

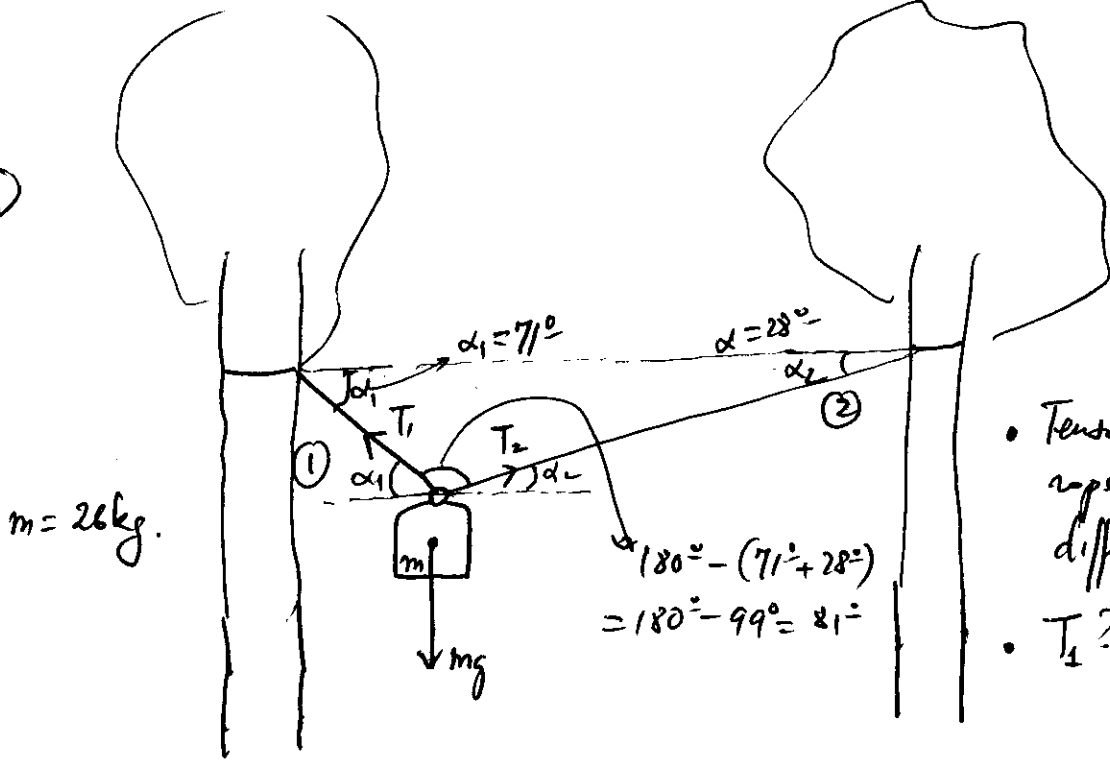
ch5. Using Newton's Law

- Static Equilibrium
- Multiple objects
- Frictional forces
- Circular motion.

Systematic approach or common strategies:

- 1) Understand the problem (making sense of the question) & make a sketch
- 2) Select a convenient coordinate system
 - ↳ most forces would point either along x- or y-axis of that coord system.
 - ↳ motion of interest is along either x- or y-axis
- 3) Make a free-body diagram of forces acting on each object → so we can derive the net force acting on each object correctly. Draw x & y components for those forces that are not already lined up along these axes.
- 4) Write 2nd Newton's Law ($\vec{F}_{net} = m \cdot \vec{a}$) for each object, for each component as needed.
- 5) Solve for what we are asked for, obtaining numeric solutions with correct units in S.I. Check if these numbers make sense.

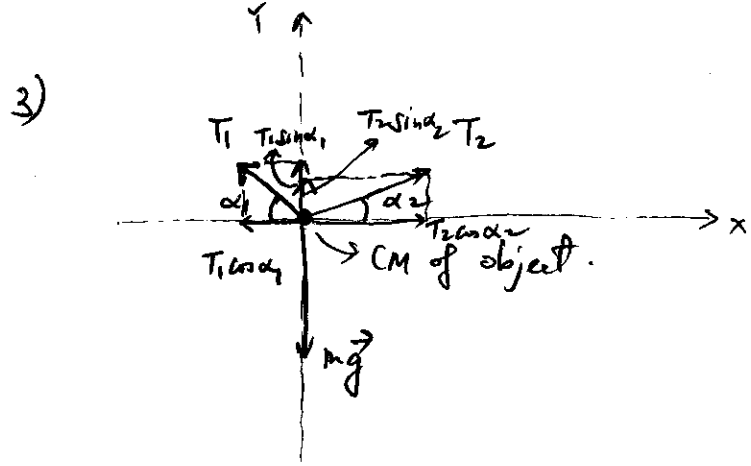
5.36



- Tensions in each rope are different: $T_1 \neq T_2$
- $T_1?$; $T_2?$

Steps.

- 1) ✓ We'll use 2nd Newton's law to find T_1 & T_2 as they balance mg .
- 2) Object: backpack: 3 forces: \vec{T}_1, \vec{T}_2, mg : none of these are perpendicular to each other. → Use standard x (horizontal), y (vertical)



This free-body diagram helps us derive the net force on the pack in each direction.

$$F_{net,x} = T_{2x} - T_{1x} = T_2 \cos 28^\circ - T_1 \cos 71^\circ$$

$$F_{net,y} = T_{2y} + T_{1y} - mg = T_2 \sin 28^\circ + T_1 \sin 71^\circ - mg$$

4) 2nd Newton's Law: $\vec{F}_{net} = m \cdot \vec{a}$

$$F_{net,x} = m \cdot a_x = m \cdot 0 = 0 \rightarrow T_2 \cos 28^\circ = T_1 \cos 71^\circ \quad (a)$$

$$F_{net,y} = m \cdot a_y = m \cdot 0 = 0 \rightarrow T_2 \sin 28^\circ + T_1 \sin 71^\circ = mg \quad (b)$$

5) Algebraic solution of a system of 2 equations with two unknowns: T_1 & T_2

$$(a) \quad T_1 = T_2 \frac{\cos 28^\circ}{\cos 71^\circ}$$

$$(b) \quad T_2 \sin 28^\circ + T_2 \frac{\cos 28^\circ}{\cos 71^\circ} \sin 71^\circ = mg$$

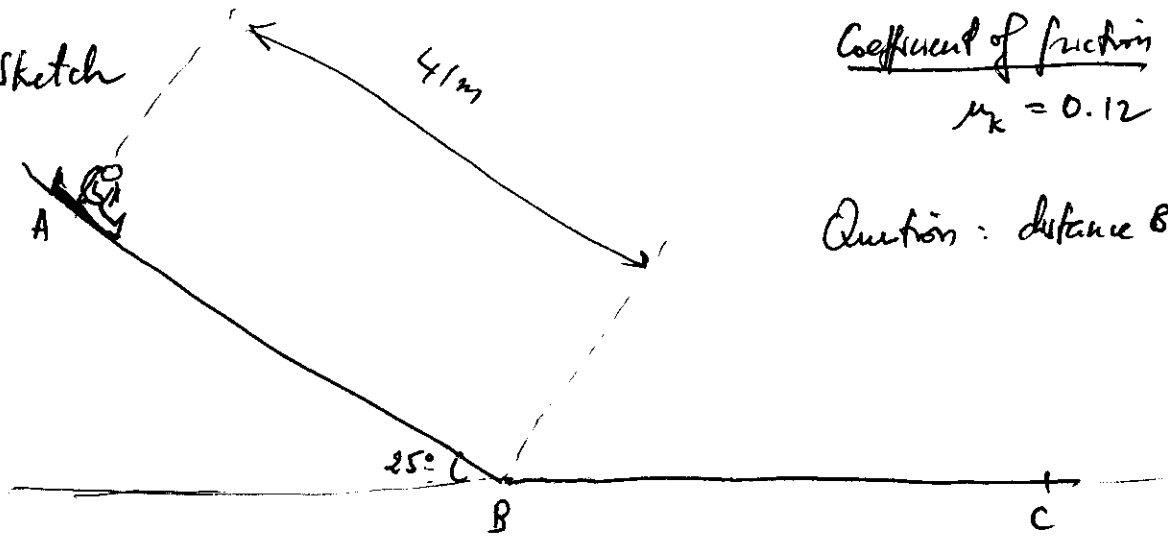
$$T_2 = \frac{26 \times 9.81}{\sin 28^\circ + \cos 28^\circ \cdot \tan 71^\circ} = 84 \text{ N}$$

$$(a) \rightarrow T_1 = 84 \frac{\cos 28^\circ}{\cos 71^\circ} = 228 \text{ N}$$

Do these numbers make sense? $T_1 > T_2 \rightarrow \text{Yes!}$

8.49

1) Sketch



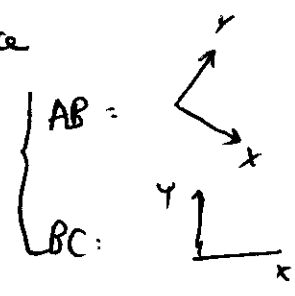
Coefficient of friction:
 $\mu_k = 0.12$

Question: distance BC?

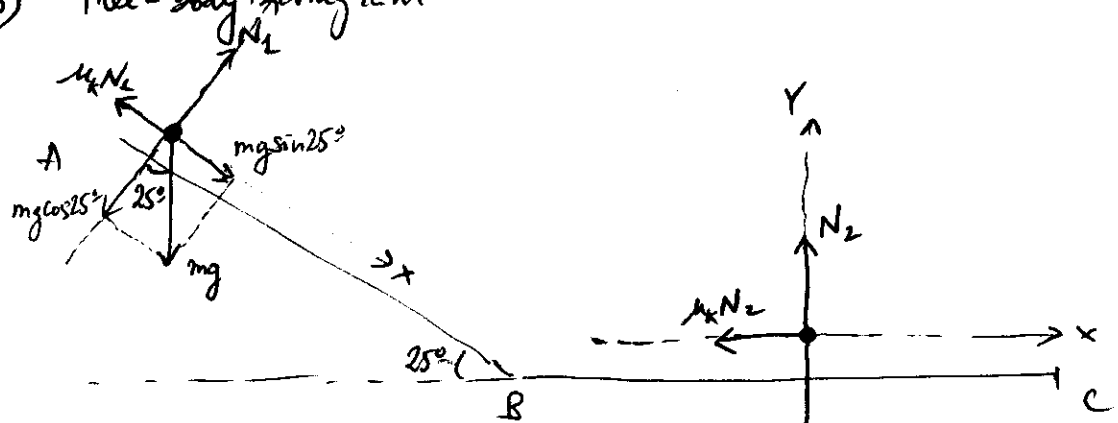
AB: constant acceleration
BC: constant deceleration due to frictional force

Will eventually stop here

2) Convenient coord. system



3) Free-body diagram.



$$\vec{F}_{net} = (mg \sin 25^\circ - \mu_k N_1) \hat{i} + (N_1 - mg \cos 25^\circ) \hat{j}$$

$$\vec{F}_{net} = -\mu_k N_2 \hat{i} + (N_2 - mg) \hat{j}$$

4) Write 2nd Newton's Law: $\vec{F}_{net} = m \cdot \vec{a}$

AB: $F_{net\ x} = m \cdot a_x \rightarrow mg \sin 25^\circ - \mu_k N_\perp = m a_x$
downhill acceleration

$F_{net\ y} = m \cdot 0 \rightarrow N_\perp - mg \cos 25^\circ = 0$

BC: $F_{net\ x} = m \cdot a_{2x} \rightarrow -\mu_k N_\perp = m \cdot a_{2x}$
deceleration
4th B&C

$F_{net\ y} = m \cdot 0 \rightarrow N_\perp - mg = 0$

5) **Solve for distance BC**: \rightarrow we'll need to use a kinematic equation (Newton's Law provides acceleration) can calculate using $v = v_0 + a \cdot t$

$$\frac{v^2 - v_0^2}{x - x_0} = 2 \cdot a_x \rightarrow \frac{0 - v_B^2}{(x - x_0)_{BC}} = 2 \cdot a_{2x}$$

unknown can calculate from Newton's equations.

- \rightarrow 3-step process
- a) Find initial velocity @ B
 - b) Find a_{2x}
 - c) Find $(x - x_0)_{BC}$

a) v_B : $\frac{v_B^2 - 0}{41} = 2 \cdot a_{1x}$

From Newton's equations in **AB**: $\frac{1}{2} g (\sin 25^\circ - \mu_k \cos 25^\circ) = \frac{1}{2} \cdot a_{1x}$

$\rightarrow v_B = \sqrt{82 \cdot 9.81 (\sin 25^\circ - 0.12 \cdot \cos 25^\circ)} = 15.9 \text{ m/s}$
 $a_{1x} = 3.08 \frac{m}{s^2}$

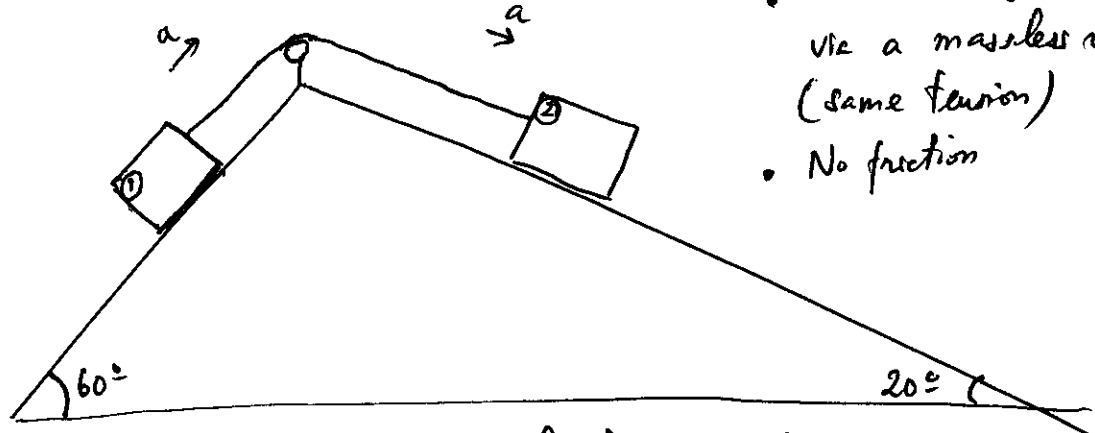
b) Calculate a_{2x} : From Newton's equations in [BC]:

$$\begin{aligned}
 -\mu_k mg &= m \cdot a_{2x} \rightarrow a_{2x} = -\mu_k g \\
 &= -0.12 \times 9.81 \\
 a_{2x} &= -1.18 \text{ m/s}^2
 \end{aligned}$$

$$c) \boxed{(x-x_0)_{BC}} = -\frac{v_B^2}{2 \cdot a_{2x}} = \downarrow \frac{15.9^2}{2 \cdot (+1.18)} = \boxed{107 \text{ m}}$$

Multiple Objects

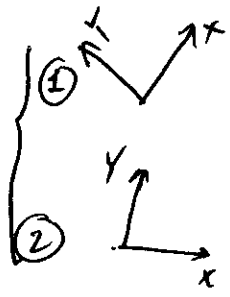
1)



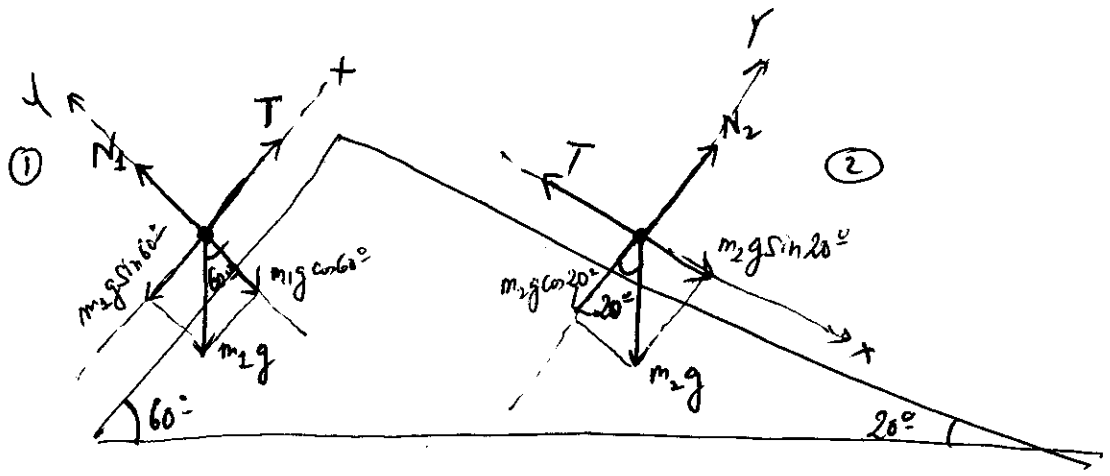
- Two boxes m_1, m_2 connected via a massless rope (same tension)
- No friction

Question: what is the acceleration of the system a ?
 If m_2 is sufficiently high the acceleration will result negative (opposite to what we assume here)

2) Convenient coord. system for each object:



3)



$$\vec{F}_{net,1} = (T - m_1 g \sin 60^\circ) \hat{i} + (N_1 - m_1 g \cos 60^\circ) \hat{j}$$

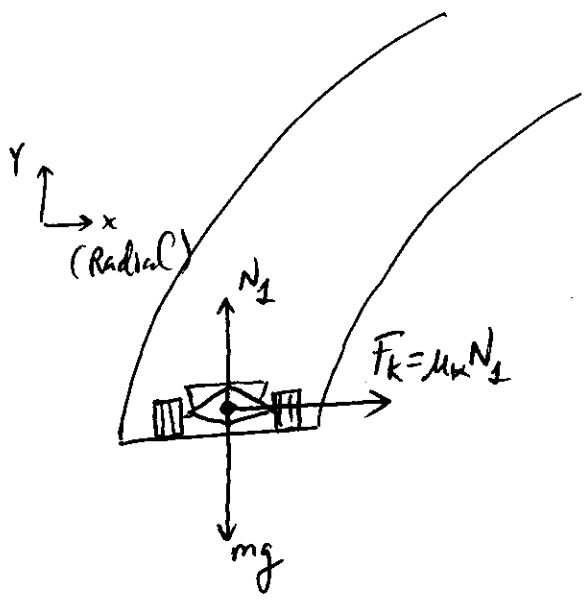
$$\vec{F}_{net,2} = (m_2 g \sin 20^\circ - T) \hat{i} + (N_2 - m_2 g \cos 20^\circ) \hat{j}$$

4) 2nd Newton's Law for each object.

$$\vec{F}_{net,1} = m_1 \cdot \vec{a} \quad \left\{ \begin{array}{l} T - m_1 g \sin 60^\circ = m_1 a \quad (1) \\ N_1 - m_1 g \cos 60^\circ = 0 \end{array} \right. \parallel \quad \vec{F}_{net,2} = m_2 \cdot \vec{a} \quad \left\{ \begin{array}{l} m_2 g \sin 20^\circ - T = m_2 \cdot a \quad (2) \\ N_2 - m_2 g \cos 20^\circ = 0 \end{array} \right.$$

Forces & Uniform Circular Motion:

Flat race track



$$\vec{F}_{net} = \mu_k N_L \hat{i} + (N_1 - mg) \hat{j}$$

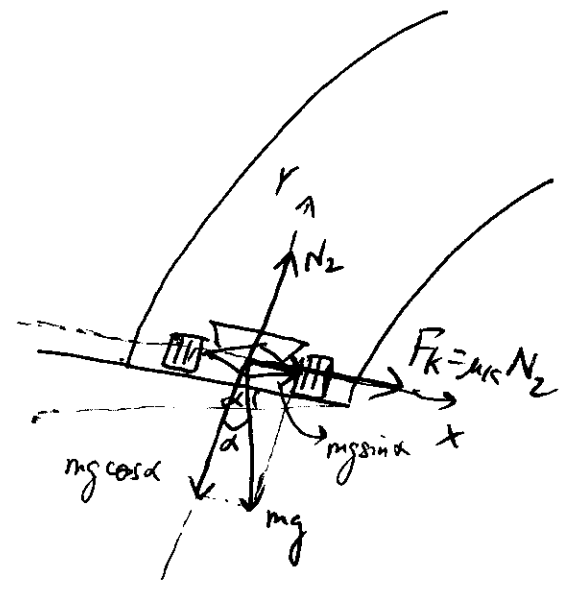
$$\vec{F}_{net} = m \cdot \vec{a} = m \cdot \frac{v^2}{R} \hat{i}$$

$$\left. \begin{aligned} N_1 - mg &= 0 \\ \mu_k N_1 &= \frac{m v^2}{R} \\ \mu_k mg &= \frac{m v^2}{R} \end{aligned} \right\}$$

UCM: object follows circular trajectory at constant speed, velocity vector has constant magnitude but changing direction due a radial acceleration $a = \frac{v^2}{R}$ (R: radius of curvature)

$$\rightarrow v_F = \sqrt{\mu_k g R} \quad (\text{flat track})$$

Slanted race track



$$\vec{F}_{net} = (\mu_k N_2 + mg \sin \alpha) \hat{i} + (N_2 - mg \cos \alpha) \hat{j}$$

$$\vec{F}_{net} = m \cdot \vec{a} = m \frac{v^2}{R} \hat{i}$$

$$\left. \begin{aligned} N_2 &= mg \cos \alpha \\ \mu_k mg \cos \alpha + mg \sin \alpha &= \frac{m v^2}{R} \end{aligned} \right\}$$

$$v_S = \sqrt{gR(\mu_k \cos \alpha + \sin \alpha)} \quad (\text{slanted track})$$

$$\frac{v_S}{v_F} = \sqrt{\frac{\mu_k \cos \alpha + \sin \alpha}{\mu_k}}$$

$$\frac{v_S}{v_F} > 1 \quad \text{if} \quad \begin{aligned} \mu_k \cos \alpha + \sin \alpha &> \mu_k \\ \sin \alpha &> \mu_k (1 - \cos \alpha) \end{aligned}$$

This allows race car to turn at a 'higher speed'!

Example:

$$\alpha = 20^\circ ; \mu_k = 0.2$$

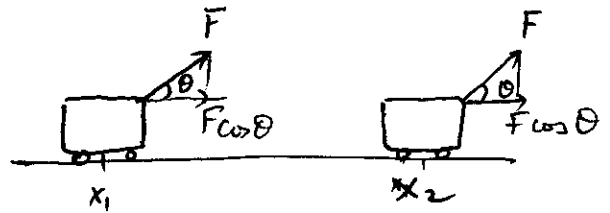
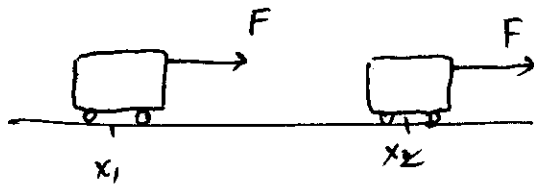
$$\frac{\sin 20^\circ}{1 - \cos 20^\circ} = 5.67 > 0.2$$

$$\frac{v_s}{v_f} = \sqrt{\frac{0.2 \times \cos 20^\circ + \sin 20^\circ}{0.2}} = 1.63$$

Ch 6 Work, Energy, and Power

Work: $\vec{F} \cdot d\vec{r}$

Applied force vector \vec{F}
 displacement vector $d\vec{r}$
 scalar product (of vectors)



Work done: Force in direction of motion x displacement

Here force in direction of motion is $F \cos \theta$

$$\text{Work} = F \cdot \Delta x = F \cdot (x_2 - x_1)$$

$$\text{Work} = F \cos \theta \cdot \Delta x = F \cos \theta (x_2 - x_1)$$

SI unit: $\text{N} \cdot \text{m} \equiv \text{J}$ (Joule)

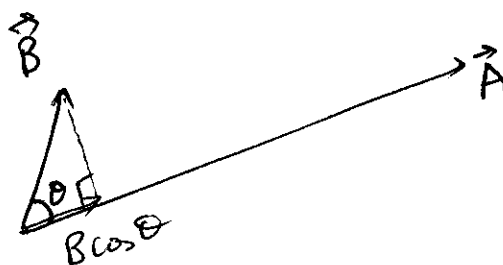
The y-component of the force applied $F \sin \theta \hat{j}$ is not doing any work: $\Delta y = 0$
 $\Rightarrow F \sin \theta \cdot \Delta y = 0$

Scalar product of vectors:

It's a product of two vectors that is a number.

A scalar is a number, such as: time, mass, energy, work etc

Scalar product of two vectors \vec{A} & \vec{B} :



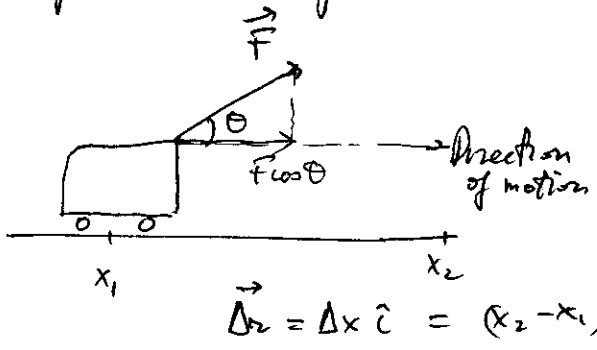
$$\vec{A} \cdot \vec{B} = A B \cos \theta$$

projection of \vec{B} onto \vec{A}

$$\vec{F} \cdot d\vec{r} = F \cos \theta \cdot dr$$

projection of \vec{F} onto direction of motion

In the previous example: $\Delta \vec{r} = \Delta x \hat{i}$:



$$\text{Work} = \vec{F} \cdot \Delta \vec{r} = F \cos \theta \cdot \Delta x$$

In general, if $\Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j}$:

$$\text{Work} = \vec{F} \cdot \Delta \vec{r} = (F \cos \theta \hat{i} + F \sin \theta \hat{j}) \cdot (\Delta x \hat{i} + \Delta y \hat{j})$$

$$\left. \begin{aligned} \hat{i} \cdot \hat{i} &= 1 \cos 0 \cdot 1 = 1 \checkmark \\ \hat{j} \cdot \hat{j} &= 1 \cos 0 \cdot 1 = 1 \checkmark \\ \hat{i} \cdot \hat{j} &= 1 \cos 90 \cdot 1 = 0 \\ \hat{j} \cdot \hat{i} &= 0 \end{aligned} \right\}$$

$$\text{Work} = F \cos \theta \cdot \Delta x + F \sin \theta \cdot \Delta y$$

So far \vec{F} : force applied is constant during the displacement $\Delta \vec{r}$. If it is changing during the displacement :

$$\text{Work} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

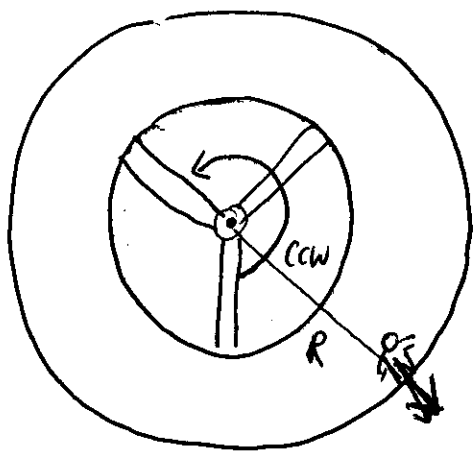
\downarrow Force applied
 \downarrow infinitesimal displacement vector

Example: work done by a spring : $F = -k \Delta x$ or $-kx$
 (if measured w.r.t. natural length)

$$\text{Work} = \int_0^x -kx \cdot dx = -k \int_0^x x dx = -\frac{1}{2} kx^2$$

\downarrow pulling it from natural length ($x=0$) to ~~pos~~ displacement x : in this case spring is receiving work (work done by us)

5.56



$R = 225\text{ m}$

Space station, view from above: it rotates in CCW direction at constant speed v
 Astronaut's mass: m

$$\frac{v^2}{R} = g$$

$$v = \sqrt{gR} = 46.9\text{ m/s}$$

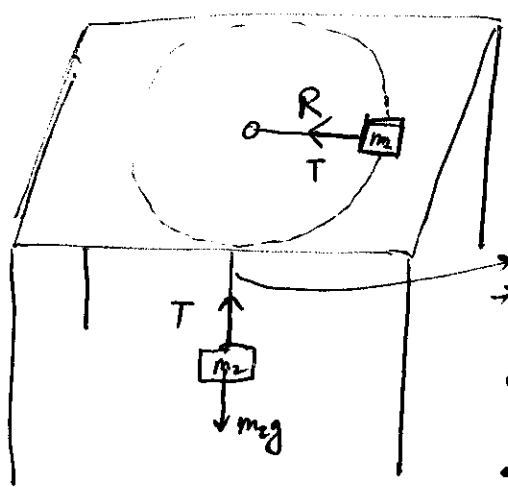
Speed: in revolution per min. \rightarrow (rpm)

1 rev is $2\pi R = 2\pi \cdot 225\text{ m}$

$$46.9 \frac{\text{m}}{\text{s}} \cdot \frac{1\text{ rev}}{2\pi \cdot 225\text{ m}} \cdot \frac{60\text{ s}}{1\text{ min}} = 1.99\text{ rpm}$$

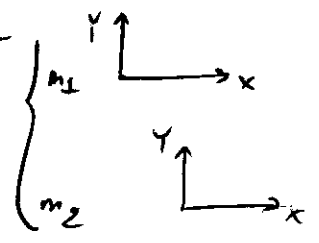
5.37

1)

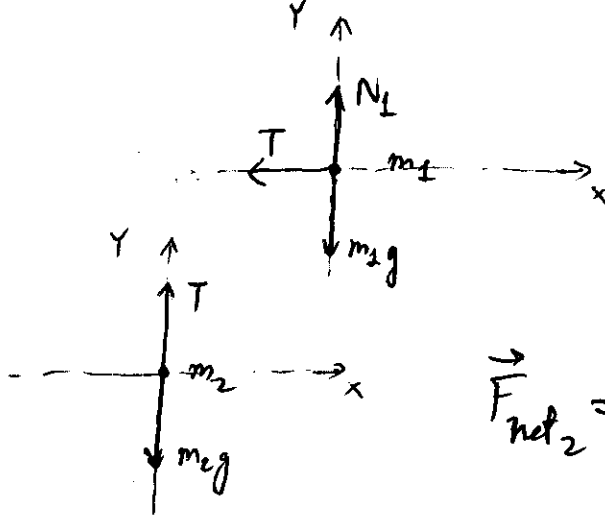


- massless string \rightarrow same tension throughout string.
- m_2 is stationary $\rightarrow a = 0$
- Tension $\left\{ \begin{array}{l} \rightarrow \text{holds } m_2 \text{ stationary} \\ \rightarrow \text{holds } m_1 \text{ in a UCM} \end{array} \right.$
- Frictionless table

2) Convenient coord. system for each object



3)



$$\vec{F}_{net_1} = -T \hat{i} + \underbrace{(N_1 - m_1g)}_0 \hat{j}$$

$$\vec{F}_{net_2} = 0 \hat{i} + \underbrace{(T - m_2g)}_0 \hat{j}$$

4) 2nd Newton's law for each object:

$$\vec{F}_{net_1} = -T \hat{i} = m_1 \cdot \vec{a}_1 = m_1 \cdot \frac{v^2}{R} (-\hat{i})$$

$$\boxed{T = m_1 \cdot \frac{v^2}{R}}$$

$$\vec{F}_{net_2} = 0 \rightarrow \boxed{T = m_2g} \quad \left. \vphantom{\vec{F}_{net_2}} \right\} m_2g = m_1 \frac{v^2}{R}$$

5) a) solve for $T \rightarrow T = m_2g$

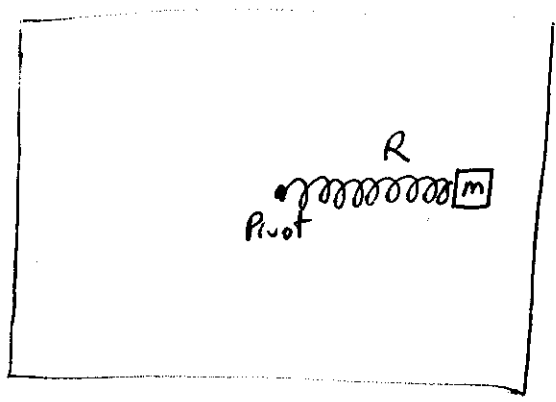
b) Period of circular motion: (also called T !): time to complete one circumference $2\pi R \rightarrow \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{m_2}{m_1}gR}}$

$$m_2g = m_1 \frac{v^2}{R} \rightarrow v = \sqrt{\frac{m_2}{m_1}gR}$$

$$\text{Period: } T = 2\pi \sqrt{\frac{m_1 R}{m_2 g}}$$

5.62

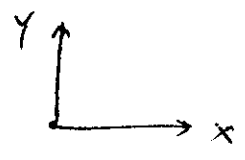
1)



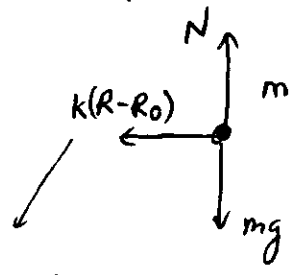
- View from above
- Frictionless table
- Spring has a natural length of $R_0 = 18 \text{ cm}$
- $K = 150 \frac{\text{N}}{\text{m}}$; $m = 2.1 \text{ kg}$

When m undergoes UCM (with $v = 1.4 \frac{\text{m}}{\text{s}}$) it needs a force towards the center of curvature, this force is provided by the spring. By the 3rd Newton's law, m will pull on the spring with a same force in opposite direction (away from center) \rightarrow spring get stretched to a new length R .
 \rightarrow Question $\rightarrow R$?

2) Convenient coord. system for m :



3)



$$\vec{F}_{\text{net}} = -k(R-R_0)\hat{i} + \underbrace{(N-mg)}_0 \hat{j}$$

Spring force is only proportional to the stretch from the natural length.

4) 2nd Newton's law for m : $\vec{F}_{\text{net}} = m \cdot \vec{a}$

$$-k(R-R_0)\hat{i} = m \cdot \frac{v^2}{R} (-\hat{i})$$

$$k(R-R_0) = \frac{mv^2}{R}$$

5) Solve for R : $kR^2 - kR_0R - mv^2 = 0$

$$150R^2 - \underbrace{150 \cdot 0.18}_{27} R - \underbrace{2.1 \cdot 1.4^2}_{4.12} = 0$$

$$R_0 = 18 \text{ cm} = 0.18 \text{ m}$$

$$R = \frac{27 \pm \sqrt{27^2 + 4 \cdot 150 \cdot 4.12}}{300}$$

↑ $\left\{ \begin{array}{l} + \rightarrow 0.279 \text{ m} \\ - \text{ gives a negative } R \rightarrow \text{ does not make sense} \end{array} \right.$

$R = 0.279 \text{ m}$

Energy: same dimension and so units as Work

For motions → Kinetic energy or K.E.:

Starting with 2nd Newton's Law:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m \cdot \frac{d\vec{v}}{dt}$$

m is constant

by net force on the object

Work applied to change the motion of an object of mass m: (between initial position \vec{r}_1 & final position \vec{r}_2)

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net} \cdot d\vec{r} = m \int_{\vec{r}_1}^{\vec{r}_2} \frac{d\vec{v}}{dt} \cdot d\vec{r} = m \int_{\vec{r}_1}^{\vec{r}_2} d\vec{v} \cdot \frac{d\vec{r}}{dt} \cdot dt$$

↓
scalar product

$$= m \int_{\vec{r}_1}^{\vec{r}_2} \vec{v} \cdot d\vec{v} = m \int_{\vec{r}_1}^{\vec{r}_2} v dv = \frac{1}{2} m v^2 \Big|_{\vec{r}_1}^{\vec{r}_2}$$

scalar product is commutative

in linear motion $\vec{v} \parallel d\vec{v}$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = KE_2 - KE_1$$

→ Work applied to change motion = K.E. = $\frac{1}{2} m v^2$.

Power: P : work or energy per unit time → SI unit: $\frac{J}{s}$

Also $\frac{J}{s}$ is called W (Watts)

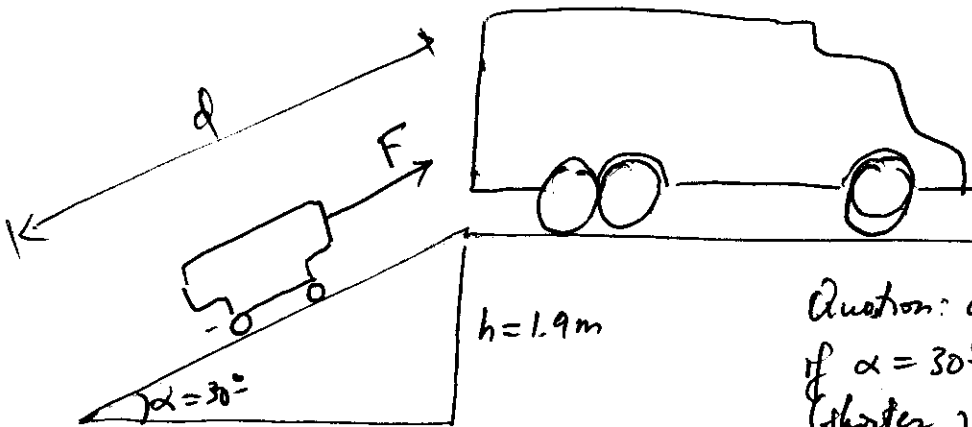
Car: { when you start it: $KE_1 = 0$
when it runs at 70mph → KE_2 } However if you reach KE_2 in less time → need more power!

Average Power: $\bar{P} = \frac{\Delta \text{Work}}{\Delta t}$

Instantaneous Power $P = \frac{d \text{Work}}{dt}$

Power & velocity: $P = \frac{d \text{Work}}{dt} = \frac{d(\vec{F} \cdot \Delta \vec{r})}{dt} = \vec{F} \cdot \frac{d \Delta \vec{r}}{dt} = \vec{F} \cdot \vec{v}$
 \vec{F} is constant

Application: Work & Force in moving a piano: load a piano onto a truck.



m = 460 kg
mu_k = 0.62

Question: compare work done if alpha = 30 degrees (shorter ramp) or 15 degrees (longer ramp) (same h = 1.9m)?

We apply a force F to move it up the ramp:

Work = F * d = F * (h / sin alpha)
length of ramp

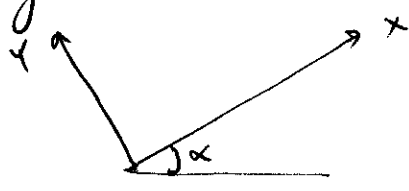
h/d = sin alpha

h = 1.9m
alpha = 15 degrees or 30 degrees
Need to find F!

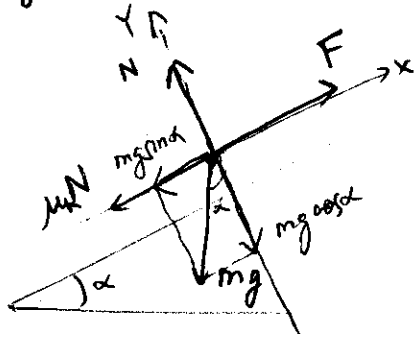
5 step strategy.

1) Sketch.

2) Convenient coord. system:



3) FBD for plane:



$$\vec{F}_{net} = (F - \mu_k N - mg \sin \alpha) \hat{i} + (N - mg \cos \alpha) \hat{j}$$

4) 2nd Newton's Law for plane: $\vec{F}_{net} = m \cdot \vec{a} = 0$
Minimum: moving it up with only constant speed $\rightarrow \vec{a} = 0$

$$\begin{cases} F = \mu_k N + mg \sin \alpha \\ N = mg \cos \alpha \end{cases} \Rightarrow \boxed{F = mg (\mu_k \cos \alpha + \sin \alpha)}$$



→ Work done = $F \cdot \frac{h}{\sin \alpha}$

| | |
|--------------------------------------|---|
| longer ramp ($\alpha = 15^\circ$) | $= mgh \frac{\mu_k \cos \alpha + \sin \alpha}{\sin \alpha}$ |
| | $= 460 \cdot 9.81 \cdot 1.9 \frac{0.62 \cos 15 + \sin 15}{\sin 15}$ |
| | $= 28.5 \text{ kJ}$ |
| shorter ramp ($\alpha = 30^\circ$) | $= 460 \cdot 9.81 \cdot 1.9 \frac{0.62 \cos 30 + \sin 30}{\sin 30}$ |
| | $= 17.8 \text{ kJ}$ |

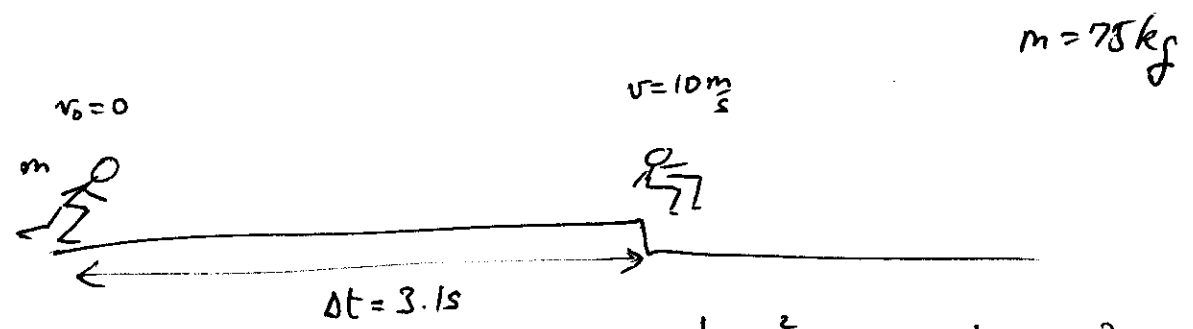
Steeper slope ($\alpha = 30^\circ$) \rightarrow less work!

Normally we prefer to do it at a smaller angle! Why?

$$\text{Force applied (minimum required)} = \begin{cases} \alpha = 15^\circ : F = 460 \cdot 9.81 (0.62 \cdot \cos 15^\circ + \sin 15^\circ) \\ = 3.88 \text{ kN} \\ \alpha = 30^\circ : F = 4.68 \text{ kN} \end{cases}$$

(a steeper ramp require less work but also a higher minimum force applied!)

6.38



$$P ? = \frac{\text{Work}}{\text{time}} = \frac{\Delta KE}{\text{time}} = \frac{\frac{1}{2}mv^2 - 0}{3.1} = \frac{\frac{1}{2} \cdot 75 \cdot 10^2}{3.1} = 1.21 \text{ kW}$$

Cars & trucks; H.P. ("Horse Power") 1 H.P. = 746 W

$$P_{\text{ jumper}} = 1.21 \text{ kW} \cdot \frac{1 \text{ HP}}{0.746 \text{ kW}} = 1.6 \text{ HP}$$

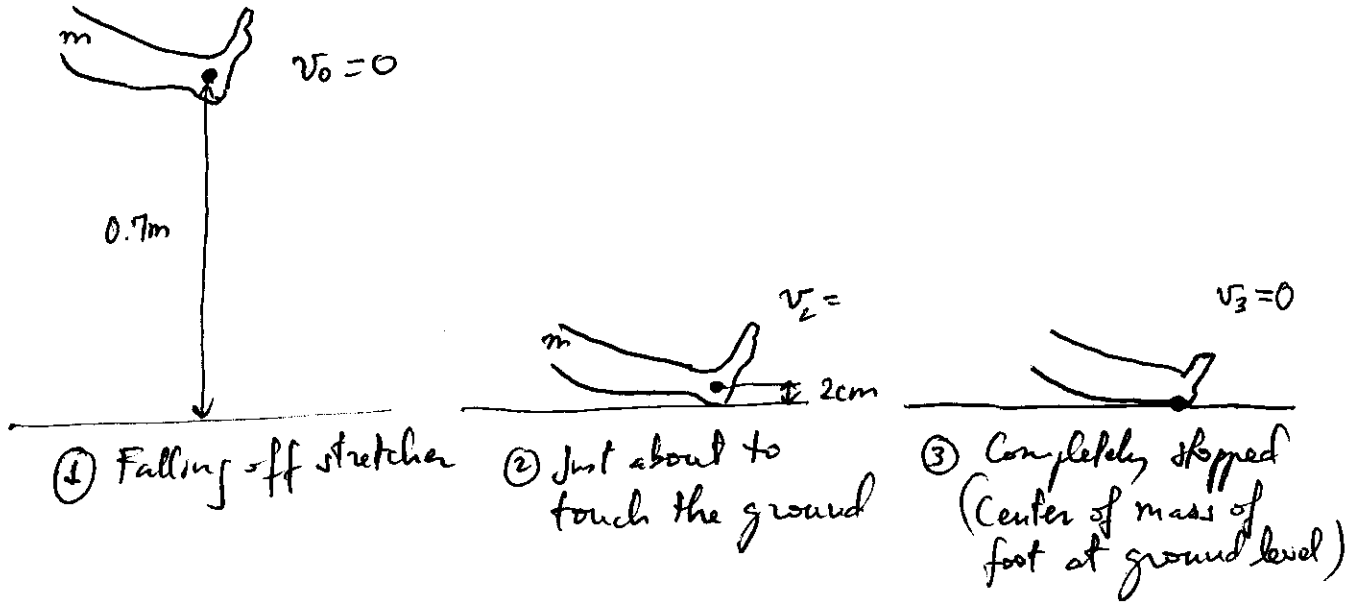
6.81

Similar to dropping an egg: 2 parts free fall

- 1) Const. accel. until almost touching the ground
- 2) Crashing until completely stopped (const. decel), very quick, where damages happen!

3 snapshots to describe the 2 parts of the free fall: (7L)

$m = 8 \text{ kg}$, falling off a stretcher $\rightarrow v_0 = 0$



constant accel.
or free fall

Stopping distance (vertical)
is 0.02 m

constant deceleration
 $KE_3 = 0$

$$KE_1 = 0$$

$$KE_2 = \frac{1}{2} m v_2^2$$

$$KE_3 = 0$$

Building up KE from
① to ② during
time Δt

Some KE is gone
from ② to ③
in very short time!
into leg (bones, tissues, etc.)

↓
Damage!

Want to calculate the stopping force F_{stop} (to see
if it is 80N or much more)

Alternative #1: use work/energy:

① → ② Find energy acquired by leg during free fall:

Alternative a) $KE_2 - KE_1 = \frac{1}{2}mv_2^2$

Alternative b) work done on leg by force of gravity:

$$\text{Work} = F \cdot h = mgh$$

② → ③ This energy is gone during the stopping distance of 0.02m

$$mgh = F_{\text{stopping}} \cdot 0.02\text{m} \rightarrow F_{\text{stop}} = \frac{mgh}{0.02\text{m}} = \frac{8 \cdot 9.81 \cdot 0.7}{0.02}$$

$$\boxed{F_{\text{stop}} = 2744\text{N}}$$

Alternative #2: use kinematic equations & Newton's Law:

① → ② Find $v_2 =$
constant accel.

$$\frac{v_2^2 - 0^2}{h} = 2 \cdot g \rightarrow v_2 = \sqrt{2 \cdot g \cdot h}$$

$$= \sqrt{2 \cdot 9.81 \cdot 0.7}$$

$$= 3.7 \frac{\text{m}}{\text{s}}$$

② → ③
const. decel. a

$$\frac{0^2 - v_2^2}{d_{\text{stop}}} = -2a \rightarrow a = \frac{v_2^2}{2 \cdot d_{\text{stop}}}$$

$$= \frac{3.7^2}{2 \cdot 0.02}$$

$$= 343.35 \frac{\text{m}}{\text{s}^2}$$

(deceleration!)

$$\boxed{F_{\text{stop}} = m \cdot a = 8 \cdot 343.35 = 2747\text{N}} \gg 80\text{N}!$$