Ch 1 Doing Physics

Dimensional Analysis:

- Dimension of speed: $[v] = \dfrac{[\Delta s]}{[\Delta t]} = \dfrac{L}{T}$ (dimension of length) / (dimension of time)

"Δ" delta = $\underbrace{\text{"change of"}}_{+\text{ or }-}$ or $\underbrace{\text{"increment of"}}_{+}$

Δs : change of position
Δt : increment of time

- Dimension of acceleration: $[a] = \dfrac{[\Delta v]}{[\Delta t]} = \dfrac{\frac{L}{T}}{T} = \dfrac{L}{T^2}$

$[v]^2 = \left(\frac{L}{T}\right)^2$

- Dimension of energy: $[E] = [\frac{1}{2} m v^2] = [m]\,[v^2] = M \dfrac{L^2}{T^2}$

Kinetic energy
- sedan @ 60 mph : lower kinetic energy (smaller mass)
- truck @ 60 mph : higher kinetic energy (larger mass m)

Application of dimensional analysis:

Decide on
$$v = \tfrac{1}{2} g h^2 \rightarrow [\tfrac{1}{2} g h^2] = [g]\cdot[h]^2 = \frac{L}{T^2}\cdot L^2 = \frac{L^3}{T^2}$$
$$v = \sqrt{gh} = (gh)^{\frac{1}{2}} \rightarrow [gh]^{\frac{1}{2}} = ([g]\,[h])^{\frac{1}{2}} = \left(\frac{L}{T^2}\cdot L\right)^{\frac{1}{2}} = \left(\frac{L^2}{T^2}\right)^{\frac{1}{2}} = \frac{L}{T} \checkmark$$

g: acceleration of gravity
h: height (length)

Units:

S.I. (system of international units)

L : m (meter)

T : s (second)

M : kg (kilogram $\Rightarrow$ kilo : $1000 = 10^3$)

Area : m^2

Volume : m^3

L,

Energy : $\frac{kg\, m^2}{s^2} = J$ (Joules)

Unit conversion:

	nano	micro	centi						
	1 nm	1 μm	1 cm	1 mm	1 km	1 light-year	1 mi	1 ft	1 in.
in m	10^{-9} m	10^{-6} m	10^{-2} m	10^{-3} m	10^{3} m	9.46×10^{15} m	1609 m	0.3048 m	2.54 cm
								0.3048m	

$1\, lb = 0.454\, kg$

$1\, h = 3600\, s$; $1\, day = 86\,400\, s$; etc.

$1\, km^2 = 10^6\, m^2$; $1\, cm^2 = 10^{-4}\, m^2$; $1\, mm^2 = 10^{-6}\, m^2$

$1\, km^3 = 10^9\, m^3$; $1\, cm^3 = 10^{-6}\, m^3$; $1\, mm^3 = 10^{-9}\, m^3$

Accuracy & Significant Figures:

- Scientific notation: $\Delta s = 3\,105\,000\,m = \underbrace{\underbrace{3.105}_{\text{Coefficient} < 10} \times \underbrace{10^6}_{\text{power of } 10}}_{\text{scientific notation}}\ m$

(calculator: 3.105E+6)

$\Delta t = 3000s = 3 \times 10^3 s$

speed: $v = \dfrac{3.105 \times 10^6 \ m}{3 \times 10^3 \ s} = \dfrac{3.105}{3} \times 10^{6-3}$

$= 1.035 \times 10^3 \ \frac{m}{s}$

- Accuracy: π : 3.1416 is more accurate than 3.14

\# of decimal digits

Addition & Subtraction: $3.1416 - 2.14 = \begin{cases} \text{Calculator: } 1.0016 \\ \text{You: } 1.00 \end{cases}$

↳ Keep lowest accuracy
(due to limit of precision of measuring instruments in Physics)

Significant figures: $\underbrace{11\,275\,000}_{\text{5 significant figures}}$ (zeroes at the end are not significant) $\underbrace{11\,275\,002}_{\text{8 significant figures.}}$

↳ Multiplication & division: keep smallest number of significant figures (except of numeric constants which are not measured)

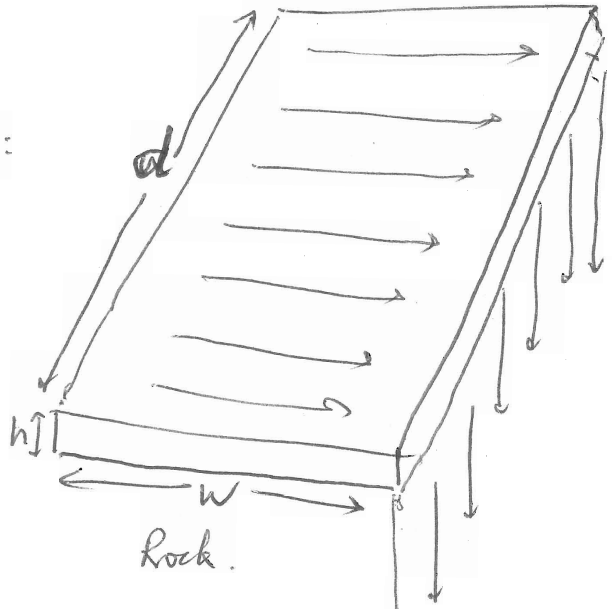
↳ Earth's circumference: $2\pi R_E = 2 \cdot 3.1416 \cdot 6.37 \times 10^6 \ m$
3 s.f's
$= 4.002398 \times 10^7 = 4.00 \times 10^7 m$

1.140 a) Estimate volume of water flowing over Niagara Falls in one second

Guess: $6\times10^4 \ \frac{m^3}{s}$; $10^6 \ \frac{m^3}{s}$;

Estimation: simple calculations using simple geometry & algebra.

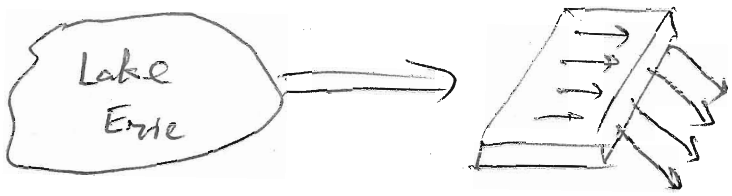
Top of Falls:
Rectangular slab:



Flow rate: $\frac{\text{volume}}{\text{unit time}} = \frac{hwd}{t}$ $\begin{cases} 1)\ \frac{h}{t}\cdot w\cdot d \\ 2)\ h\cdot\frac{w}{t}\cdot d \ \checkmark \\ 3)\ h\cdot w\cdot\frac{d}{t} \end{cases}$ $\frac{w}{t}$ → speed of water toward the Falls

Estimation: $\begin{cases} h \sim 1m \quad 10m \quad 100m \\ \frac{w}{t} \sim 1 m/s \quad 5 m/s \quad 50 m/s \\ d \sim 100m \quad 1000m \quad 10000m \end{cases}$ $1\cdot5\cdot1000 = 5000\frac{m^3}{s}$

b)



If Falls are shut off → water in Lake Erie will rise

Question : how long for it to rise 1 m ?

Info available :

→ Flow rate (Estimate) $= \frac{\text{Vol}}{\text{Time}} = \frac{5000\,m^3}{1\,s}$

→ Volume of water as it rises 1m :

$\text{Area}_{\text{Erie}} \cdot 1m = 75 \cdot 10^3 \times 375 \cdot 10^3 \times 1$

$= 75 \times 375 \times 10^6\, m^3$

$\text{Time} = \frac{\text{Volume}}{\text{Flow rate}} = \frac{\text{Volume}}{\frac{\text{Volume}}{\text{time}}}$

$t = \frac{75 \times \overset{75}{\cancel{375}} \times 10^6}{\cancel{5} \times 10^3}\, s = 75^2 \times 10^3\, s = 9 \times 10^6\, \cancel{s}\; \frac{1\,\text{day}}{84600\, \cancel{s}}$

$= 100$ days

~ 3 months.

Ch2 Motion in a Straight Line:

Average motion:

↳ $\text{speed} = \dfrac{\boxed{\text{distance}}}{\text{time}}$; $\text{velocity} = \dfrac{\boxed{\text{displacement}}}{\text{time}}$

$$W \bullet \xrightarrow{10\,mi} \bullet E$$

Total time: 2hr.

$\xleftarrow{}$ 2mi

$$\text{speed} = \frac{12\,mi}{2\,hr} = 6\,mph$$

$$\text{velocity} = \frac{8\,mi}{2\,hr} = 4\,mph$$

(since it takes into consideration the direction of motion)

Average velocity: $\bar{v} = \dfrac{\Delta x}{\Delta t}$

Δx: change of position or displacement

Δt: time increment

Instantaneous velocity: $v = \underbrace{\lim_{\Delta t \to 0}\frac{\Delta x}{\Delta t}} = \dfrac{dx}{dt}$

(Time derivative of the position x)

Example: $x = a_1 t^4$: instantaneous velocity:

a_1 ↓ constant

$$v = \frac{dx}{dt} = \frac{d\,(a_1 t^4)}{dt} = 4a_1 t^3$$

$$\frac{dt^n}{dt} = n\,t^{n-1}$$

Units
$$\begin{cases} x \to m \\ v \to \dfrac{m}{s} \\ a_1 \to \dfrac{m}{s^4} \end{cases}$$

(Dimensional analysis helps write the units for ~~numeric~~ constants.)

Velocity can change over time $\to$ acceleration

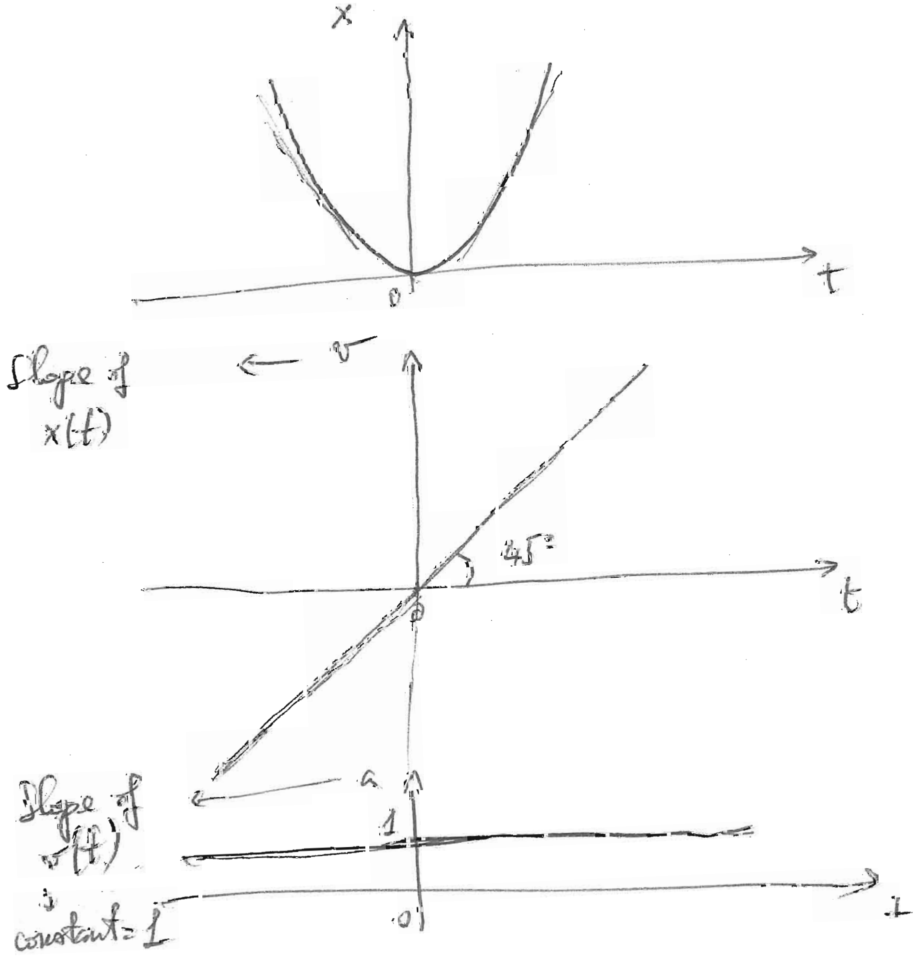
<u>Average acceleration</u>: $\bar{a} = \dfrac{\Delta v}{\Delta t}$ $\begin{cases} \Delta v: \text{change of velocity} \\ \Delta t: \text{change of time} \end{cases}$

<u>Instantaneous acceleration</u>: $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$
(Time derivative of the velocity)

<u>Example</u>: $x = at^4 \to \begin{cases} v = 4at^3 \\ a = 12a_1t^2 \end{cases}$

Units $\begin{cases} a \to \frac{m}{s^2} \\ a_1 \to \frac{m}{s^4} \end{cases}$

<u>Another example</u>: an object whose position as a function of time is given as: $\boxed{x = a_1 t^2}$
$\left(a_1 = \frac{1}{2} \frac{m}{s^2} \right)$



$v = \dfrac{dx}{dt} = 2a_1 t$

$a = \dfrac{dv}{dt} = 2a_1 = 1$

So: objects whose position varies quadratically in time follows a constant acceleration!

Constant acceleration motion in a straight line:

Average acceleration = Instaneous' acceleration

$$\boxed{\bar{a} = a}$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$$

$\begin{cases} v = \text{final velocity} \\ v_0 = \text{initial velocity} \end{cases}$

$\begin{cases} t : \text{final time} \\ 0 : \text{initial time} \end{cases}$

Can conclude: $v - v_0 = a \cdot t \rightarrow \boxed{v = v_0 + a \cdot t}$ (1)

1st Kinematic equation

Derivation of 2nd Kinematic Equation:

Average velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \longrightarrow x = x_0 + \bar{v} \cdot t \quad (A)$$

Mathematically: $\bar{v} = \frac{1}{t-0}\int_0^t v\,dt \overset{(1)}{=} \frac{1}{t}\int_0^t (v_0 + a \cdot t)\,dt$

$$= \frac{v_0}{t}\underbrace{\int_0^t dt}_{[t]_0^t = t} + \frac{a}{t}\underbrace{\int_0^t t\,dt}_{\left[\frac{1}{2}t^2\right]_0^t = \frac{1}{2}t^2}$$

$$= v_0 + \frac{1}{2}at$$

$$= \frac{1}{2}v_0 + \frac{1}{2}a.0 + \frac{1}{2}v_0 + \frac{1}{2}at$$

$$= \frac{1}{2}\left[(v_0 + a.0) + (v_0 + a.t)\right]$$

$$\bar{v} \overset{(1)}{=} \frac{1}{2}[\, v_0 \quad + \quad v \,] \quad (B)$$

Plug (B) into (A):

$$(A) \quad x = x_0 + \bar{v}\cdot t \overset{(B)}{=} x_0 + \underbrace{\tfrac{1}{2}(v_0 + v)}_{\bar{v}}\cdot t$$

$$\overset{(1)}{=} x_0 + \frac{1}{2}\left(v_0 + v_0 + a\cdot t\right)\cdot t$$

$$\boxed{x = x_0 + v_0\cdot t + \frac{1}{2}a\cdot t^2} \quad (2)$$

2nd Kinematic equation
(for constant acceleration)

Comes from
- Definitions: $\bar{v}, v, \bar{a}, a$ $(\bar{a} = a)$
- Constant acceleration
- Math expression for $\bar{v}$ & some calculus.

Constant acceleration motion in 1D:

$$v = v_0 + a\cdot t \quad (1)$$

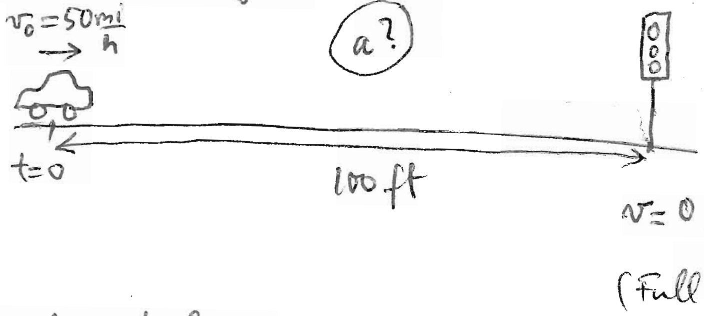
$$x = x_0 + v_0\cdot t + \frac{1}{2}a\cdot t^2 \quad (2)$$

Can derive from (1) & (2): $\quad \dfrac{v^2 - v_0^2}{x - x_0} = 2\cdot a \quad (3)$

Note: a) No time in eq. (3)
b) final quantities: x, v,
Initial quantities: x_0, v_0

2.35 Deceleration : is a negative acceleration

1) Sketch with given information : car in a straight line



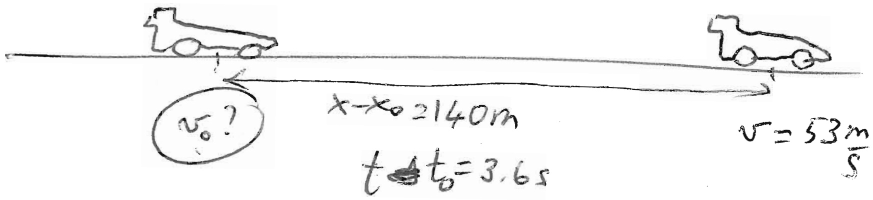
2) Decide which equation (1&2 or 3) to apply:
Since there is no time information let's try (3)

$$\frac{0^2 - 22.35^2}{30.48} = 2 \cdot a \rightarrow a = \frac{-22.35^2}{(30.48) \cdot 2} \simeq \boxed{-8.192\ \frac{m}{s^2}}$$

Unit conversion :
$$50\frac{mi}{h} \cdot \frac{1609m}{mi} \cdot \frac{h}{3600s} = 22.35\frac{m}{s}$$
$$x - x_0 = 100ft \cdot \frac{0.3048m}{ft} = 30.48\ m$$

2.61 Constant acceleration

a) 1) Sketch with given information : racing car in 1D



Eq.1 : $v = v_0 + a.t \rightarrow \boxed{v_0 = v - a.t}$

Eq.2 : $x - x_0 = v_0 . t + \frac{1}{2}at^2$

Alternative#1 : Find a, then v_0 :

↳ eliminate v_0 : $x - x_0 = (v - at).t + \frac{1}{2}at^2$

$x - x_0 = vt - \frac{1}{2}at^2$

$$\rightarrow \boxed{v_0 = v - a.t = 53 - 7.83 \cdot 3.6 = 24.8\frac{m}{s}} \rightarrow a = \frac{x - x_0 - v.t}{-\frac{1}{2}t^2} = \frac{140 - 53 \cdot 3.6}{-\frac{1}{2} \cdot 3.6^2}$$

$a = 7.83\ m/s^2$

Alternative #2: Eliminate a (unknown)

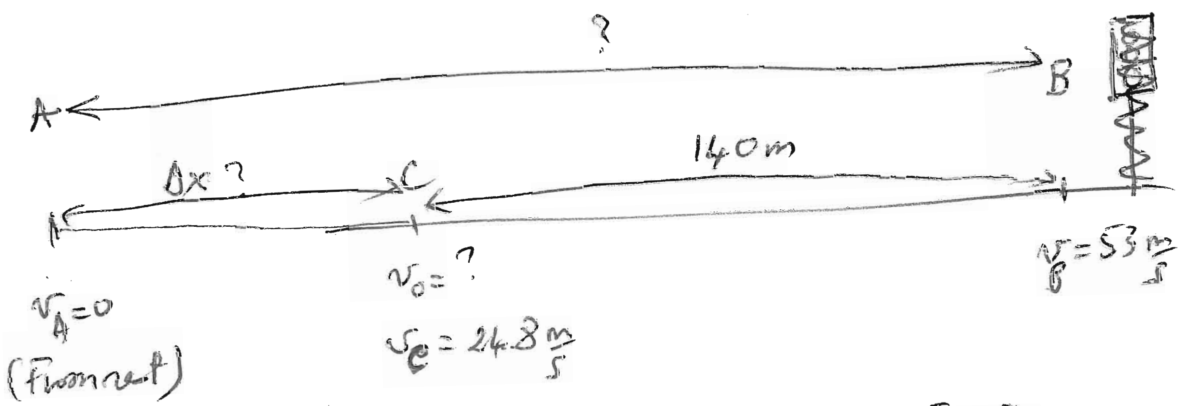
(1) $a = \dfrac{v - v_0}{t}$

(2) $x - x_0 = v_0 t + \frac{1}{2}\left(\frac{v - v_0}{\cancel{t}}\right) \cdot t^{\cancel{2}} = \frac{1}{2} v_0 t + \frac{1}{2} v \cdot t$

$\frac{1}{2} v_0 t = x - x_0 - \frac{1}{2} v \cdot t \rightarrow \boxed{v_0 = \frac{2(x - x_0)}{t} - v}$

$= \dfrac{2 \cdot 140}{3.6} - 53 = \boxed{24.8 \frac{m}{s}}$

b) How far did it travel $\boxed{\text{from rest}}$ to end of 140m distance.



Alternative #1: Need distance AC or $\underline{\Delta x_{AC}}$ given $\begin{cases} v_A = 0 \\ v_c = 24.8 \frac{m}{s} \end{cases}$

Eq(3): $\dfrac{v_C^2 - v_A^2}{\Delta x_{AC}} = 2 \cdot a$ $\begin{cases} a = \text{acceleration, same as} \\ \text{that b/w C \& B} \end{cases}$

From part a) $a = 7.83 \frac{m}{s^2}$

$$\Delta x_{AC} = \frac{24.8^2}{2 \cdot 7.83} = 39.37 \text{ m}.$$

$$\Delta x_{AB} = \Delta x_{AC} + 140 \text{ m} = 179.37 \text{ m}.$$

Alternative #2: Can also apply eq(3) to AB:

$$\frac{v_B^2 - v_A^2}{\Delta x_{AB}} = 2 \cdot a \rightarrow \Delta x_{AB} = \frac{53^2 - 0^2}{2 \cdot 7.83} = 179.37 \text{ m}$$

Alternative #3 :

$\boxed{\text{Focus on AC}}$

Eq (2):

$$\underbrace{(x-x_0)}_{\Delta x_{AC}} = \underbrace{v_0}_{v_A=0}\cdot t + \frac{1}{2}at^2 \rightarrow \Delta x_{AC} = \frac{1}{2}\cdot \underbrace{a}_{7.83\,\frac{m}{s^2}}\, \underbrace{t^2}_{\text{calculate using eq (1)}}$$

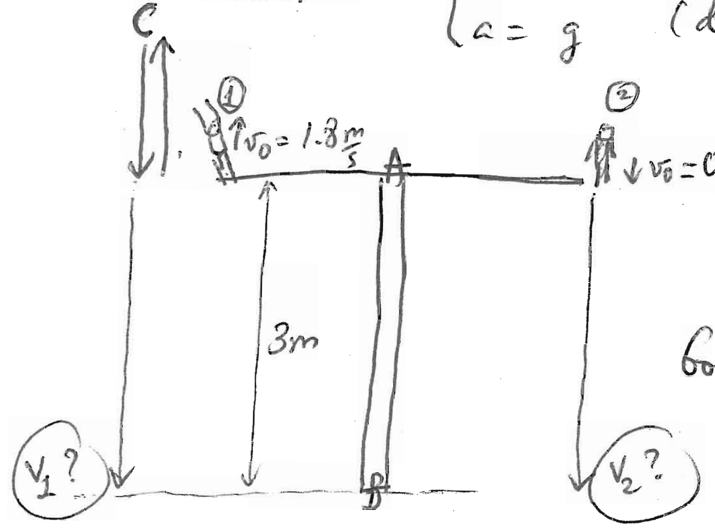
Eq (1):

$$\underbrace{v_C}_{24.8\frac{m}{s}} = \underbrace{v_A}_{0} + \underbrace{a}_{7.83\frac{m}{s^2}}\cdot t \rightarrow t = \frac{v_C}{a} = 3.17\,s.$$

$$\Delta x_{AC} = \frac{1}{2}\, 7.83 \cdot 3.17 = 39.37\ m.$$

$$\Delta x_{AB} = \Delta x_{AC} + 140 = 179.37\ m.$$

(2.69) Constant acceleration $\begin{cases} a = -g & \text{(upward motion)} \\ a = g & \text{(down-ward motion)} \end{cases}$



② steps off when ① passes platform on way down.

Goal: compare motion b/w A & B (3m vertical)

Before calculation: a) diver ① will reach water at higher speed ($v_1 > v_2$) since he starts @ 0 speed @ C so has more time to reach water.

b) If we look b/w A & B diver ① starts with $v_0 = 1.8\,m/s$ (on way down) while diver ② starts with $v_0 = 0$

a) Driver ①

Eq 3: $\dfrac{v_{1B}^2 - v_{1A}^2}{3} = 2 \cdot g$

$v_{1B} = \sqrt{2 \cdot 9.81 \cdot 3 + 1.8^2}$

$= 7.88 \frac{m}{s}$

Driver ②

$\dfrac{v_{2B}^2 - \cancel{v_{2A}^2}^{\,0}}{3} = 2 \cdot g$

$v_{2B} = \sqrt{2 \cdot 9.81 \cdot 3}$

$= 7.67 \frac{m}{s}$

b) Which hits the water first and by how much?
Both start @ A downward @ same time.

Driver ①

Eq 1: $v_{1B} = v_{1A} + g t_1$

$t_1 = \dfrac{v_{1B} - v_{1A}}{g}$

$t_1 = \dfrac{7.88 - 1.80}{9.81}$

$= 0.62\ s$

Driver ②

$v_{2B} = v_{2A} + g\, t_2$

$t_2 = \dfrac{v_{2B} - 0}{g}$

$t_2 = \dfrac{7.67}{9.81}$

$= 0.78\ s.$

① hits water $\sim 0.16\ s$ before ②.

If we start counting time when diver #1 jumps up = Need to add $t_{1\,up\,\&\,down}$ for diver ①

$t_{1\,up}$:

C $v_{1C} = 0$

A $v_{1A} = 1.8\ m/s$

$\gg$ Eq (1): $\cancel{v_{1C}}^{\,0} = v_{1A} - g \cdot t_{1up}$

$t_{1up} = \dfrac{v_{1A}}{g} = \dfrac{1.8}{9.81}$

$= 0.18\ s$

$t_{1\,up\,\&\,down} = 2 \times t_{1up} = 0.36\ s.$

$\rightarrow t_1 = 0.62 + 0.36 = 0.98\ s > t_2 = 0.78\ s.$

Ch3: Motion in Two & Three Dimensions

1D: We Describe motion using: position (x), velocity (v), acceleration (a)

1D	2D	3D
x	$\vec{r} = (x, y) = (r, \theta)$	$(x, y, z) = (r, \theta, \varphi)$
v	$\vec{v} = (v_x, v_y) = (v, \theta_v)$	$(v_x, v_y, v_z) = (v, \theta_v, \varphi_v)$
a	$\vec{a} = (a_x, a_y) = (a, \theta_a)$	$(a_x, a_y, a_z) = (a, \theta_a, \varphi_a)$

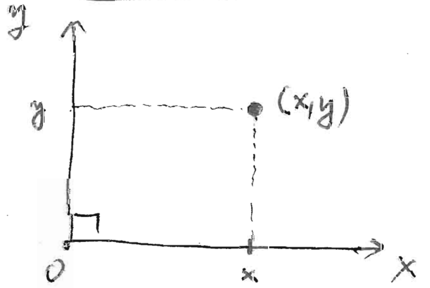
vectors { Length or magnitude; Direction

$\vec{r}$ = position vector $= (x, y)$ (Cartesian Coordinates) $= (r, \theta)$ (polar coordinates)

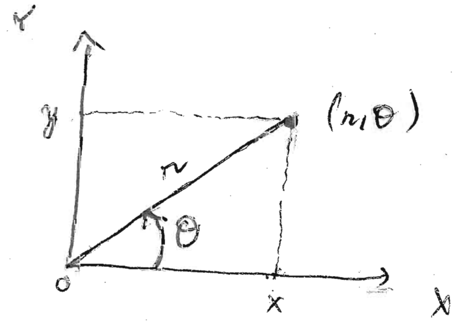
$\vec{v}$: velocity vector $= (v_x, v_y) = (v, \theta_v)$

$\vec{a}$: acceleration vector $= (a_x, a_y) = (a, \theta_a)$

Cartesian



Polar



(Coordinates are the projections of the position on to the Cartesian axes)

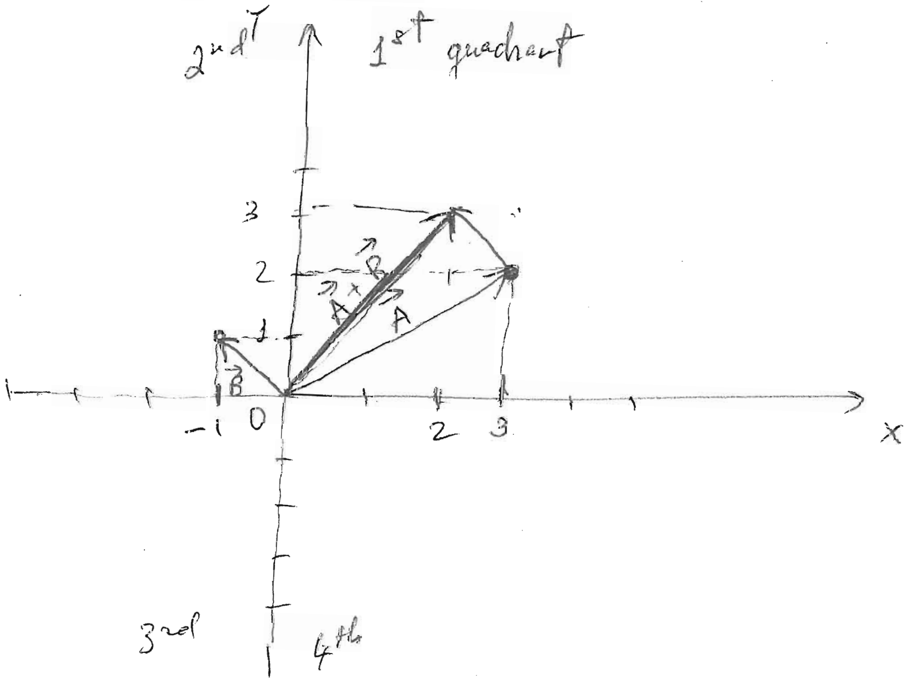
Cartesian $\longrightarrow$ Polar: $\begin{cases} r = \sqrt{x^2 + y^2} & \text{Pythagoras Theorem} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{cases}$

Polar $\longrightarrow$ Cartesian $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Vector operation : addition & subtraction

- → Graphical
- → Mathematical (using unit vectors) — length is 1

Graphical add & subtract:



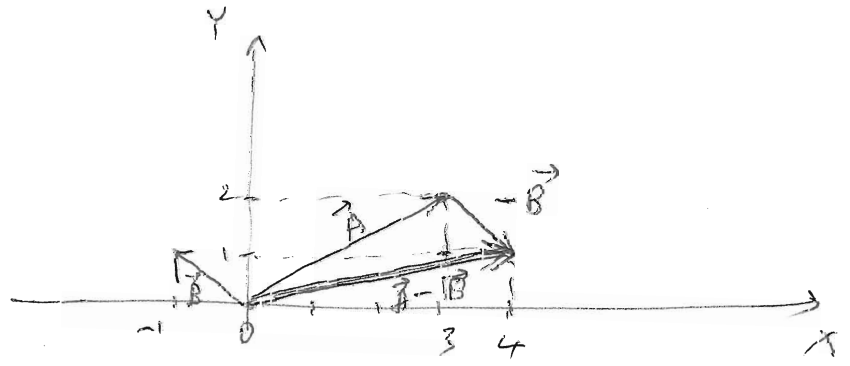
$\vec{A} = (3, 2)$

$\vec{B} = (-1, 1)$

$\vec{A}+\vec{B}$: 1) Draw a copy of $\vec{B}$ from tip of $\vec{A}$

2) $\vec{A}+\vec{B}$ is from origin of $\vec{A}$ to the tip of the copy of $\vec{B}$

$\vec{A}+\vec{B} = (2, 3)$



$\vec{A}-\vec{B}$: similar 2-step process except reversing direction of $\vec{B}$

$\vec{A}-\vec{B} = (4, 1)$

Math method:

using unit vectors: Length 1; x-axis → $\hat{i}$; y-axis → $\hat{j}$; z-axis → $\hat{k}$ — used to describe directions

$\vec{A} = 3\hat{i} + 2\hat{j} = A_x\hat{i} + A_y\hat{j}$ $(A_x = 3;\ A_y = 2)$

$\vec{B} = -\hat{i} + \hat{j} = B_x\hat{i} + B_y\hat{j}$ $(B_x = -1;\ B_y = 1)$

$\vec{A}+\vec{B} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} = 2\hat{i} + 3\hat{j}$

$\vec{A}-\vec{B} = (A_x-B_x)\hat{i} + (A_y-B_y)\hat{j} = 4\hat{i} + \hat{j}$