightarrow \boxed{\Delta y = \frac{mg}{k}}$$

3-34 Boat to cross a river { Water flow: x ; Across river: y }



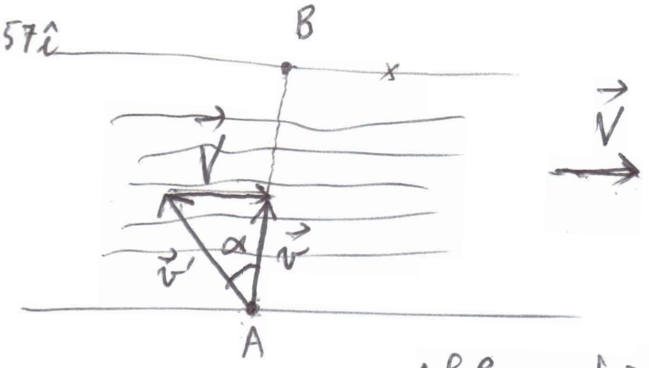
AB = 63 m

$\boxed{\vec{v} = \vec{v}' + \vec{V}}$

1) Velocity of water $\vec{V} = 0.57\hat{i}$

2) Velocity boat wrt water $\vec{v}'$ ($v' = 1.3$ m/s; $\theta_{v'}$?)

3) Velocity of boat wrt ground: $\vec{v} = v\hat{j}$

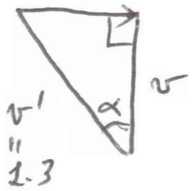


A & B are fixed points on river banks.

$$v\hat{j} = \vec{v}' + 0.57\,\hat{i}$$

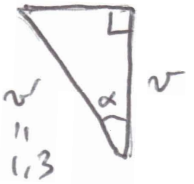
a) In what direction should you head so a boat starting from A will arrive at B. We need to bend to the left of AB (y-direction). How much will depend on how large is the velocity of water.

Method #1 $V = 0.57$



$$\sin\alpha = \frac{V}{v'} \rightarrow \alpha = \sin^{-1}\left(\frac{0.57}{1.3}\right) = 26^\circ$$

Method #2: $V = 0.57$



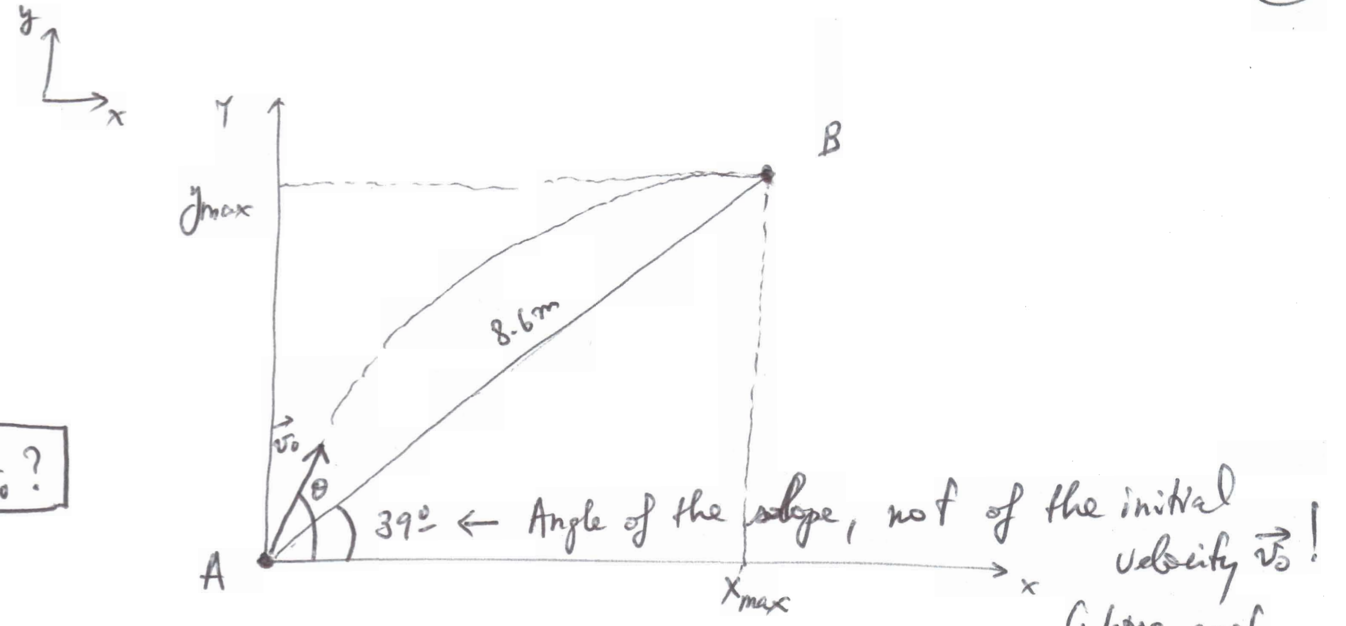
$$\tan\alpha = \frac{V}{v} \rightarrow \alpha = \tan^{-1}\frac{V}{v} = \tan^{-1}\left(\frac{0.57}{1.17}\right) = 26^\circ$$

Pythagoras Th: $v'^2 = v^2 + V^2$

$$v = \sqrt{v'^2 - V^2} = \sqrt{1.3^2 - 0.57^2} = 1.17 \text{ m/s}$$

b) How long would it take you to cross the river = $t = \dfrac{63\text{m}}{1.17\frac{\text{m}}{\text{s}}} = 53.9\text{s}.$

3.66



$\boxed{\vec{v}_0 ?}$

- Final velocity of chocolate bar @ B is horizontal → it will follow 1st half of the parabolic projectile motion.

$$\begin{cases} 1) & v_y = 0 \text{ @ B} \\ 2) & \text{B is the max. altitude point} \end{cases} \begin{cases} x_{max} = 8.6\cos 39° = 6.68m \\ y_{max} = 8.6 \sin 39° = 5.4m \end{cases}$$

Method #1

$$x_{max} = \frac{v_0^2 \sin 2\theta}{2g} = 6.68m$$

$$y_{max} = \frac{v_0^2 \sin^2\theta}{2g} = 5.4m$$

Two equations with two unknowns v_0 & θ.

Method #2: or instead using kinematic equations

Eq#3

$$\begin{cases} 3a) & \frac{v_x^2 - v_{0x}^2}{x - x_0} = 2a_x \\ 3b) & \frac{v_y^2 - v_{0y}^2}{y - y_0} = 2a_y \end{cases}$$

$a_x = 0$

$a_y = -g$ (upward half)

Final velocities are @ B

Initial velocities are @ A

3b) $0 - v_{0y}^2 = 2a_y \underbrace{(y - y_0)}_{y_{max}} \Rightarrow -2g \cdot y_{max} \rightarrow v_{0y} = \sqrt{2gy_{max}} = \sqrt{2 \cdot 9.81 \cdot 5.4} = 10.3 \, m/s$

Now find $v_{ox} = v_x = \frac{x-x_o}{t} = \frac{x_{max}}{t} = \frac{6.68m}{\frac{10.3}{9.81}s} = 6.36\ m/s$

$\downarrow$

$a_x = 0$

$t = ?$ look @ motion along y-direction:

Eq 1) $\underbrace{v_y}_{B} = \underbrace{v_{oy}}_{A} - g \cdot t \rightarrow 0 = v_{oy} - g \cdot t$

$t = \frac{v_{oy}}{g} = \frac{10.3}{9.81}$

$\rightarrow \vec{v}_o = 6.36\hat{i} + 10.3\hat{j}\ \frac{m}{s}$ (1st quadrant)

$$\boxed{\vec{v}_o = (12.1,\ \theta = 58.3^\circ)}$$

Method #3: use eqs (1) & (2)

1) $\vec{v} = \vec{v}_0 + \vec{a} \cdot t$ $\begin{cases} v_x = v_{ox} & (a_x = 0) & (1a) \\ v_y = v_{oy} - gt & (a_y = -g) & (1b) \end{cases}$

2) $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2}\vec{a} \cdot t^2$ $\begin{cases} x - x_0 = v_{ox} t & (2a) \\ y - y_0 = v_{oy} t - \frac{1}{2} g t^2 & (2b) \end{cases}$

Goal is to solve for v_{ox} & v_{oy}:

solve for t using (1b) then plug t into (2b):

$t = \frac{v_{oy}}{g} \rightarrow$ (2b): $y_{max} = \frac{v_{oy}^2}{g} - \frac{1}{2} \cancel{g} \frac{v_{oy}^2}{g^{\cancel{2}}} = \frac{v_{oy}^2}{2g}$

$\rightarrow v_{oy} = \sqrt{2g\, y_{max}}$

$= \sqrt{2 \cdot 9.81 \cdot 5.4} = 10.3 \frac{m}{s}$

(2a) $x_{max} = v_{ox} \cdot t \rightarrow v_{ox} = \frac{x_{max}}{t} = \frac{6.68}{\frac{v_{oy}}{g}} = \frac{6.68}{\frac{10.3}{9.81}}$

$= 6.36\ m/s.$

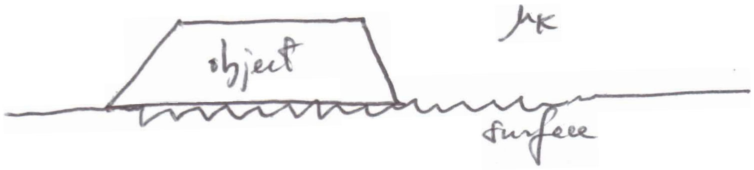
$\rightarrow \vec{v}_o = 6.36\hat{i} + 10.3\hat{j} = \left(\sqrt{6.36^2 + 10.3^2},\ \tan^{-1}\frac{10.3}{6.36}\right)$

$= (12.1\ \frac{m}{s},\ 58.3^\circ)$

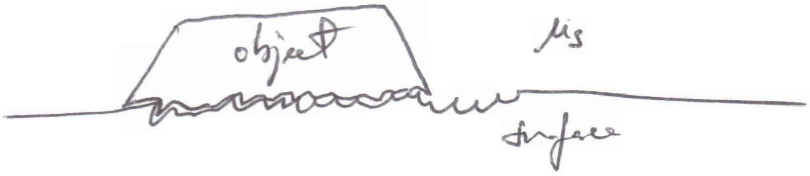
Friction force: present when an object is in contact with a surface

- Static: in contact with surface, not moving:
  $F_s = \mu_s N$ $\begin{cases} \mu_s: \text{coefficient of static friction} \\ N: \text{normal force by surface on object.} \end{cases}$
- Kinetic: moving
  $F_k = \mu_K N$ $\begin{cases} \mu_K = \text{coefficients of kinetic friction} \\ N: \text{normal force.} \end{cases}$

Microscopically, surface is rough:



object $\begin{cases} \text{moving or} \\ \text{recently placed on surface.} \end{cases}$



object static on surface.

$\mu_K < \mu_s$

3.29 Car eastward then at constant speed v_0 it turns southward. Direction of average acceleration?

$$\overline{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{\Delta t}$$

North y

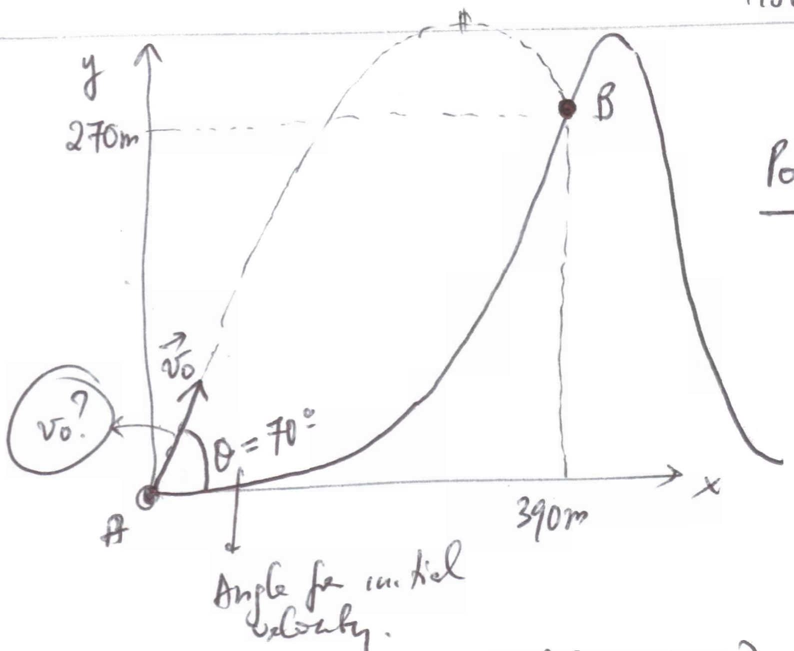
x East

$$\left\{ \begin{array}{l} \vec{v}_0 = v_0 \hat{i} \\ \vec{v} = -v_0 \hat{j} \end{array} \right\} \rightarrow \overline{\vec{a}} = \frac{-v_0 \hat{j} - v_0 \hat{i}}{\Delta t} = \underbrace{-\frac{v_0}{\Delta t}\hat{i} - \frac{v_0}{\Delta t}\hat{j}}_{3^{rd}\ \text{quadrant}}$$

$$\theta_a = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\left(\frac{-\frac{v_0}{\Delta t}}{-\frac{v_0}{\Delta t}}\right) = \tan^{-1} 1$$

$$= 45° \xrightarrow{+180°} \boxed{225°}$$

3.73



Point B: $(x = 390, y = 270)$

↳ Belongs to the parabolic trajectory of projectile.

Trajectory equation (projectile motion): $y = x \cdot \tan\theta - \frac{g}{2}\frac{x^2}{v_0^2 \cdot \cos^2\theta}$

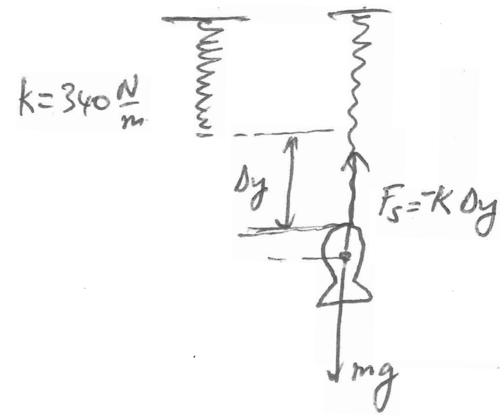
→ Solve for v_0: → $v_0^2 = \frac{g}{2}\frac{x^2}{(x\tan\theta - y)\cos^2\theta}$

$$v_0 = \sqrt{\frac{9.81}{2}\frac{390^2}{(390\tan 70° - 270)\cos^2 70°}} = 89.2\ \frac{m}{s}$$

$N - mg$ ...

$N = m(g-a) = 74(9.81-1.7) = 599N$

4.38

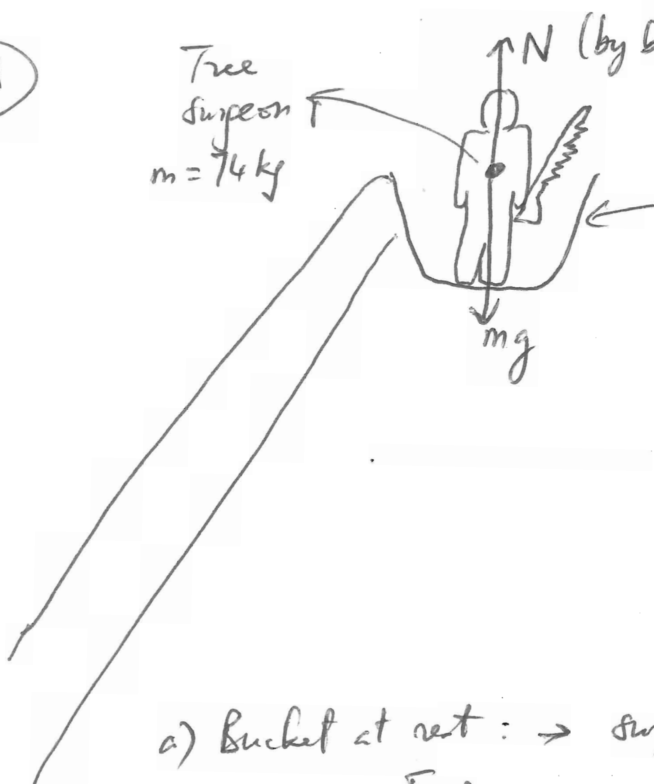


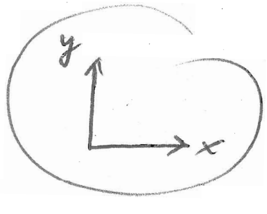
Forces on fish: $F_{net} = m\cdot a = 0$

$$mg - k\Delta y = 0$$

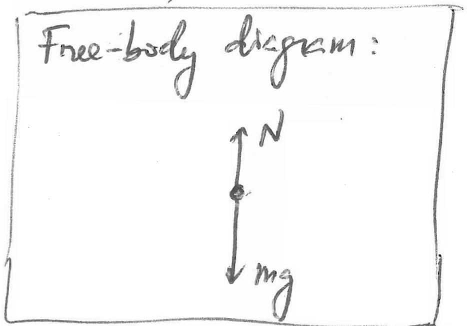
$$\Delta y = \frac{mg}{k} = \frac{6.7\times 9.81}{340} = 0.193\,m$$

4.41





forces by bucket on surgeon:



a) Bucket at rest: → surgeon at rest → $a = 0$

$$F_{net\ on\ surgeon} = N - mg = m\cdot a = 0.$$

$$N = mg = 74\times 9.81 = 725\,N$$

b) Up @ 2.4 m/s → $a=0$ → $N = 725\,N$

c) Down @ 2.4 m/s → $a=0$ → $N = 725\,N$

d) Accelerating up @ 1.7 m/s² $= a$

$$F_{net\ on\ surgeon} = N - mg = ma$$

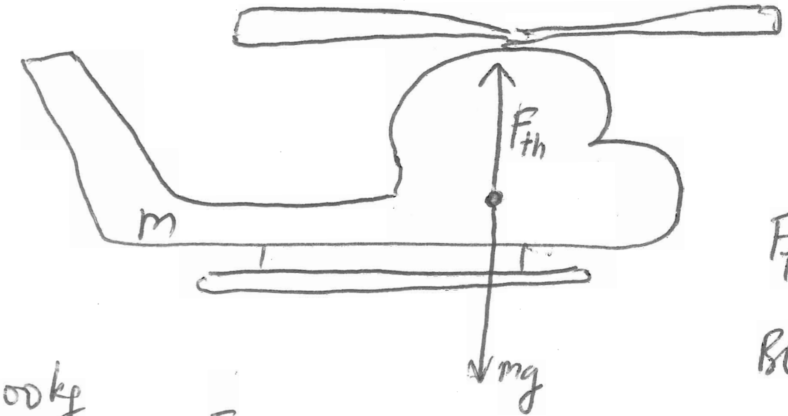
$$N = m(g+a) = 74(9.81+1.7) = 851\,N$$

e) Accelerating down @ 1.7 m/s² $= a$

$$N - mg = -ma \quad \text{or} \quad mg - N = ma$$

$$N = m(g-a) = 74(9.81-1.7) = \boxed{599\,N}$$

4.53

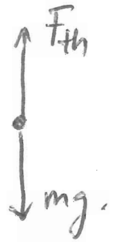


F_{Th} : Thrust force:

Blades apply a force on air (downward) by action & reaction air applies equal & opposite (upward) force on blades. → F_{th}.

$m = 4300\,kg$

FBD:



a) Hovering @ constant altitude

$a = 0$

$F_{net\ on\ helicopter} = F_{th} - mg = ma = 0$

$F_{th} = mg = 4300 \cdot 9.81 = 42\,kN$ (upward)

$F_{on\ air} = 42\,kN$ downward

b) ~~Up @~~ Dropping @ $21\frac{m}{s}$, speed decreasing at $3.2\frac{m}{s^2} = a$

speed decreasing in downward direction ; same as an upward acceleration.

$F_{th} - mg = +ma$

$F_{th} = m(g+a) = 4300\,(3.2 + 9.81)$

$= 55.9\,kN$ upward.

$F_{on\ air} = 55.9\,kN$ downward.

c) Rising @ $17\frac{m}{s}$, speed decreasing at $3.2\,m/s^2$

↳ upward accel. → $a = +3.2\frac{m}{s^2}$

→ $F_{on\ air} = 55.9\,kN$ downward

d) Rising w/o acceleration $\rightarrow a = 0$ $\rightarrow$ same as a)

$F_{on\ air} = 42\,kN$ downward.

e) Rising with speed decreasing = upward deceleration

= downward acceleration

$a = +3.2\,m/s^2$

$$F_{th} - mg = -ma$$

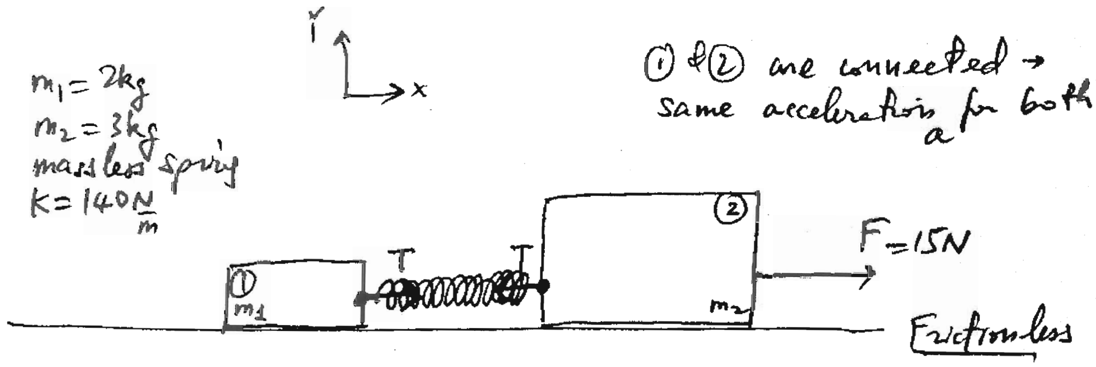
$$F_{th} = m(g-a) = 4300\,(9.81 - 3.2)$$

$$= 28.4\,kN \text{ upward.}$$

$F_{on\ air} = 28.4\,kN$ downward.

4.51

$m_1 = 2kg$
$m_2 = 3kg$
massless spring
$K = 140\frac{N}{m}$



① & ② are connected → same acceleration a for both!

Let's look at forces & motion along x.

→ Let's focus on object #①:

$F_{net①} = m_1 \cdot a$ (2nd Newton's law)

1) $T = m_1 a$

→ Let's focus on object #②:

$F_{net②} = m_2 \cdot a$

2) $F - T = m_2 \cdot a$

To find Δx for the spring I need T (because by 3rd Newton's law: if spring is pulling the object with tension T, object is pulling on the spring with same force in opposite direction!)

Solve for our system of 2 equations and 2 unknowns: (T & a)

1) $T = m_1 a$

2) $F - T = m_2 a$

Eliminate a: $a = \frac{T}{m_1}$ (1) $\rightarrow$ (2) $F - T = \frac{m_2}{m_1} T$

$$T = \frac{F}{1 + \frac{m_2}{m_1}} = \frac{15}{1 + \frac{3}{2}} = 6N$$

$$\boxed{\Delta x = \frac{T}{k} = \frac{6N}{140\frac{N}{m}} = 0.0429\,m}$$