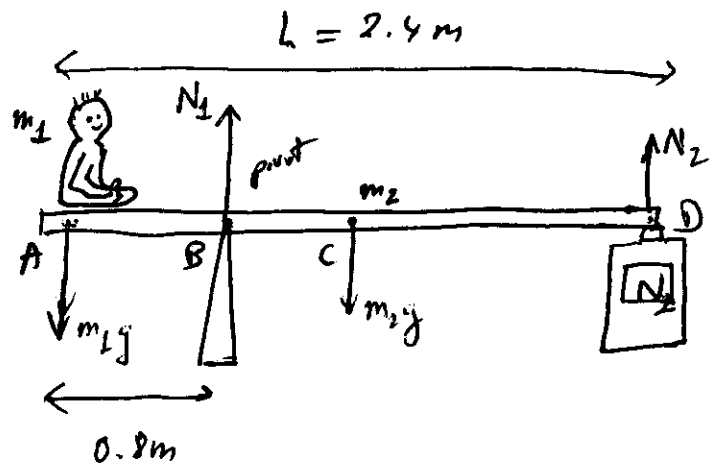


Ch 12 Static Equilibrium

$$\begin{cases} 1) \sum_i \vec{F}_i = 0 & \text{not moving (Net force on system is 0)} \\ 2) \sum_i \vec{\tau}_i = 0 & \text{not rotating (Net torque on system is also 0)} \end{cases}$$

12.22



What is the position of the child for a given value of N so system is in static equilibrium?

Data: $m_1 = 40 \text{ kg}$; $m_2 = 60 \text{ kg}$
 $L = 2.4 \text{ m}$; pivot at 0.8 m from left.
 $N_2 = 1000 \text{ N}$ or 3000 N .

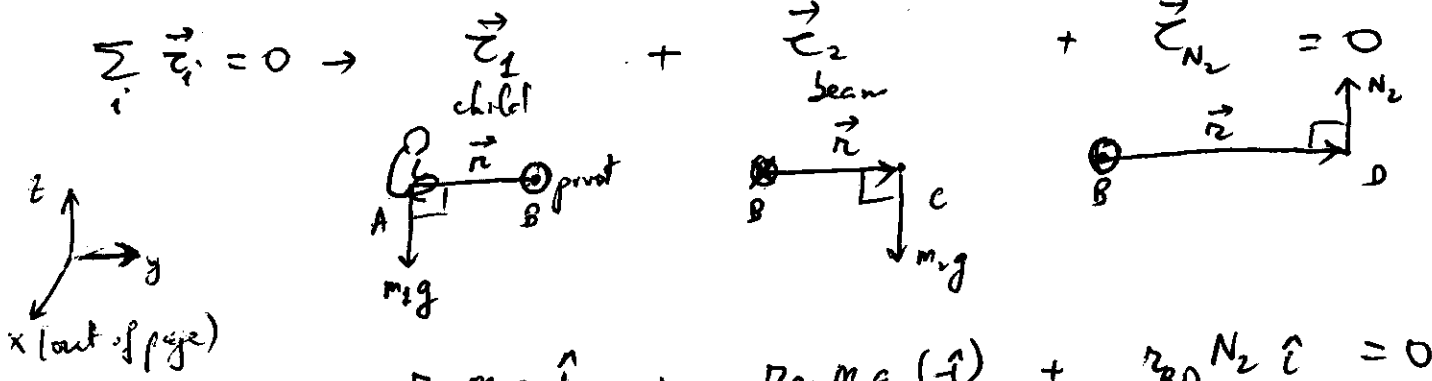
(out page) \rightarrow beam connects every thing \rightarrow Focus on beam:

Static equilibrium $\begin{cases} \sum_i \vec{F}_i = 0 \rightarrow N_1 + N_2 - m_1 g - m_2 g = 0 \\ \sum_i \vec{\tau}_i = 0 \rightarrow \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_{N_2} = 0 \end{cases}$

Note: There are 4 forces but only 3 torques since \vec{N}_1 is applied right on the pivot point!

In general any force applying on pivot has $\vec{r} = 0 \rightarrow$ no torque contribution

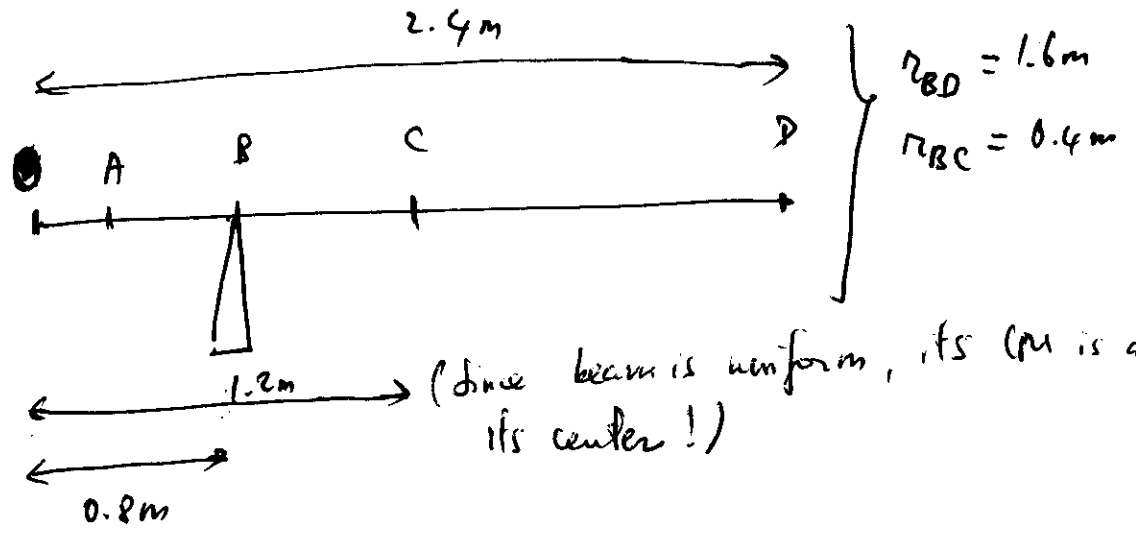
\rightarrow A common technique in static equilibrium is to place a pivot at the application point of the force we know least about



$$r_{BA} m_1 g \hat{i} + r_{BC} m_2 g (-\hat{i}) + r_{BD} N_2 \hat{i} = 0$$

$$r_{BA} m_1 g + r_{BD} N_2 - r_{BC} m_2 g \geq 0$$

Figure out r_{BD} & r_{BC} , then find r_{BA} from this eq.



$$a) \rightarrow r_{BA} = \frac{r_{BC} m_2 g - r_{BD} N_2}{m_1 g} = \frac{0.4 \cdot 60 \cdot 9.81 - 1.6 \cdot 100}{40 \cdot 9.81} = 0.19m$$

$N_2 = 100N$
From left edge of beam: child sits @ $0.8 - 0.19 = 0.61m$

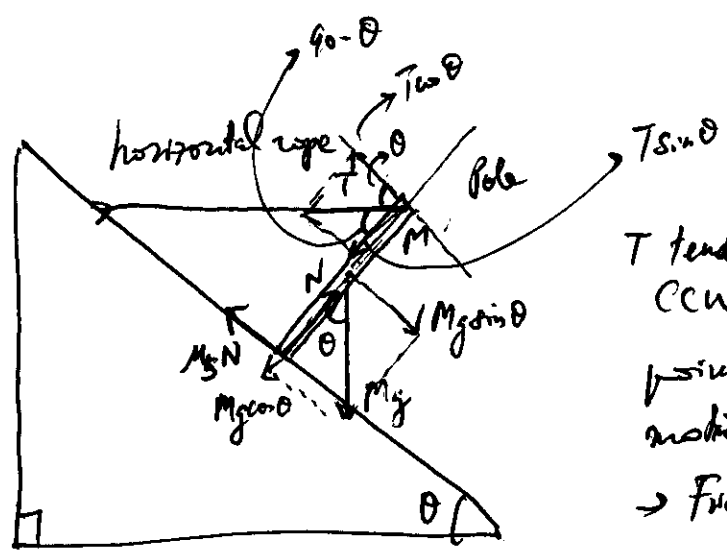
$$b) N_2 = 300N \quad r_{BA} = \frac{0.4 \cdot 60 \cdot 9.81 - 1.6 \cdot 300}{40 \cdot 9.81} = -0.62m$$

Child sits 0.62m to the right of pivot (B)

From left edge of beam: child sits @ $1.42m$

12.57

Min μ_s
so pole not slipping



T tends to rotate pole in CCW direction \rightarrow @ contact point b/w pole & incline motion tends to be downhill \rightarrow Friction will be uphill!

\rightarrow Focus on pole (four):

- Mg (its weight at its CM)
- N (normal force by incline)
- T (by rope)
- $\mu_s N$ (by friction w/ incline)

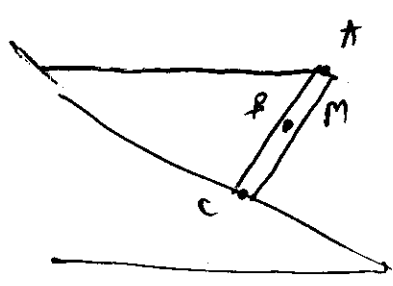
Static equilibrium:

$$\sum_i \vec{F}_i = 0$$

$$\left\{ \begin{array}{l} x: Mg \sin \theta - T \cos \theta - \mu_s N = 0 \quad (1) \\ y: N - Mg \cos \theta - T \sin \theta = 0 \quad (2) \end{array} \right.$$

$\sum_i \vec{C}_i = 0$ Pivot: place pivot @ application point of force we know the least about.

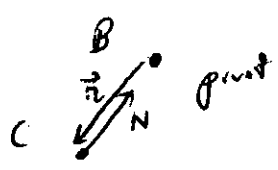
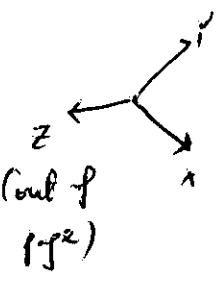
Candidates for the pivot point:



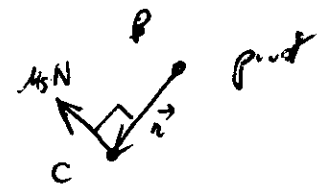
A
 B
 C (can't since μ_s will not be in our equation!)

Let's pick B: proof

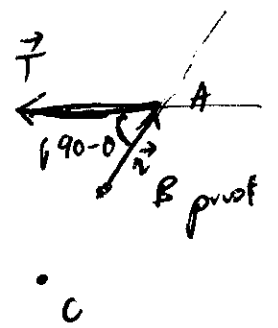
→ 3 torques: not from Mg;



$\sin 180^\circ = 0$
(No torque from N either)



$\tau_{BC} = \mu_s N (-\hat{k})$
into page
 $\frac{L}{2}$
(L length of pole)



$\tau_{BA} = T \sin(90 - \theta) (\hat{k})$
 $\frac{L}{2}$
(L length of pole) out of page

$$-\frac{L}{2} \mu_s N + \frac{L}{2} T \cos \theta = 0$$

$$-\mu_s N + T \cos \theta = 0 \quad (3)$$

Eliminate tension T:

(3) solve for T then plug it into (1):

$$T = \frac{\mu_s N}{\cos \theta}$$

$$\rightarrow Mg \sin \theta - \frac{\mu_s N}{\cos \theta} \cos \theta - \mu_s N = 0$$

$$Mg \sin \theta - 2\mu_s N = 0$$

$$N = \frac{Mg \sin \theta}{2\mu_s}$$

$$(2) \quad N - Mg \cos \theta - T \sin \theta = 0$$

$$\frac{Mg \sin \theta}{2\mu_s} - Mg \cos \theta - \frac{\mu_s N \sin \theta}{\cos \theta} = 0$$

$$\frac{Mg \sin \theta}{2\mu_s} - Mg \cos \theta - \mu_s \tan \theta \frac{Mg \sin \theta}{2\mu_s} = 0$$

$$\frac{1}{\sin \theta} \times \left(\frac{\sin \theta}{2\mu_s} - \cos \theta - \frac{\tan \theta \sin \theta}{2} = 0 \right)$$

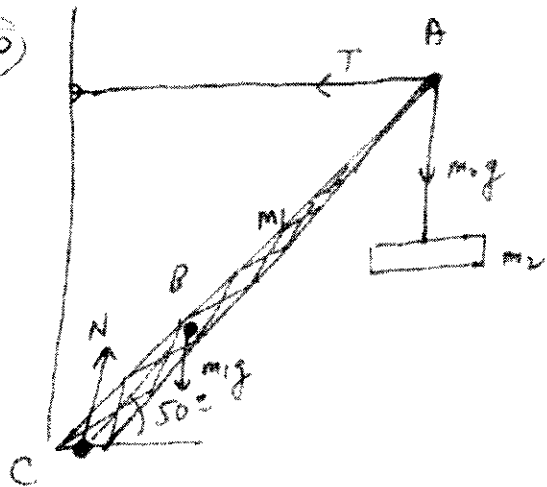
$$\frac{1}{2\mu_s} - \frac{1}{\tan \theta} - \frac{\tan \theta}{2} = 0$$

$$\frac{1}{2\mu_s} = \frac{1}{\tan \theta} + \frac{\tan \theta}{2} = \frac{2 + \tan^2 \theta}{2 \tan \theta}$$

$$\mu_s \geq \frac{\tan \theta}{2 + \tan^2 \theta} \quad (\text{Min to hold the pole})$$

$$\boxed{\mu_s \geq \frac{\tan \theta}{2 + \tan^2 \theta}}$$

12.40



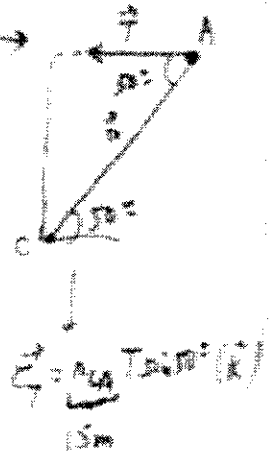
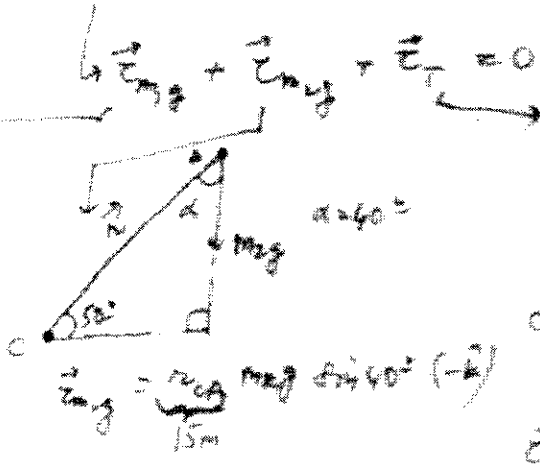
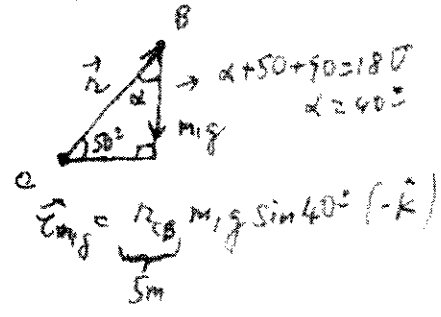
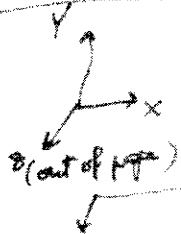
$AC = 20\text{ m}$, $CB = 5\text{ m}$
 B @ CM of boom (mass m_1)
 $m_1 = 860\text{ kg}$
 $m_2 = 2500\text{ kg}$

$\sum \vec{F}_i = 0 \rightarrow$ net force on boom is zero

Static equilibrium, for m_1 :

$\sum \vec{\tau}_i = 0 \rightarrow$ pivot: A B C

we don't know N & its angle

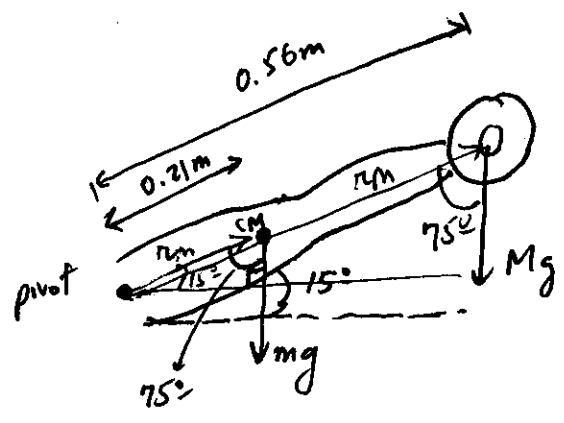


$$- 5 \times 860 \times 9.81 \sin 40^\circ - 15 \times 2500 \times 9.81 \sin 40^\circ + 15 T \sin 50^\circ = 0$$

$$T = \frac{5 \times 860 \times 9.81 \sin 40^\circ + 15 \times 2500 \times 9.81 \sin 40^\circ}{15 \sin 50^\circ}$$

$$T = 22900\text{ N}$$

12-28



$m = 4.2 \text{ kg}$
 $M = 6 \text{ kg}$

z (out of page)

(a) Torque about shoulder (pivot) : $\vec{\tau} = \vec{\tau}_m + \vec{\tau}_M$

$\vec{\tau} = \vec{r} \times \vec{F}$

↳ From pivot to application point \vec{F}

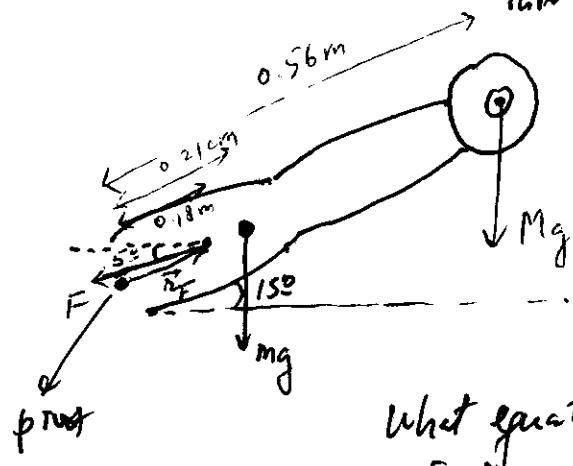
$\vec{\tau} = r_m mg \sin 75^\circ (-\hat{i}) + r_M Mg \sin 75^\circ (-\hat{i})$

$= (0.21 \cdot 4.2 + 0.56 \cdot 6) 9.81 \sin 75^\circ (-\hat{i}) \text{ Nm}$

$= 40.2 \text{ Nm } (-\hat{i})$

↓ into the page.

(b)

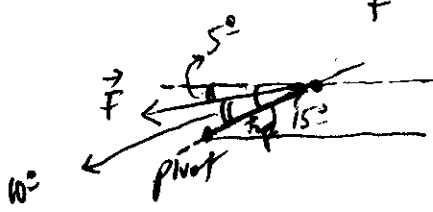


F : force exerted by deltoid muscle 5° below horizontal @ 0.18 m from pivot.

What equation can we use to find F ?
 → Static equilibrium : $\sum \vec{F}_i = 0$
 $\sum \vec{\tau}_i = 0$

Since we have $\vec{\tau}$ by m & M (arm) (weight) → let's calculate $\vec{\tau}_F$

$\vec{\tau}_F = r_F F \sin 10^\circ (\hat{i}) = 0.18 F \sin 10^\circ (\hat{i})$



$\sum \vec{\tau}_i = 0 \Rightarrow 0.18 F \sin 10^\circ - 40.2 = 0$

$F = \frac{40.2}{0.18 \cdot \sin 10^\circ} = 1280 \text{ N}$
 $= 1.28 \text{ kN}$

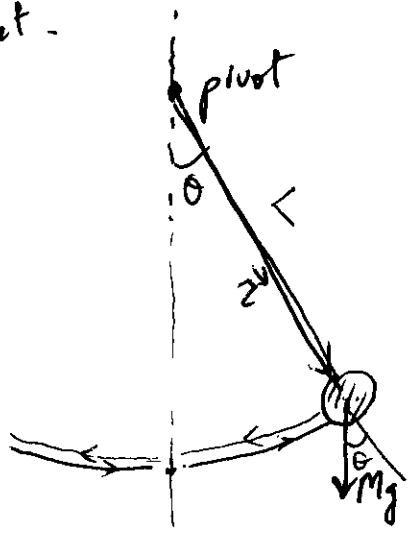
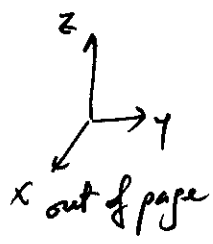
Ch 13 Oscillatory Motion

→ A different type of motion: not linear, not exactly rotational

Examples:

1) pendulum:

bob attached to a string (having negligible mass compared to the bob), attached to a fixed point which is the center of rotation or pivot point.



As the bob is released, θ decreases to 0 (vertical), then increases again to the same value (if friction Bob, mass m & air resistance are ignored) and then returning to the original position, and repeating → oscillations

Almost = rotational motion:

$$\tau = I \alpha \quad \sim \quad \alpha = \frac{\tau}{I}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \underbrace{LMg \sin \theta}_{\tau} (-\hat{i})$$

$$\rightarrow \alpha = \frac{\tau}{I} = -\frac{LMg \sin \theta}{ML^2} = -\frac{g}{L} \sin \theta$$

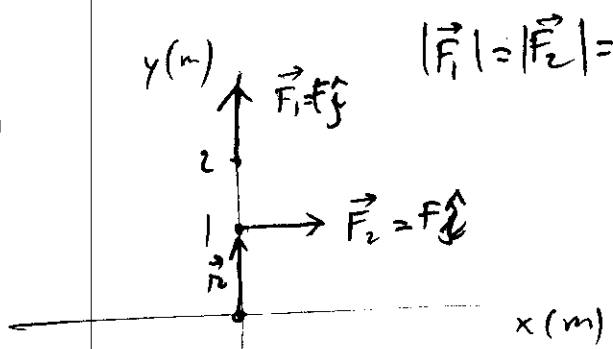
$$\frac{d^2\theta}{dt^2} = -\frac{g}{L} \sin \theta$$

approximation: $\sin \theta \approx \theta$ → This happens when θ is small → "Small Angle Approx."

(angular acceleration of pendulum is the torque by gravity divided by the moment of inertia of the bob wrt the pivot point: ML^2)

(2.16)

(A)



$|\vec{F}_1| = |\vec{F}_2| = F$

Can we add \vec{F}_3 so the three

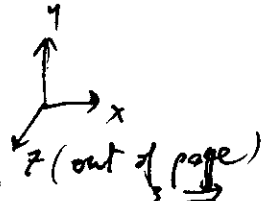
$$\begin{cases} \sum_{i=1}^3 \vec{F}_i = 0 & (1) \\ \sum_{i=1}^3 \vec{\tau}_i = 0 & (2) \end{cases}$$

(1) Yes

(2) Yes :

$\vec{F}_3 = -F\hat{i} - F\hat{j}$

$\vec{r} \times \vec{F}$

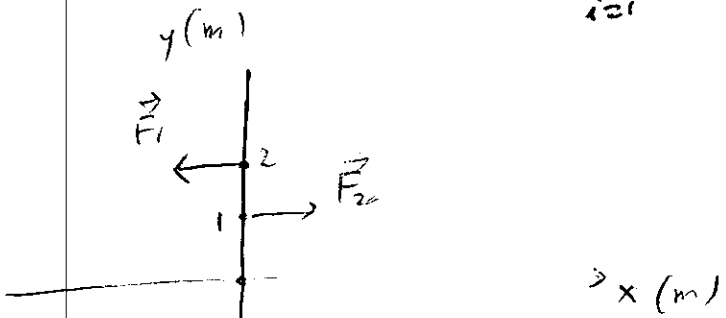


$\sum_{i=1}^3 \vec{F}_i = 0$

Let's pick, for example origin as the pivot point. Torque by \vec{F}_2 is $1 \cdot F(-\hat{k})$. Torque by \vec{F}_3 is torque by $-F\hat{j}$, which is zero, plus torque by $-F\hat{i}$, which is equal and opposite to torque by \vec{F}_2 .

$\rightarrow \sum_{i=1}^3 \vec{\tau}_i = 0$

(B)



(1) $\sum_{i=1}^3 \vec{F}_i = 0$ No

(2) $\sum_{i=1}^3 \vec{\tau}_i = 0$ Yes. ($\vec{F}_3 = -F\hat{i}$ or $\vec{F}_3 = F\hat{j}$...)

Small Angle Approximation. $\sin \theta \approx \theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

2nd order linear differential equation.

$$\theta = \theta_m \cos \omega t$$

θ_m = amplitude of oscillation
 ω : angular frequency

ω : # angular oscillations per second :

$$\frac{d\theta}{dt} = -\omega \theta_m \sin \omega t$$

$$\frac{d^2\theta}{dt^2} = -\omega^2 \theta_m \cos \omega t = -\frac{g}{L} \theta_m \cos \omega t$$

$$\omega = \sqrt{\frac{g}{L}}$$

Ang. freq of a pendulum is determined by L (inversely) & g .

\hookrightarrow $\left\{ \begin{array}{l} \text{large } L \rightarrow \text{lower } \omega \\ \text{lower } L \Rightarrow \text{higher } \omega \end{array} \right.$

For a pendulum (where we can apply the "small angle approx."), the number of oscillations per second or angular frequency depends on g and L (but not on m)

$$\omega = \sqrt{\frac{g}{L}}$$

less ω for longer pendulum
more osc. per second for shorter pendulum

2) Torsional pendulum

Not a swinging motion, more of a twisting back & forth



I - moment of inertia for rotation about the axis

$$\tau = I \alpha$$

Twisting: $\tau = -K\theta$ → twisting angle

"kappa" = torsional constant (Analogy of Hooke's Law for twisting motion)

$$-K\theta = I \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{K}{I} \theta$$

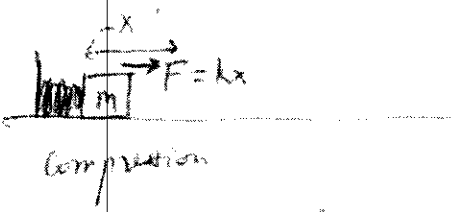
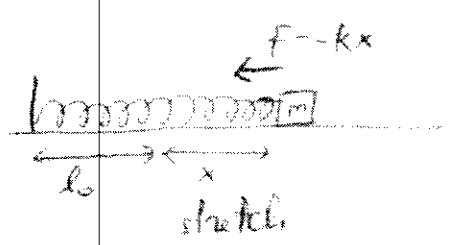
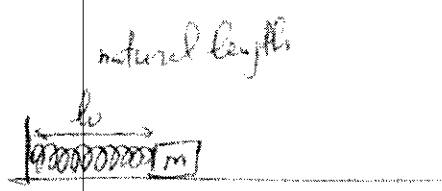
2nd order linear differential equation

$$\theta = \theta_m \cos \omega t$$

amplitude of oscillation

$$\omega = \sqrt{\frac{K}{I}}$$

3) Spring & bob $\left\{ \begin{array}{l} k = \text{spring constant} \\ m = \text{mass of bob} \end{array} \right.$



Spring always opposes the motion of m
 $\rightarrow m$ undergoes oscillatory motion

Newton's 2nd Law -

$$F = ma$$
$$-kx = m \frac{d^2x}{dt^2}$$

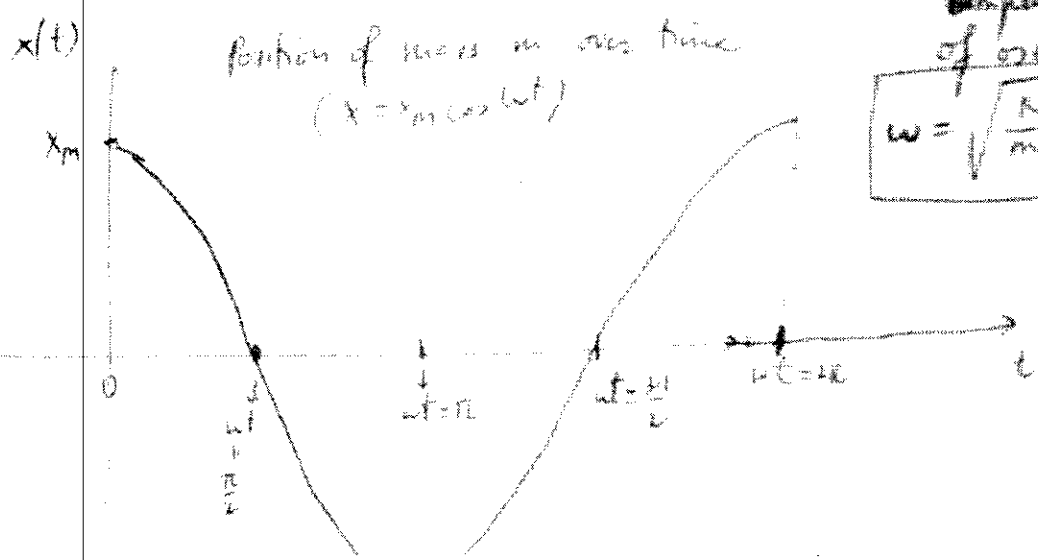
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

2nd order linear differential of

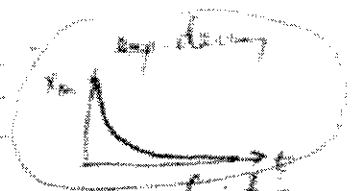
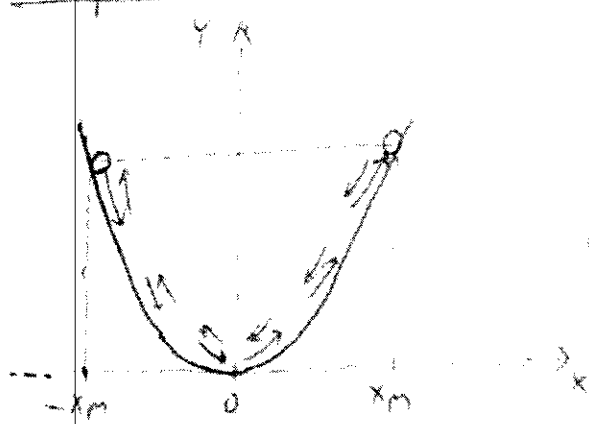
$$x = x_m \cos(\omega t)$$

amplitude of oscillation

$$\omega = \sqrt{\frac{k}{m}}$$

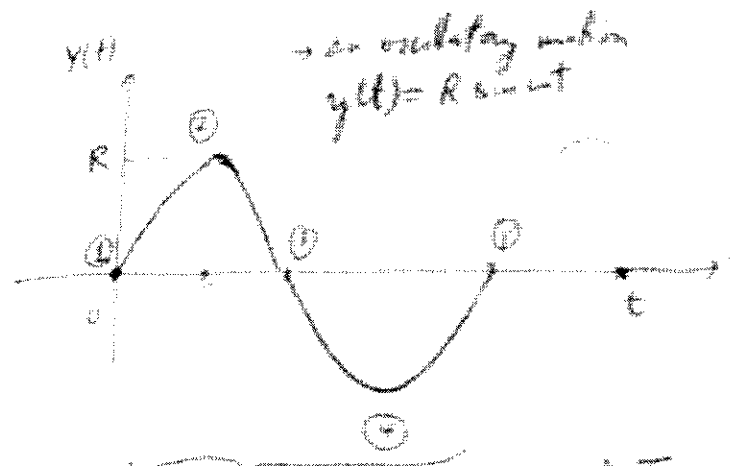
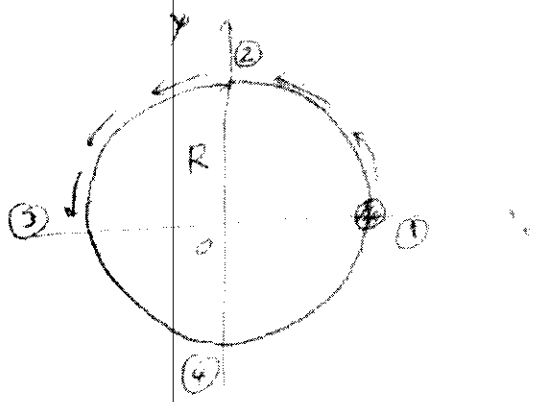


4) A particle trapped in a potential well

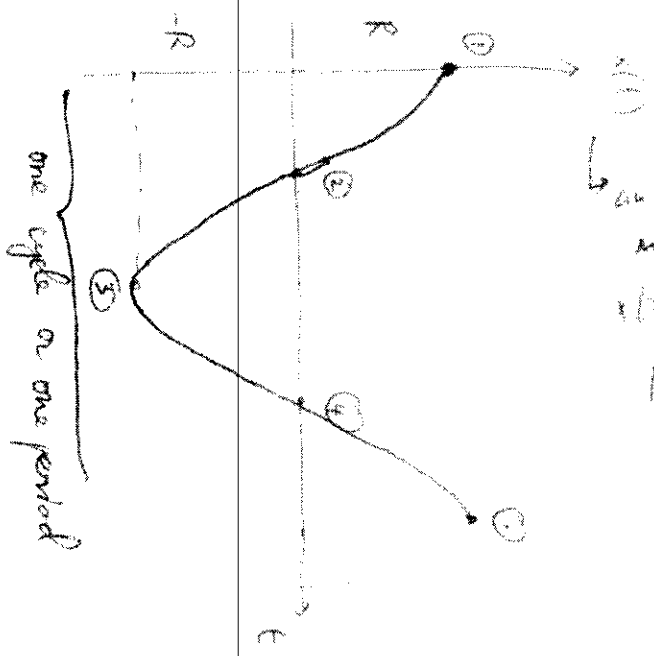


→ Assume there is no friction
 Simple Harmonic Motion
 $x(t) = x_m \cos(\omega t)$ → Motion SHM
 → If there is friction - damped oscillatory motion
 $x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega t + \phi)$
 amplitude decaying exponentially

5) Coordinates x & y of a particle undergoing a Uniform Circular Motion or UCM



→ an oscillatory motion
 $y(t) = R \sin \omega t$
 one cycle or one period T



→ an oscillatory motion
 $x(t) = R \cos(\omega t)$ (x & y are coords of a particle in UCM) → are shifted by a phase of 90° or $\frac{\pi}{2}$

$$T = \frac{2\pi}{\omega}$$

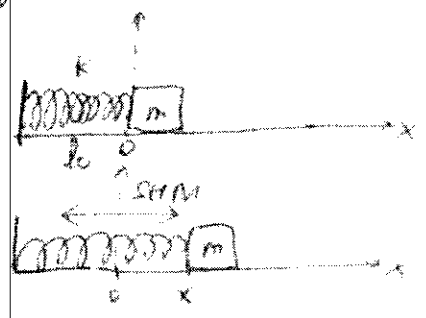
Summary: For a spring & bob $\frac{k}{m}$
 SHM: $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ (x position of mass m)

↳ solution is $x(t) = x_m \cos \omega t$; $\omega = \sqrt{\frac{k}{m}}$

Damped SHM: $\frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{b}{m} \frac{dx}{dt}$

↳ solution is $x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega t + \phi)$
 damping term $-\frac{bt}{2m}$

Energy in SHM: For a spring & bob



SHM: $x(t) = x_m \cos \omega t$
 \hookrightarrow displacement w.r.t. equilibrium length
 $\omega = \sqrt{\frac{k}{m}}$

Total energy since system laying on a horizontal surface
 \rightarrow some you potential energy at all times
 \rightarrow can ignore

↳ $E = KE + \text{Potential} = \underbrace{\frac{1}{2} m v^2}_{\text{mass } m} + \underbrace{\frac{1}{2} k x^2}_{\text{spring } k}$

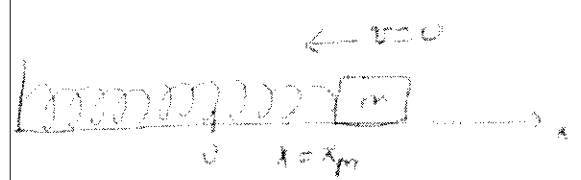
SHM: $\left. \begin{aligned} x(t) &= x_m \cos \omega t \\ v(t) &= \frac{dx}{dt} = -x_m \omega \sin \omega t \end{aligned} \right\} \rightarrow E = \frac{1}{2} m \omega^2 x_m^2 \sin^2 \omega t + \frac{1}{2} k x_m^2 \cos^2 \omega t$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 = \frac{k}{m}$$

$$E = \frac{1}{2} m \dot{x}_m^2 + \frac{1}{2} k x_m^2$$

$$= \frac{1}{2} k x_m^2 \left[\sin^2 \omega t + \cos^2 \omega t \right]$$

Total energy in a mass + spring system is constant over time! equal to $\frac{1}{2} k x_m^2$ in SHM



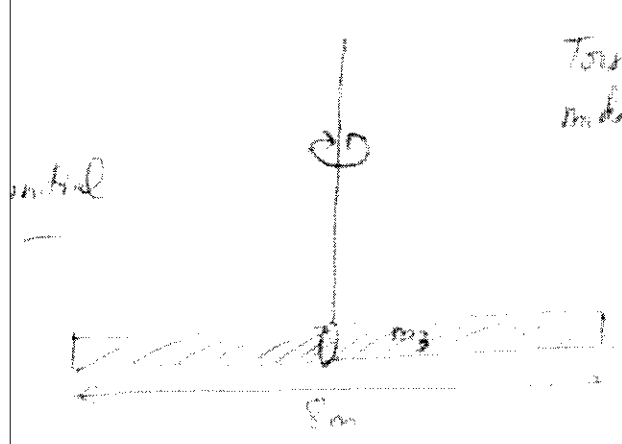
$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$E = \frac{1}{2} m 0^2 + \frac{1}{2} k x_m^2 = \frac{1}{2} k x_m^2$$

$v = v_m$

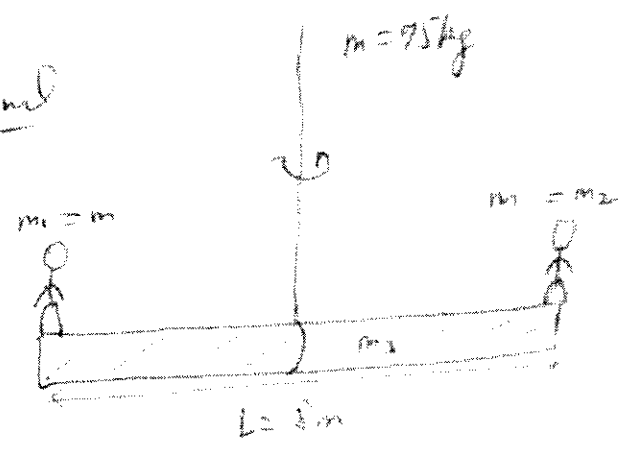


$$E = \frac{1}{2} m v_m^2 + \frac{1}{2} k 0^2 = \frac{1}{2} m x_m^2 \omega^2 = \frac{1}{2} k x_m^2 \frac{k}{k} = \frac{1}{2} k x_m^2$$



Torsional oscillations about middle axis $\Rightarrow \omega = \sqrt{\frac{k}{I}}$

And



$$\omega_f = \sqrt{\frac{K}{I_f}}$$

K - material of spring
 torsional constant
 $\rightarrow K_i = K_f = K$

$$I_f = I_i + 2m\left(\frac{L}{2}\right)^2$$

$$I_f > I_i \rightarrow \omega_f < \omega_i$$

$$\frac{\omega_f}{\omega_i} = 0.8 = \frac{\sqrt{\frac{K}{I_f}}}{\sqrt{\frac{K}{I_i}}} = \sqrt{\frac{I_i}{I_f}}$$

ω_f diminishes by 20% $\omega_f = \omega_i - 0.2\omega_i = 0.8\omega_i$

When there is a square root \rightarrow square up both sides

$$0.8^2 = \frac{I_i}{I_f} = \frac{I_i}{I_i + m\frac{L^2}{2}}$$

$$0.8^2 I_i + mL^2 \frac{0.8^2}{2} = I_i$$

$$I_i = \frac{mL^2 \frac{0.8^2}{2}}{1 - 0.8^2}$$

Also: $I_i =$ moment of inertia of a beam $\rightarrow L, m_3$
 center axis $I_i = \frac{1}{12} m_3 L^2$

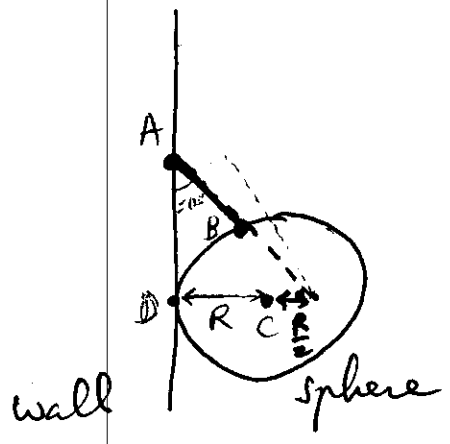
$$\frac{1}{12} m_3 L^2 = \frac{mL^2}{2} \frac{0.8^2}{1 - 0.8^2}$$

$$m_3 = 6m \frac{0.8^2}{1 - 0.8^2} = 650 \frac{0.8^2}{1 - 0.8^2}$$

$$m_3 = 800\text{kg}$$

12.29

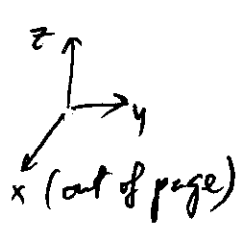
→ Smallest coef of friction μ_s b/w sphere & wall



Static Equilibrium

$$\sum \vec{F}_i = 0 \quad \begin{cases} y: & N - T \cos 60^\circ = 0 \quad (1) \\ z: & T \sin 60^\circ + \mu N - Mg = 0 \quad (2) \end{cases}$$

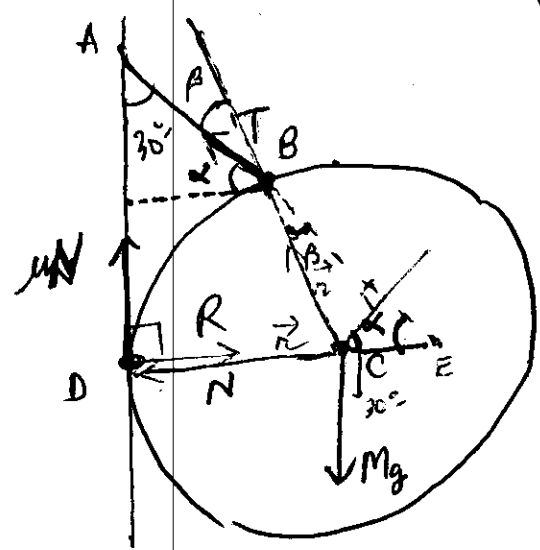
$\sum \vec{\tau}_i = 0 \rightarrow$ Pivot: C (no information on M !!)



$$\vec{\tau}_N + \vec{\tau}_{Mg} + \vec{\tau}_T = 0$$

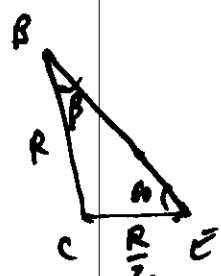
$$R \mu N (-\hat{z}) + RT \sin 25.6^\circ (\hat{z}) = 0$$

$$\boxed{-\mu N + T \sin 25.6^\circ = 0 \quad (3)}$$



$\alpha = 90 - 30 = 60^\circ$

Law of Sines:



$$\frac{\sin 60}{R} = \frac{\sin \beta}{\frac{R}{2}} \rightarrow 2 \sin \beta = \sin 60 \rightarrow \beta = \sin^{-1} \left(\frac{\sin 60}{2} \right) = 25.6^\circ$$

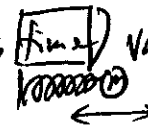
Use (1) in (3) : (1) $\rightarrow N = T \cos 60^\circ$

\rightarrow (3) $-\mu T \cos 60^\circ + T \sin 25.6^\circ = 0$

$$\mu \geq \frac{\sin 25.6^\circ}{\cos 60^\circ}$$

$$\mu \geq 0.86$$

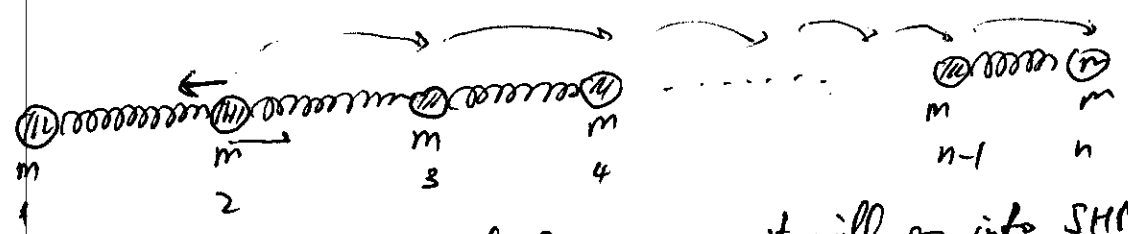
Ch 14 Wave Motion:

Oscillatory motion: periodic (regularly repeating in time) variation of a position or angle: 

Wave motion: the oscillation/periodic variation/perturbation is propagated in space

1) Propagation: Propagation of a perturbation is such that the objects involved (spring & mass for example) are local while the perturbation reaches as far as ~~the~~ possible.

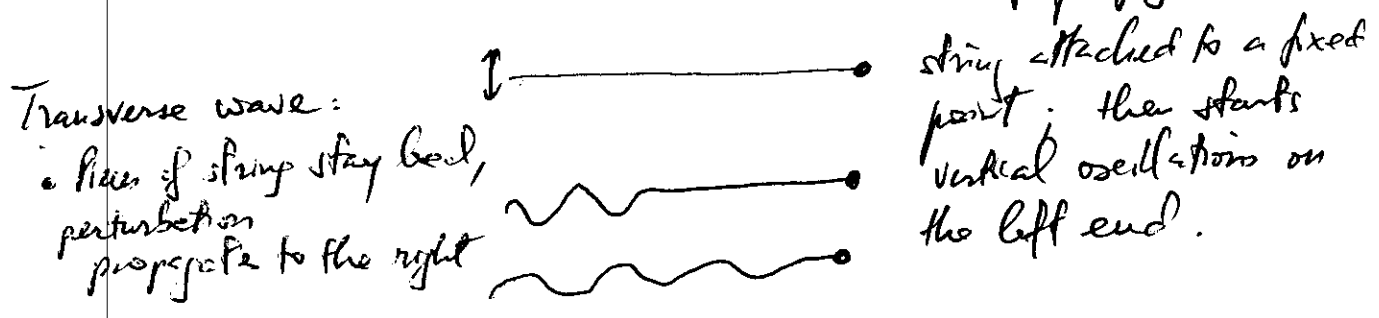
An example of a longitudinal wave →



If a perturbation is applied on m_2 → it will go into SHM being still local. However m_3 will start to do same thing, m_4 ... m_n : the perturbation is propagated while the objects involved are still local

2) Waves involve both time & space variation!

3) Two types:
 { Transverse waves: perturbation is perpendicular to direction of propagation.
 Longitudinal waves: perturbation is parallel to direction of propagation



Math description of a transverse wave:



$$y(x,t) = A \sin(kx - \omega t)$$

• Wave w/ time & space variation

• Perturbation about y

• Propagation is about x

A : amplitude of the wave & of the perturbation or oscillation

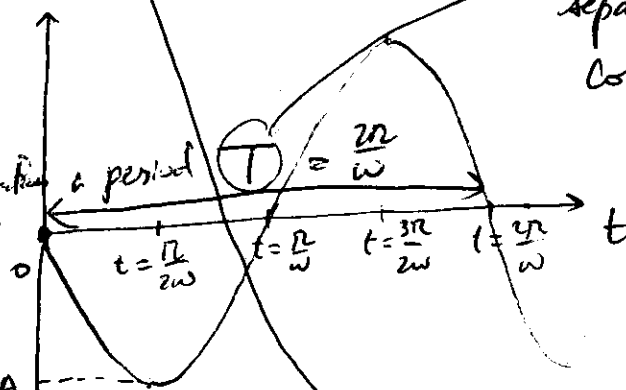
k : wave number = $\frac{2\pi}{\lambda}$ (m^{-1})

λ : wave length (separation in space b/w 2 consecutive peaks) (m)

ω : angular frequency = by plotting $\cos \omega t$ we found $\omega = \frac{2\pi}{T}$ (T : period or the separation in time b/w 2 consecutive peaks) (s^{-1})

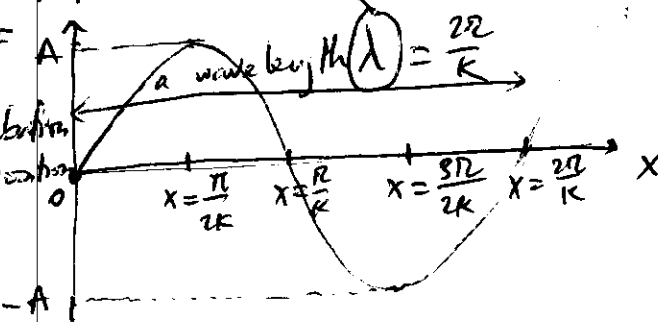
$$y(0,t) = A \sin(-\omega t) = -A \sin \omega t$$

Transverse perturbation @ $x=0$, over time



$$y(x,0) = A \sin(kx)$$

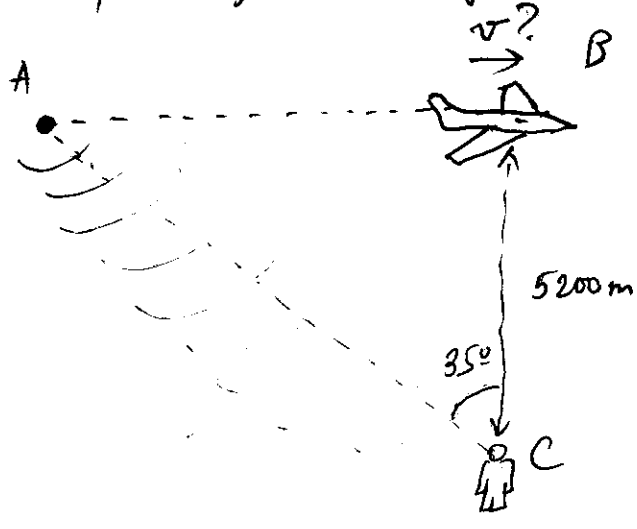
Transverse perturbation @ $t=0$ versus position



14.61

July 4th
Fireworks!

Sound wave speed $v_{\text{sound}} = 330 \text{ m/s}$
(Curiosity: light wave speed $v_{\text{light}} = c = 300000000 \text{ m/s}$)



When you see the fighter jet @ B (light wave reached your eyes) the sound from B did not reach your ears yet. Sound from A reached your ears when light from B reached your eyes.

$$v = \frac{d_{AB}}{t_{\text{sound AC}}} = \frac{d_{AB}}{\frac{d_{AC}}{v_{\text{sound}}}} = v_{\text{sound}} \frac{d_{AB}}{d_{AC}} = v_{\text{sound}} \cdot \sin 35^\circ = 330 \cdot \sin 35^\circ = 189 \frac{\text{m}}{\text{s}}$$

time it takes the sound to travel AC

(Curiosity: $v_{\text{aircraft}} = 189 \cdot 3.6 = 680 \frac{\text{km}}{\text{h}}$)