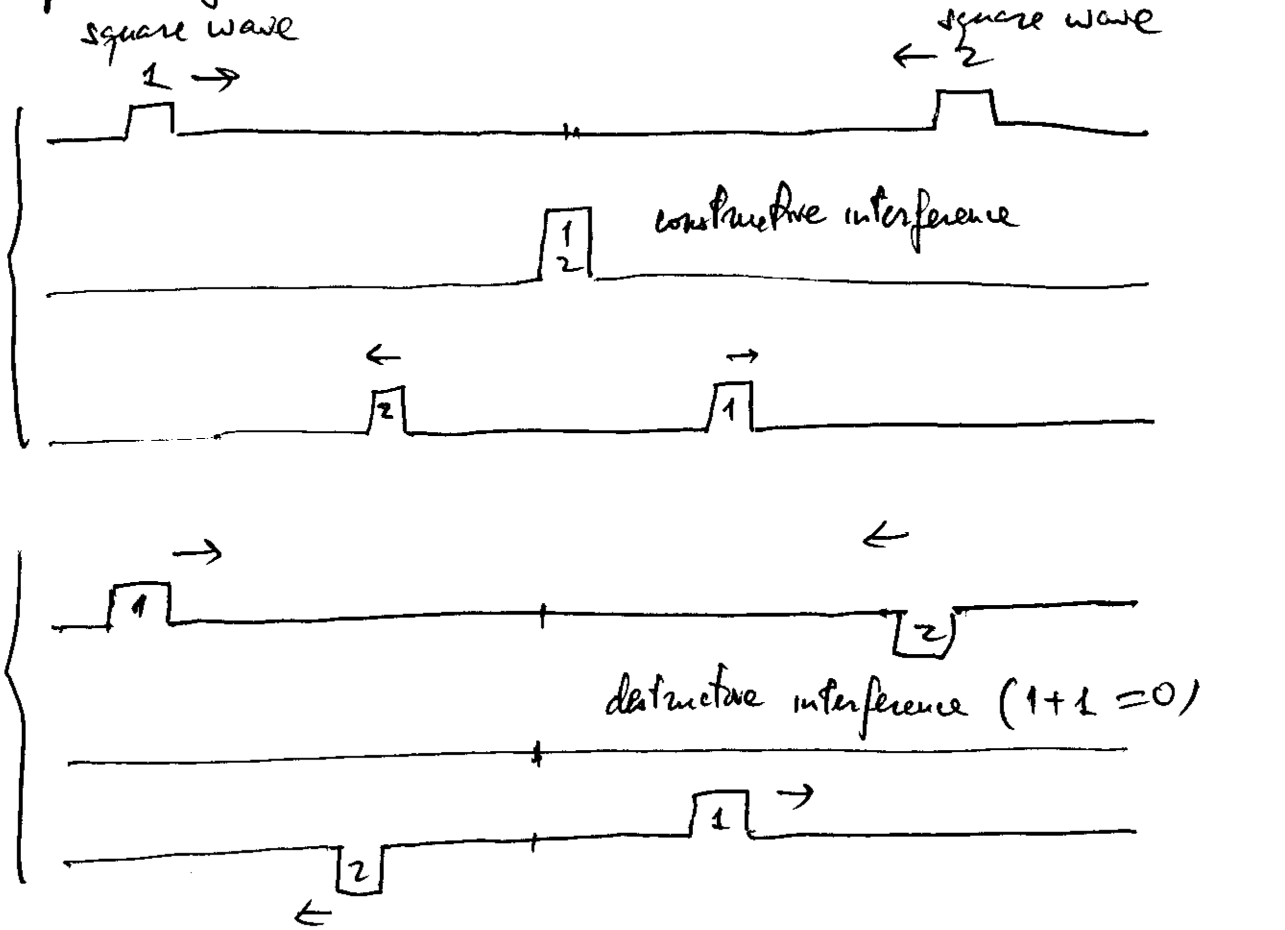
Ch 14 Wave Motion (Cont.)

Beat Phenomenon & Standing Waves & Doppler Effect

Superposition of Waves: constructive & destructive interference



Beat Phenomenon: (superposition of 2 sound waves with slightly different frequencies)

- Physical observations: a) Twin engine airplanes (propellers): should go at equal angular frequencies, however there is normally a small difference in frequencies. We can't hear the oscillations from either engine but we can hear the beating between the two.
  b) Old fashion way of tuning string instruments
- Math description

↳ Combine 2 transverse waves: $y(x,t) = A\sin(kx - \omega t)$

2 waves @ $x=0$ with frequencies ω_1 & ω_2

$$\begin{cases} y_1(0,t) = A \sin(-\omega_1 t) \\ y_2(0,t) = A \sin(-\omega_2 t) \end{cases}$$

For twin-engine & string tuning $x=0$ is location of our ear drum.

↓

Superposition of these 2 waves @ $x=0$:

$$y_1(0,t) + y_2(0,t) = -A\left[\sin\omega_1 t + \sin\omega_2 t\right]$$

$$\boxed{\text{Trigonometry Identity:} \quad \sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$= -2A \underbrace{\sin\left(\frac{\omega_1+\omega_2}{2}\right)t}_{\text{high frequency osc.}} \cdot \underbrace{\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)}_{\text{low frequency modulation}}$$

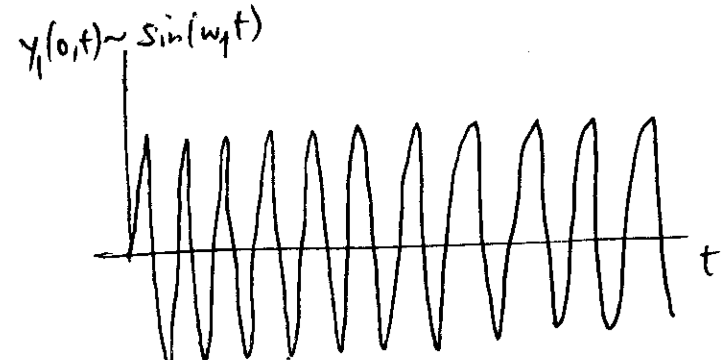
When ω_1 & ω_2 are very similar

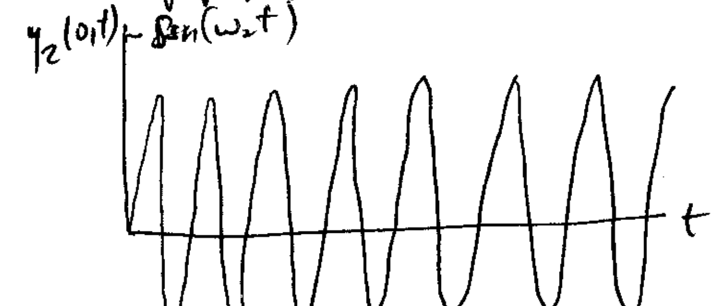
1) The average $\simeq$ either one

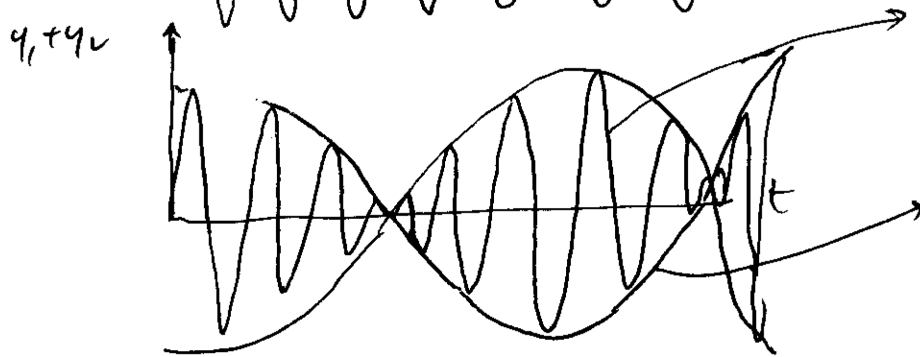
2) The difference is $\ll$ either one

↓

The high frequency oscillations are modulated by a very low frequency envelope which we can hear → these are the <u>beats</u>.







$\sin\left(\frac{\omega_1+\omega_2}{2}t\right)$ → High frequency oscillations

$\cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)$ → low frequency envelope.

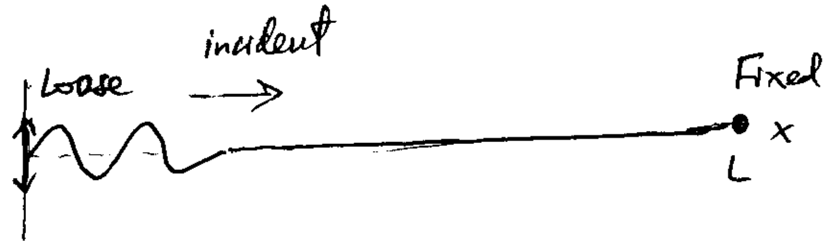
↓

Beats.

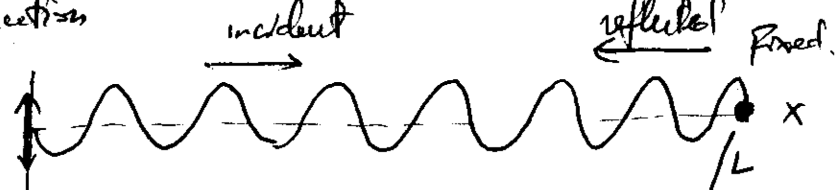
Standing Waves:

Physical observation:

- String along x-axis of length L. Fixed at $x=L$
- Transverse perturbation along y-axis @ loose end @ $x=0 \rightarrow$ transverse wave propagating in x-direction



- Reflected wave travels in $-x$ direction
- Incident & reflected waves add up to standing waves (not moving)



wave is inverted or reflected: a wave is being sent back in $-x$ direction.

Math description: $y_T(x,t) = y_{inc}(x,t) + y_{refl.}(x,t)$

$$= A\cos(kx - \omega t) - A\cos(kx + \omega t)$$

$(kx - \omega t)$: propagation in $+x$

$-$: Reflected wave @ fixed end $x=L$

$(kx + \omega t)$: propagation in $-x$

$$\boxed{\text{Trigonometry identity: } \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}$$

$$y_T(x,t) = -2A\sin\left(\frac{kx - \cancel{\omega t} + kx + \cancel{\omega t}}{2}\right)\sin\left(\frac{\cancel{kx} - \omega t - \cancel{kx} - \omega t}{2}\right)$$

$$= -2A\sin kx \sin(-\omega t) = 2A\sin kx \sin\omega t$$

Note: there is no further $(kx-\omega t)$ in the argument of $\sin$ or $\cos$ → $\boxed{\text{no propagation}}$, only oscillation in space ($\sin kx$) & time ($\sin \omega t$).
↳ Standing wave.

A very important consequence from math description of standing wave:

The string is fixed @ $x = L$: $y_T(L,t) = 2A \boxed{\sin kL} \sin \omega t = 0$
@ all time t.

$$\rightarrow \sin kL = 0 \iff kL = n\pi \quad (n = 1, 2, 3, \ldots)$$ (a multiple of π)

$$\frac{2\pi}{\lambda} L = n\pi$$

$$\boxed{\lambda = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots)}$$

→ Only wave of certain wavelengths can stand in a string of length L

$$\lambda_n = \frac{2L}{1}, \frac{2L}{2}, \frac{2L}{3}, \frac{2L}{4}, \ldots$$

→ For example, wave of wavelength $\lambda = 2.5L$ can't stand in this string!

$\lambda_1 = 2L$ → in one L we have half wavelength

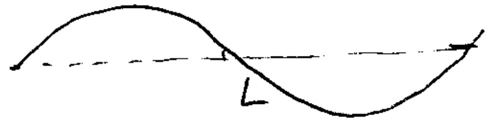
$\hookrightarrow \omega_1 = v\frac{\pi}{L}$

$$\omega = kv \rightarrow \omega_1 = \frac{2\pi}{\lambda_1} v = \frac{2\pi}{2L} v$$

L

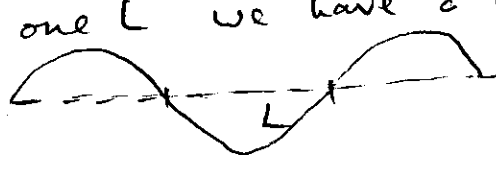
$\lambda_2 = L$ → in one L we have a full wavelength

$\hookrightarrow \omega_2 = 2\omega_1$



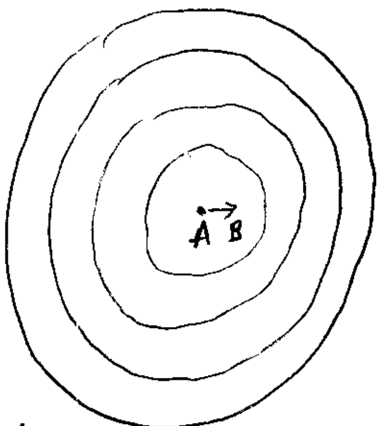
$\lambda_3 = \frac{2L}{3}$ → in one L we have a wavelength + a half wavelength.

$\hookrightarrow \omega_3 = 3\omega_1$

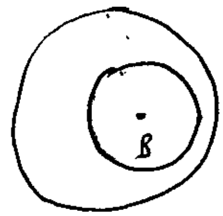


Doppler Effect: source of a wave is not static, it moves at constant speed u.

Wave takes some time to propagate → wave emitted by moving source at previous time is still travelling which is catched up by a new wave emitted by the same source:



$t = 0$



spread out, $t = t_1$ → wave fronts are further apart

$\lambda' > \lambda_0$

$\lambda' = \lambda_0 + uT$

piled up → wave fronts are closer together

$\lambda' < \lambda_0$

$\lambda' = \lambda_0 - uT$

T: Periods of wave

u: speed of moving source

λ_0: original wavelengths of wave

$$f' = \frac{f_0}{1 + \frac{u}{v}}$$

receding source

$$f' = \frac{f_0}{1 - \frac{u}{v}}$$

approaching source

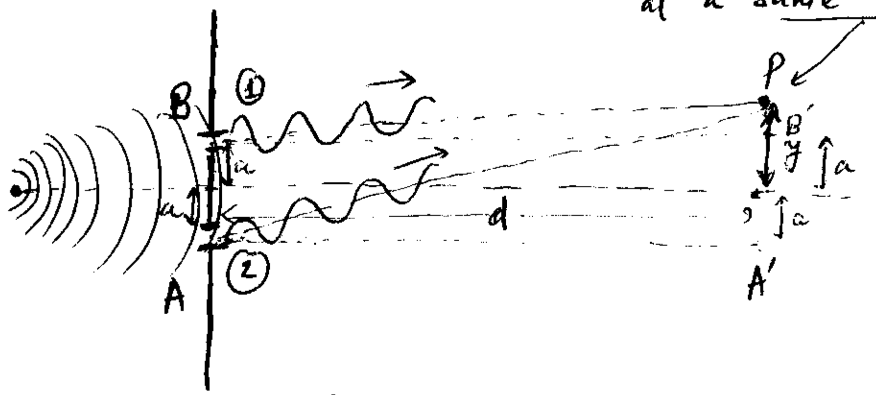
f_0: original frequency of wave; v: wave propagation speed

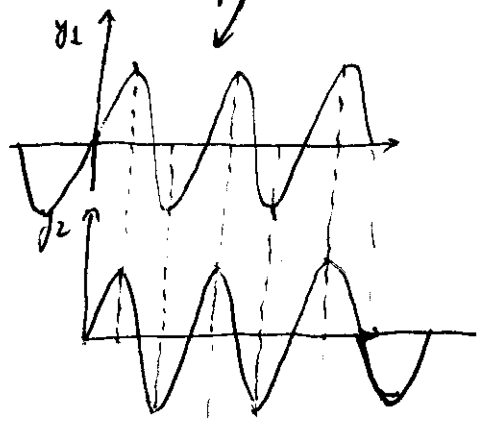
u = source speed.

Wave Motion (Cont.):

Constructive & Destructive Interference

Constructive Interference: Two identical waves ~~with~~ arriving at a same location **in phase**

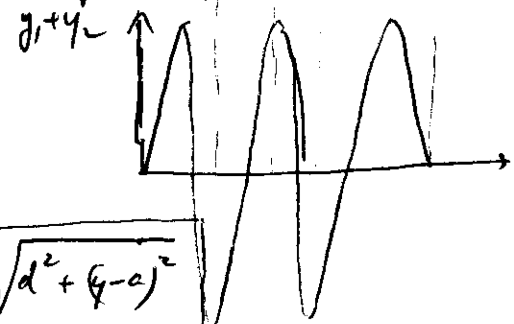




Two identical waves of wavelength λ travelling out from A & B towards P.

In phase: Arrival. $\quad AP - BP = 0, \lambda, 2\lambda, 3\lambda, \ldots$

path difference $= n\lambda \quad (n = 0, 1, 2, \ldots)$



$AA'P$: $\quad AP = \sqrt{d^2 + (y+a)^2}$

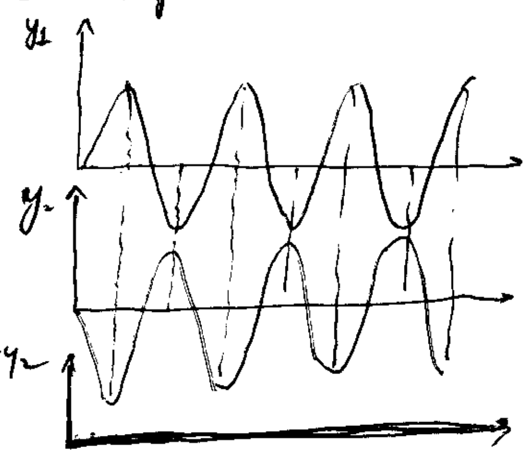
$BB'P$: $\quad BP = \sqrt{d^2 + (y-a)^2}$

In-phase arrival: $AP - BP = \sqrt{d^2 + (y+a)^2} - \sqrt{d^2 + (y-a)^2}$

(Constructive interference @ P) $\quad = n\lambda \quad (n = 0, 1, 2, 3, \ldots)$

P on midline b/w A & B.

Destructive Interference: Two identical waves arriving a same location out of phase



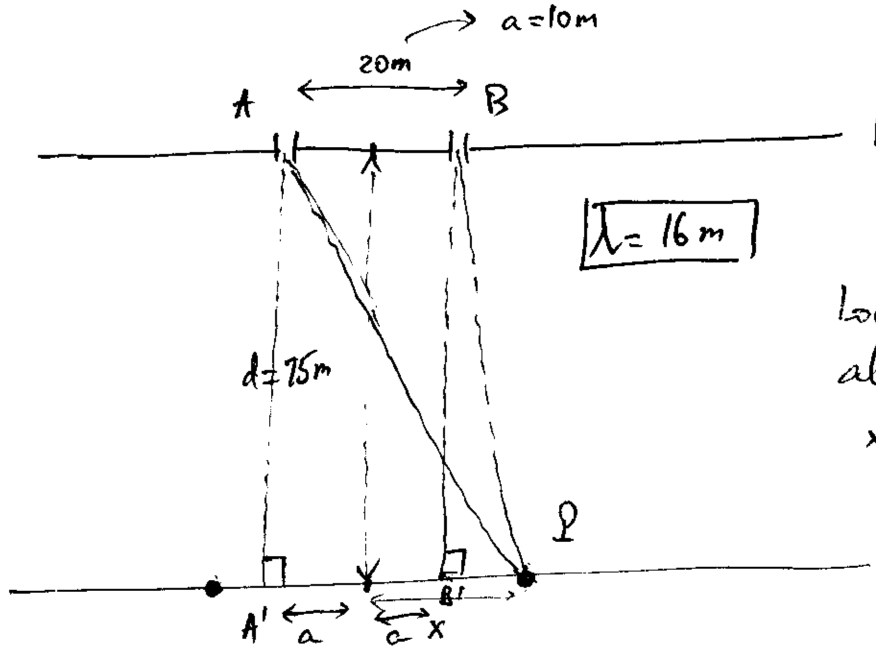
Out of phase arrival:

$$AP - BP = \underbrace{(2n+1)\frac{\lambda}{2}}_{\text{odd number of half wavelengths}} \quad (n = 0, 1, 2, 3, \ldots)$$

$$\boxed{\sqrt{d^2 + (y+a)^2} - \sqrt{d^2 + (y-a)^2} = (2n+1)\frac{\lambda}{2} \quad n = 0, 1, 2, 3 \ldots}$$

Destructive interference @ P.

Constructive & Destructive Interference in Water Wave:



Breakwater w/ 2 openings A&B

$\boxed{\lambda = 16\,m}$

Location of $\boxed{1^{st}\text{ maximum}}$ ($n=1$) along observation line x?

1st max: $AP - BP = n\lambda = \lambda \quad (n=1)$

$$\sqrt{d^2 + (x+a)^2} - \sqrt{d^2 + (x-a)^2} = \lambda$$

$$\sqrt{75^2 + (x+10)^2} - \sqrt{75^2 + (x-10)^2} = 16$$

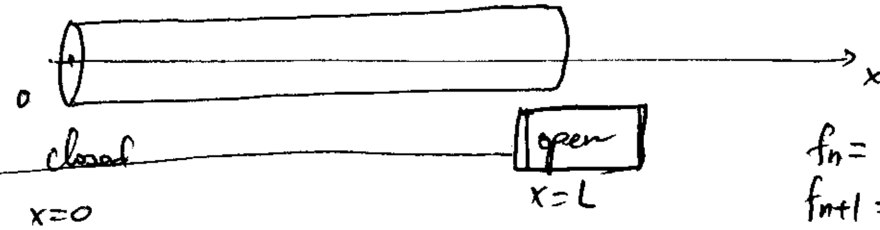
Solve for x: 1) Elevate both sides to power of 2
2) Isolate cross term w/ square roots → Repeat step 1.

$$75^2 + (x+10)^2 + 75^2 + (x-10)^2 - 2\sqrt{75^2 + (x+10)^2} \cdot \sqrt{75^2 + (x-10)^2} = 16^2$$

$$\left[2 \cdot 75^2 + 2x^2 + 200 - 16^2\right]^2 = 4\left(75^2 + (x+10)^2\right)\left(75^2 + (x-10)^2\right)$$

Polynomial of order 4 → $x = \pm 33\,m$

14.72



$L = 1.5\,m$

$f_n = 225\,Hz$ → f_0 (lowest freq)

$f_{n+1} = 375\,Hz$ → v ?

Pipe with one end closed → sound waves
A closed end for sound waves is similar to a fixed end on a string → Only certain wavelengths can stand in the pipe:

$$y_T = y_{inc} + y_{reflected}$$
$$= A\cos(kx - \omega t) - A\cos(kx + \omega t) = 2A \sin kx \sin \omega t$$

One end is closed → $y_T(L,t) = 0$ → $\sin kL = 0$ → $kL = n\pi$

$$\frac{2\pi}{\lambda_n} L = n\pi \rightarrow \boxed{\lambda_n = \frac{2L}{n}} \quad (n = 1, 2, 3, \dots)$$

one end is ~~open~~ closed

→ Pipe with one end open: → $y_T(L,t) = \max$ → $\sin kL = 1$

$$kL = \underbrace{(2n+1)\frac{\pi}{2}}_{\text{odd multiple of } \frac{\pi}{2}} \quad (n = 0, 1, 2, 3, \dots)$$

$$\frac{2\pi}{\lambda_n} L = (2n+1)\frac{\pi}{2} \rightarrow \boxed{\lambda_n = \frac{4L}{2n+1}} \quad (n = 0, 1, 2, 3, \dots)$$

one end is open.

Rewrite in terms of frequencies: $\boxed{v = \frac{\omega}{k} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f \cdot \lambda}$

$$f = \frac{v}{\lambda} \rightarrow \boxed{f_n = \frac{v}{\frac{4L}{2n+1}} = \frac{2n+1}{4L} v \quad (n = 0, 1, 2, 3, \dots)}$$

$$\frac{f_{n+1}}{f_n} = \frac{(2(n+1)+1)/\cancel{4L} \cdot \cancel{v}}{\frac{2n+1}{\cancel{4L}} \cancel{v}} = \frac{2n+3}{2n+1} = \frac{375}{225} = \frac{125 \times \cancel{3}}{75 \times \cancel{3}} = \frac{5}{3}$$

→ Solve for n → $\boxed{n = 1}$ $f_1 = 225\,Hz$ → $f_2 = 375\,Hz$

f_0 ?

a) $\frac{f_1}{f_0} = \frac{\frac{3}{4L}\cancel{v}}{\frac{1}{4L}\cancel{v}} = 3 \rightarrow \boxed{f_0 = \frac{f_1}{3} = \frac{225}{3} = 75\,Hz}$

b) Sound speed: v

$f_1 = \frac{3}{4L} v \rightarrow \boxed{v = \frac{f_1 4L}{3} = \frac{225 \cdot 4 \cdot 1.5}{3} = 450\,\frac{m}{s}}$

Ch 15 Fluid Motion

Fluid: Gas — density ρ (rho) is variable $\longleftrightarrow$ compressible

Liquid — density ρ is constant $\longleftrightarrow$ incompressible

In a gas: molecules are far apart which can be brought closer together (compressing the gas)

Describe a fluid:

- Density $\rho = \dfrac{M \text{ (mass)}}{V \text{ (Volume)}} = \dfrac{dm}{dV}$ (SI unit: $\frac{kg}{m^3}$)
- Pressure P: normal force per unit area $P = \dfrac{F}{A} \sim \dfrac{dF}{dA}$

(SI: $\frac{N}{m^2}$ = Pa or Pascal)

other unit: Atm (Atmosphere)

$1\,\text{Atm} = 1.013 \times 10^5\,\text{Pa}$.

$\boxed{\text{Crushed barrel:}}$ experiment: a) open barrel was heated to a certain temperature so fewer gas molecules inside barrel will exert same pressure (atmospheric pressure) (rest molecules were pushed out) Lower ρ, same P

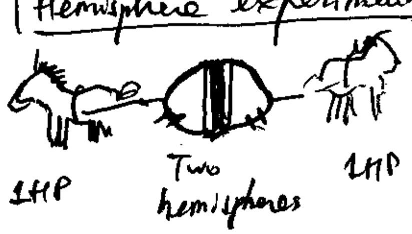
b) barrel is closed tight (no further flow of gas molecules either way)

c) ice were poured on barrel: lowering inside gas temperature: $\downarrow P$, same ρ.

- Outside barrel: atmospheric pressure
- Inside barrel: lower than atmospheric pressure

$\hookrightarrow$ Barrel is crushed by force of atmospheric pressure.

$\boxed{\text{Hemisphere experiment:}}$



a) Air from inside the hemisphere is pump out $\downarrow P$

b) Higher P outside is keeping the hemisphere together

- Hydrostatic equilibrium $\leftrightarrow$ water transportation
- Conservation of mass
- Conservation of energy

$\}$ $\rightarrow$ Bernoulli's principle $\begin{cases} \text{gas} \rightarrow \text{air transportation} \\ \text{water} \rightarrow \text{hydro power plants.} \end{cases}$

<u>Hydrostatic equilibrium</u>: fluid pressure compensating force of gravity
$\rightarrow$ floating objects in air (balloons) or in water (ships, boats, ...). Astronaut training in water

- <u>The difference in pressure</u> above (less) and below (more) on an element of fluid exactly cancels the weight of that element of fluid.

$\rightarrow$ element of fluid of weight $g\,dm$ and cross sectional area A & volume $dV = A\,dh$ $\Big\}$ $\rightarrow dm = \rho\,dV$

Container filled with water $\boxed{\text{on Earth}}$

ρ is $\begin{cases} \text{higher at bottom} \\ \text{lower at top} \end{cases}$ $\leftrightarrow$ $P \begin{cases} \text{higher at bottom} \\ \text{lower at top.} \end{cases}$

$\downarrow$

$$A\,dP - g\,dm = 0$$

$$\cancel{A}\,dP - g\rho \underbrace{dV}_{\cancel{A}dh} = 0$$

$$dP = g\rho\,dh$$

$$\boxed{\frac{dP}{dh} = g\rho}$$

$\downarrow$

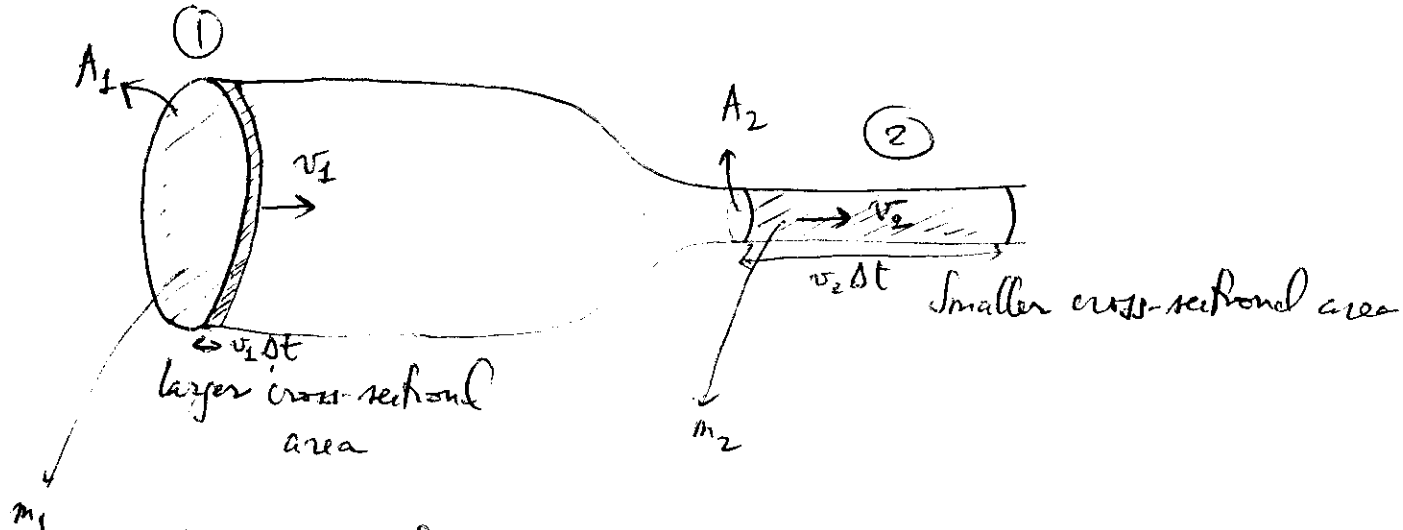
Bouyancy:

$$\underset{\substack{\downarrow \\ \text{boyant force}}}{F_b} = P.A = \underbrace{g\rho h}_{P} A$$

Conservation of Mass:

→ Assuming no leaking

→ Consequence : $\underset{\text{speed of fluid}}{v} \cdot \underset{\text{cross-sectional area}}{A} = \text{constant}$



Conservation of mass : $m_1 = m_2$

$$(\rho V)_1 = (\rho V)_2$$

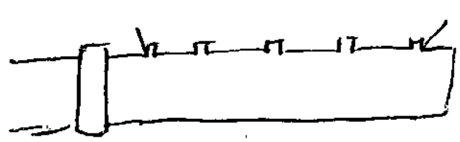
$$\cancel{\rho}\, v_1 \cancel{\Delta t} \cdot A_1 = \cancel{\rho}\, v_2 \cancel{\Delta t} \cdot A_2$$

$$v_1 A_1 = v_2 A_2$$

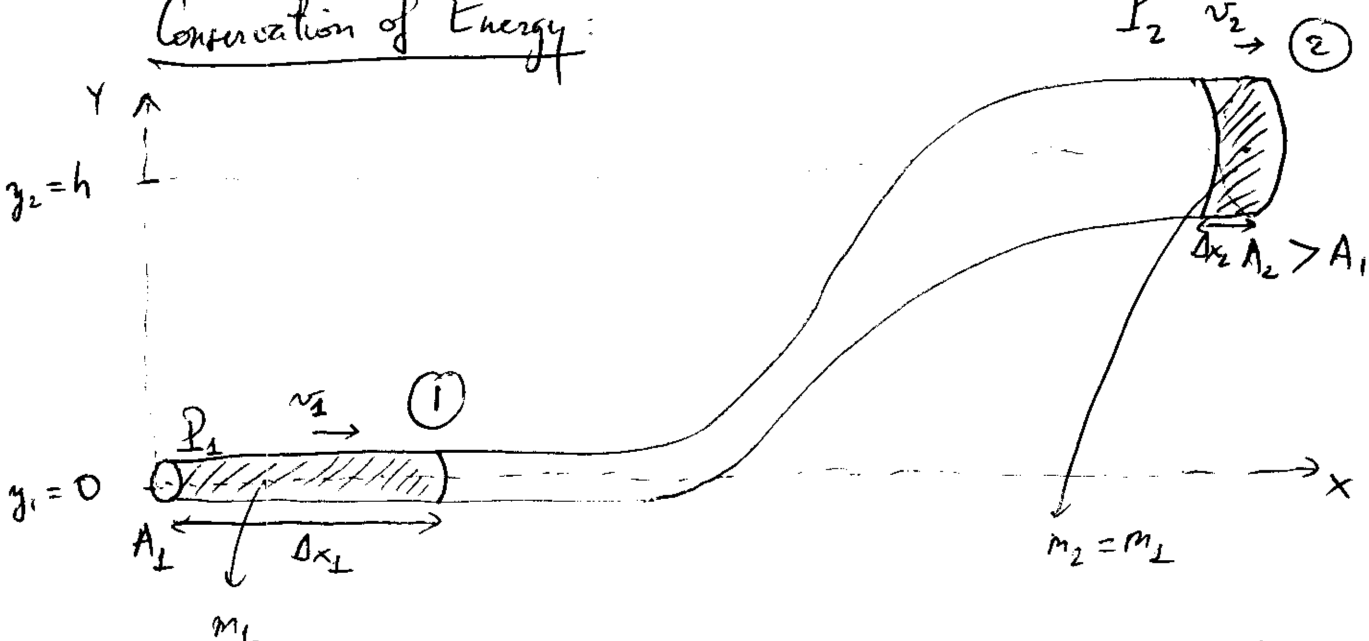
$$\boxed{vA \text{ is constant}}$$

↳ Larger cross-sect. area → lower speed
Smaller " " " → higher speed.

Sprinkler:



Conservation of Energy:



This is possible only if $P_1 > P_2$: pressure has done work in moving column of liquid from ① to ②:

$\Delta W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$

Work done by pressure ($F.\Delta x$)

↳ Is also $\Delta KE + \Delta P.E$

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \frac{1}{2} m v_2^2 + mg y_2 - \frac{1}{2} m v_1^2 - mg y_1$$

↳ Consequence: grouping terms at position ① & position ②:

$$\boxed{\frac{1}{2} m v^2 + mgy + PA\Delta x = \text{constant}}$$

Divide by volume: $V = A\Delta x$

$$\frac{1}{2} \left(\frac{m}{V}\right) v^2 + \left(\frac{m}{V}\right) gy + P \frac{A\Delta x}{A\Delta x} = \text{constant}.$$

density

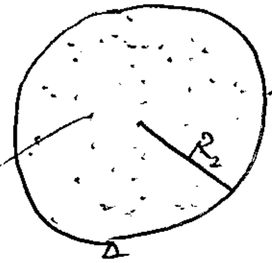
$$\boxed{\frac{1}{2} \rho v^2 + \rho g y + P = \text{constant}}$$ Bernoulli's equation.

15.60 Helium balloon : bouyancy :

$$F_b = g\rho \underbrace{hA}_{V} \rightarrow \boxed{F_b = g\rho V}$$

$m_3 = 1g$.

Hellium → ①
Rubber → ②



$m_2 = 0.85g$
(mass of rubber)
$R_2 = 0.15m$

How many paper clips (each has mass 1g) can we hang before this balloon looses its bouyancy?

$\rho_{He} = \rho_1 = 0.18 \frac{kg}{m^3}$

Bouyancy in this case: He has lower density than air

When N clips are attached: weigh by { Rubber, He, Clips

$$\left(Nm_3 + m_2 + \underbrace{\rho_1 \frac{4}{3}\pi R_2^3}_{m_1}\right) g = F_b = g \underbrace{\rho_{air} V_{balloon}}_{\text{mass of air displaced by He balloon}}$$

$$N = \frac{\rho_{air} \frac{4}{3}\pi R_2^3 - m_2 - \rho_{He} \frac{4}{3}\pi R_2^3}{m_3}$$

Data

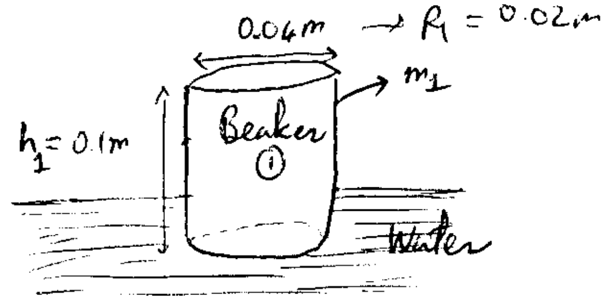
$\boxed{\rho_{air} = 1.2 \frac{kg}{m^3}}$

$$N = \frac{(1.2 - 0.18)\frac{4}{3}\pi\, 0.15^3 - 0.85 \cdot 10^{-3}}{10^{-3}} = \underbrace{13.6 \text{ clips}}$$

↓

13 clips before balloon looses bouyancy.

15.50 Similar to 15.60 but now in a liquid (but also a fluid!)



Beaker floating in water with $\frac{h}{3}$ submerged:
weight of beaker is hold by a bouyant force F_b

$F_b = g\rho_{water} V$ where V = vol. of water displaced by beaker

(in 15.60: V here was the volume of the entire balloon since whole balloon was submerged in air)

in 15.50: $V = \frac{1}{3} V_1 = \frac{1}{3} \pi R_1^2 h_1$

$$\to F_b = g\rho_{water} \frac{1}{3} \pi R_1^2 h_1 = m_1 \cdot g$$ → here m_1 is obtained.

How many rocks (each of mass $m_2 = 15g$) can be placed in beaker before it sinks ⟷ if we submerge another $\frac{2}{3}h$ of beaker can hold N rocks: additional bouyant force is $\tilde{F}_b$

$$\tilde{F}_b = g\rho_{water} \frac{2}{3} \pi R_1^2 h_1 = N m_2 g$$

$$N = \frac{\rho_{water} \frac{2}{3} \pi R_1^2 h_1}{m_2}$$

$$= \frac{1000 \frac{2}{3} \pi \cdot 0.02^2 \cdot 0.1}{15 \cdot 10^{-3}}$$

$$N = 5.59$$

↓

5 rocks before beaker sinks.

Data:

$\rho_{water} = 1000 \frac{kg}{m^3}$

15.22 4300 kg elephant on one foot approximated as a circle 

$R = 0.3\,m$

$$P = ? = \frac{F}{Area} = \frac{m \cdot g}{\pi R^2} = \frac{4300 \cdot 9.81}{\pi \cdot 0.3^2} = 596\ kPa$$

k = 10^3

(1 Atm - normal atmospheric pressure = 101300 Pa $\approx$ 101 kPa)

14.45

Exam 3 Review of Topics (Ch10 – 15)

Rotational Motion (Ch.10)

Torque : select pivot point $\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta\ \hat{\tau}$

position vector $\vec{r}$: from pivot point to force application point

θ : angle b/w $\vec{r}$ & $\vec{F}$

$\hat{\tau}$: perpendicular to both $\vec{r}$ & $\vec{F}$, given by RHR

Analog's of 2nd Newton's Law

$$\vec{\tau} = I \cdot \vec{\alpha}$$

I : moment of inertia

$\vec{\alpha}$: vector angular acceleration.

Angular Momentum (Ch.11)

$$\vec{L} = \vec{r} \times \vec{p} \quad (\vec{p} : \text{linear momentum } \vec{p} \equiv m\vec{v})$$

$$= rp\sin\alpha\ \hat{L}$$

$\vec{r}$: position vector : from pivot to position of object

α : angle b/w $\vec{r}$ & $\vec{p}$

$\hat{L}$: perpendicular to both $\vec{r}$ & $\vec{p}$, given by RHR

Round object : $L = I\omega$ (I : m. of. inertia ; ω : angular speed)

Conservation of Angular momentum :

$$\vec{\tau}_{\text{net external}} = \frac{d\vec{L}}{dt} \; ; \; \vec{\tau}_{\text{net external}} = 0 \rightarrow \boxed{\vec{L}_i = \vec{L}_f}$$

Static Equilibrium (Ch.12)

. Condition : $\sum_i \vec{F}_i = 0$

$\sum_i \vec{\tau}_i = 0 \leftrightarrow$ select pivot so force with least information offer no torque.

. Use geometry to find angles b/w $\vec{r}$ & $\vec{F}$ for different torques. Torque directions by RHR.

Oscillatory Motion (Ch. 13)

SHM
- Pendulum (Gravitational): $\frac{d^2\theta}{dt^2} = \underbrace{-\frac{g}{L}\theta}_{\text{small angle approx.}} \rightarrow \theta(t) = \theta_m \cos\omega t$, $\omega = \sqrt{\frac{g}{L}}$
- Torsional Pendulum: $\frac{d^2\theta}{dt^2} = -\frac{K}{I}\theta \rightarrow \theta(t) = \theta_m \cos\omega t$, $\omega = \sqrt{\frac{K}{I}}$
- Spring & bob: $\frac{d^2x}{dt^2} = -\frac{k}{m}x \rightarrow x(t) = x_m \cos\omega t$, $\omega = \sqrt{\frac{k}{m}}$

Total energy is constant: $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \underbrace{\frac{1}{2}kx_m^2}_{\text{constant}}$.

Wave Motion (Ch 14)

→ Types
- Longitudinal: osc. in same direction as propagation
- $\boxed{\text{Transverse}}$: osc. perpendicular to direction of propagation.
  - ↳ Wave in a string:
    - Math description: $y(x,t) = A\sin(kx-\omega t)$
    - propagation in $+x \Leftrightarrow A\sin(kx-\omega t)$
    - propagation in $-x \Leftrightarrow A\sin(kx+\omega t)$
    - $v = \sqrt{\frac{T}{\mu}}$ { v: wave speed; T: tension in string; μ: linear mass density of string.

→ Properties:

Superposition
- Beat phenomenon:
  - amplitude modulation or beats $\propto \cos\left(\frac{\omega_1-\omega_2}{2}\cdot t\right)$ ↓ low freq.
  - intensity $\propto \cos^2 \rightarrow$ frequency is $\omega_1-\omega_2$
- Standing waves: Inc + reflected, $2A\boxed{\sin kx}\sin\omega t$
  - pipe one ~~open~~ closed end: @ $x=L$: $f_n = \frac{n}{2L}v$ $(n=1,2,3,\dots)$
  - pipe one ~~closed~~ Open end: @ $x=L$: $f_n = \frac{2n+1}{4L}v$ $(n=0,1,2,3,\dots)$
- Interference
  - Constructive: in phase arrival: $AP-BP = \sqrt{d^2+(y+a)^2} - \sqrt{d^2+(y-a)^2} = n\lambda$ $(n=0,1,2,3,\dots)$
  - Destructive: out of phase arrival: $AP-BP = (2n+1)\frac{\lambda}{2}$ $(n=0,1,2,3,\dots)$

Doppler effect: source is moving @ speed u:

$$f' = \frac{f_0}{1 \pm \frac{u}{v}}$$

$+$: receding source; $-$: approaching source

Fluid Motion (Ch 15)

- Hydrostatic Equilibrium : $\frac{dP}{dh} = g\rho \rightarrow$ Buoyancy : $F_b = g\rho V$

  Buoyant force on a He balloon of volume V is given by the amount of air displaced by the balloon : $\rho_{air} V_{balloon}$

- Conservation of mass : vA = constant

  v: fluid speed; A: cross sectional area of pipe

- Conservation of energy : $\frac{1}{2}\rho v^2 + \rho g h + P$ = constant

  Bernoulli's equation.

$P = \frac{F}{A}$; SI : Pa

1 Atm = 101300 Pa

14.45

Doppler Effect:

$$f' = \frac{f_0}{1 + \frac{u}{v}} \rightarrow \frac{f'}{f_0} = \frac{1}{1 + \frac{u}{v}}$$

receding source : $f' < f_0$

$$\boxed{f = \frac{v}{\lambda}} \rightarrow \lambda = \frac{v}{f} \xrightarrow{\lambda \propto \frac{1}{f}} \frac{\lambda'}{\lambda_0} = 1 + \frac{u}{v}$$

u: Distant Galaxy speed.

v: wave → Light speed.

$$\frac{\lambda}{T} = v$$

$$\frac{708\,nm}{656\,nm} = 1 + \frac{u}{c} \rightarrow u = \left(\frac{708}{656} - 1\right) \cdot 3 \times 10^8 = 2.38 \times 10^7 \, \frac{m}{s}$$