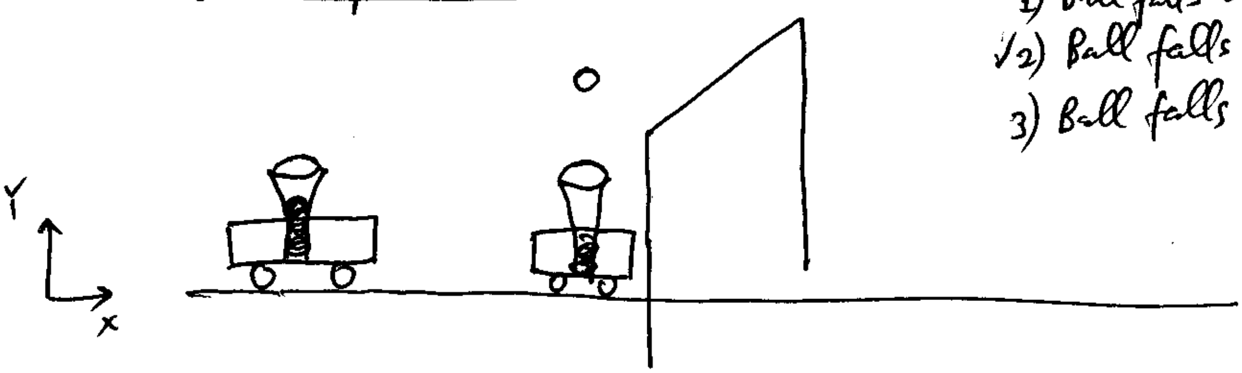
Equations of Motion in 2D:

Important assumption: $\boxed{\text{motions along perpendicular directions are independent! (For example: those along x \& y axes)}}$

Visual experiment #1

1) Ball falls behind cart
✓2) Ball falls back into funnel
3) Ball falls in front of cart



Frictionless track.
↓
(cart can go along @ constant velocity)

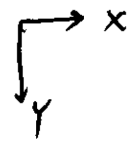
- v_x = constant
- Spring & funnel:
  I can release the spring to give the ball a vertical initial velocity:

Ball: it was travelling with the cart ⇒ has a constant velocity motion along x-direction. When it is ejected by the spring, it will acquire an additional vertical motion which is a constant acceleration motion.
Since motion along x & y are independent, ball falls back into funnel:

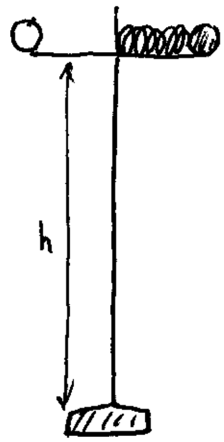
1) air resistance is ignored.
2) cart & track: no friction so cart maintains constant horizontal velocity.

Visual Experiment #2:

#1 #2



$v_{x_0} = 0$
$v_{y_0} = 0$
$a_y = g$
$y - y_0 = h$



$v_{x_0} = v_0$
$v_{y_0} = 0$
$a_y = g$
$y - y_0 = h$

Kinematic equation #2: $y - y_0 = v_{y0} t + \frac{1}{2} g t^2$

since $\left\{ \begin{matrix} y - y_0 \\ v_{y_0} \\ g \end{matrix} \right\}$ are the same for balls #1 & #2 $\rightarrow t_1 = t_2$

or both will hit the ground at the same time.

Kinematic equations in 2D for constant acceleration:

$$1D: \begin{cases} v = v_0 + at & (1) \\ x = x_0 + v_0 t + \frac{1}{2} a t^2 & (2) \end{cases} \quad \rightarrow \quad 2D \begin{cases} \left.\begin{matrix} v_x = v_{0x} + a_x t \\ v_y = v_{0y} + a_y t \end{matrix}\right\} (1) \\ \left.\begin{matrix} x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \end{matrix}\right\} (2) \end{cases}$$

$\cancel{v_x = v_{0x} + a_y t}$ v_x can't be affected by a_y!

Also: 2D $\begin{cases} \vec{v} = \vec{v}_0 + \vec{a} t & (1) \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 & (2) \end{cases}$

where: $\begin{cases} \vec{v} \equiv v_x \hat{i} + v_y \hat{j} \\ \vec{r} \equiv x \hat{i} + y \hat{j} \\ \vec{a} \equiv a_x \hat{i} + a_y \hat{j} \end{cases}$ $\Bigg\|$ $\vec{v} = \frac{d\vec{r}}{dt} = \underbrace{\frac{dx}{dt}}_{v_x} \hat{i} + \underbrace{\frac{dy}{dt}}_{v_y} \hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$

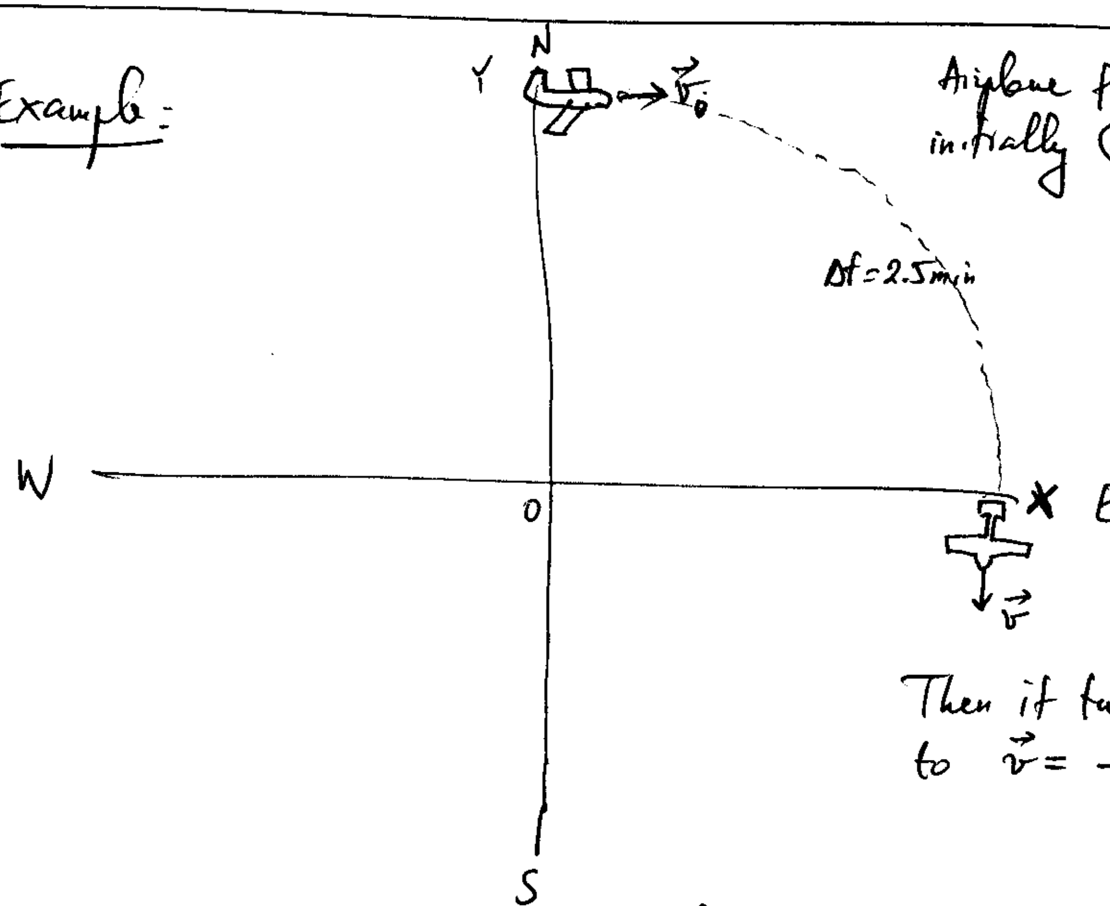
$= \frac{d^2 x}{dt^2} \hat{i} + \frac{d^2 y}{dt^2} \hat{j}$

<u>In 3D:</u> in vector notation it's the equation as with 2D:

$$\begin{cases} \vec{v} = \vec{v}_0 + \vec{a}\cdot t & (1) \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\cdot t & (2) \end{cases}$$

$$\begin{cases} \vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k} \\ \vec{v} = v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k} \\ \vec{a} = a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k} \end{cases}$$

<u>Example:</u>



Airplane flying eastward initially @ $\vec{v}_0 = 2100\,\frac{km}{h}\,\hat{i}$

Then it turns southward to $\vec{v} = -1800\,\frac{km}{h}\,\hat{j}$

What is the average acceleration during this time interval? m/s^2

<u>Conversions:</u> $2100\,\frac{km}{h}\,\frac{1000\,m}{1\,km}\,\frac{1\,h}{3600\,s} = \frac{2100}{3.6} = 583.3\,\frac{m}{s}$ | $\Delta t = 2.5\,min = 150\,s$

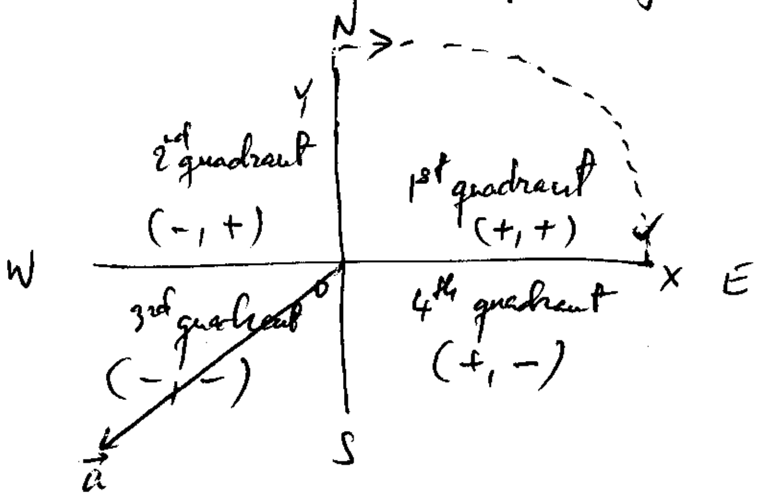
$1800\,\frac{km}{h} \longrightarrow \frac{1800}{3.6} = 500\,\frac{m}{s}$

Average acceleration: $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v} - \vec{v}_0}{\Delta t} = \dfrac{-500\hat{j} - 583.3\hat{i}}{150} \; \dfrac{(\frac{m}{s})}{(s)}$

$$\boxed{\vec{a} = -3.3\hat{j} - 3.9\hat{i} \left(\frac{m}{s^2}\right)} \qquad \underbrace{\begin{cases} a_x = -3.9 \frac{m}{s^2} \\ a_y = -3.3 \frac{m}{s^2} \end{cases}}$$

Cartesian components of the average acceleration.

What is the direction of average acceleration?



The average acceleration in the 3rd quadrant makes sense for this eastward to southward change of direction!

What are the polar coordinates of this average acceleration?

$$a = \sqrt{(-3.9)^2 + (-3.3)^2} = 5.1 \frac{m}{s^2}$$

Magnitude of the average acceleration.

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-3.3}{-3.9}\right) = \tan^{-1}\left(\frac{3.3}{3.9}\right) = \underline{40.5^\circ}$$

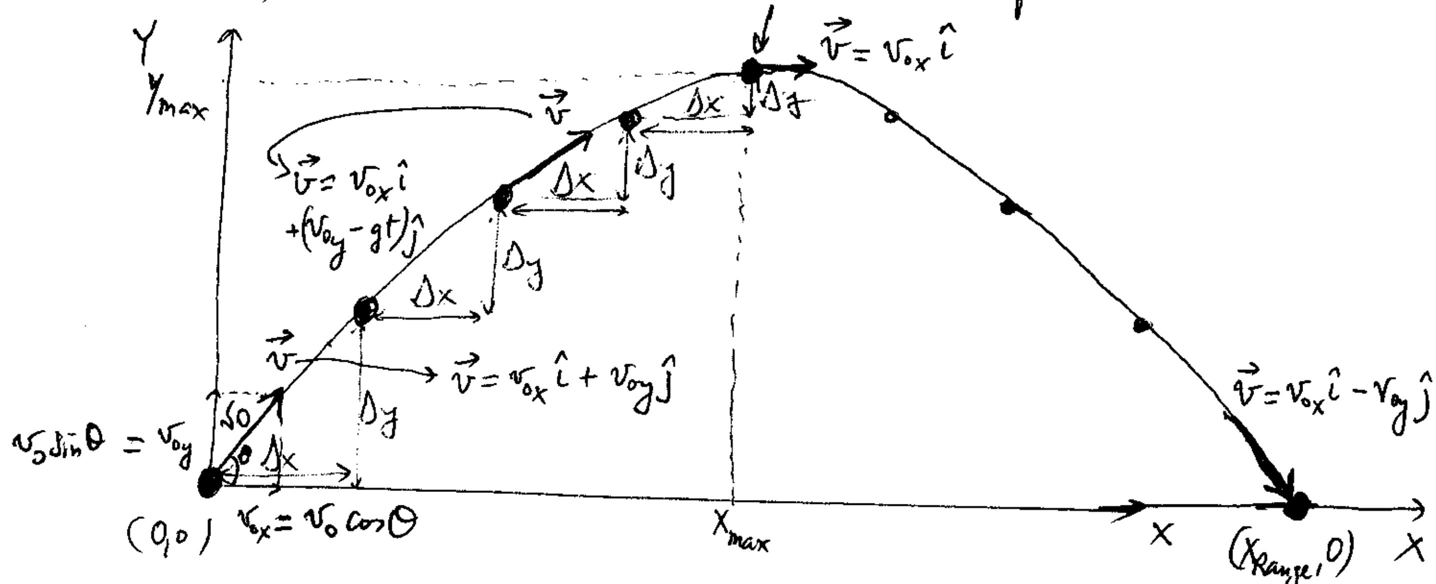
mistake by the calculator. → $\boxed{+180^\circ}$

$\boxed{220.5^\circ}$

Third quadrant!

Projectile Motion:

In the movie for the rolling cart with a ball ejected vertically by a spring: the ball followed a projectile motion. (a combination of two perpendicular motion: one at constant velocity and the other – in a perpendicular direction – at constant acceleration)



Snapshots of the object @ equal time intervals Δy's are getting smaller on the way up while Δx's stay the same

Defining features of a projectile motion:

1) Vertical motion is a constant deceleration/acceleration due to gravity
2) Horizontal motion is a constant velocity or uniform motion.

Note: • parabolic trajectory is symmetric w.r.t. the max. altitude point.

• examples of projectile motion: ball launched by a catapult or trebuchet, canon, short-range missile, etc.. baseball, etc.

Mathematical description:

Kinematic equations for constant acceleration in 2D $\left\{ \vec{a}: \begin{array}{l} a_x = 0 \\ a_y = \pm g \end{array} \right\}$

1) $\vec{v} = \vec{v}_0 + \vec{a} \cdot t \quad \begin{cases} v_x = v_{0x} \\ v_y = v_{0y} \mp g \cdot t \end{cases}$

(upward motion: −
downward motion: +)

2) $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2}\vec{a} \cdot t^2 \quad \begin{cases} x = x_0 + v_{0x} \cdot t \\ y = y_0 + v_{0y} \cdot t \mp \frac{1}{2} g \cdot t^2 \end{cases}$

- θ is useful in practical applications of projectile motion: (angle of initial velocity of the object) $\begin{cases} v_{0x} = v_0 \cos\theta \\ v_{0y} = v_0 \sin\theta \end{cases}$
- Normally we can place origin at initial position ($x_0 = 0 = y_0$)

2) $\begin{cases} x = v_0 \cos\theta \cdot t \quad \rightarrow t = \dfrac{x}{v_0 \cdot \cos\theta} \\ y = v_0 \sin\theta \cdot t \mp \frac{1}{2} g \cdot t^2 \end{cases} \longrightarrow \boxed{y = x \cdot \tan\theta \mp \dfrac{g}{2} \dfrac{x^2}{v_0^2 \cdot \cos^2\theta}}$

Relating x to y:
Trajectory Equation:
it's a parabola in XY plane

$$\boxed{\text{Max. altitude point: } (x_{max}, y_{max}) = \left(\frac{v_0^2 \boxed{\sin 2\theta}}{2g}, \frac{v_0^2 \boxed{\sin^2\theta}}{2g} \right)}$$

Proof: KE # 1): $v_y = \underbrace{v_0 \sin\theta}_{v_{0y}} - gt$

@ $(x_{max}, y_{max}) \rightarrow v_y = 0 = v_0 \sin\theta - gt \rightarrow \boxed{t_{max} = \dfrac{v_0 \sin\theta}{g}}$

KE # 2): $x_{max} = v_{0x} \cdot t_{max} = v_0 \cos\theta \dfrac{v_0 \sin\theta}{g} = \dfrac{v_0^2 \cos\theta \sin\theta}{g}$

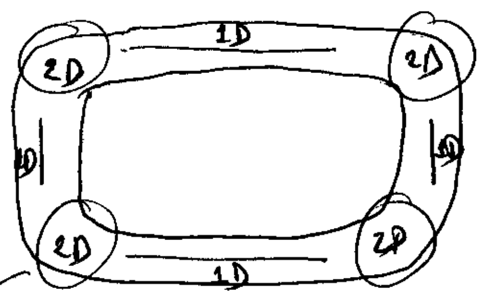
$\sin 2\theta = 2\cos\theta\sin\theta \rightarrow \boxed{x_{max} = \dfrac{v_0^2 \sin 2\theta}{2g}}$

$$y_{max} = v_{oy} \cdot t_{max} - \frac{1}{2} g \cdot t_{max}^2$$

$$= \frac{v_0^2 \sin^2\theta}{g} - \frac{1}{2}\cancel{g} \cdot \frac{v_0^2 \sin^2\theta}{g^{\cancel{2}}}$$

$$\boxed{y_{max} = \frac{v_0^2 \sin^2\theta}{2g}}$$

Range: $\begin{cases} x_{Range} = \dfrac{v_0^2 \sin 2\theta}{g} \\ y_{Range} = 0 \end{cases}$



Uniform Circular Motion: (UCM) → speed is constant but direction is changing.

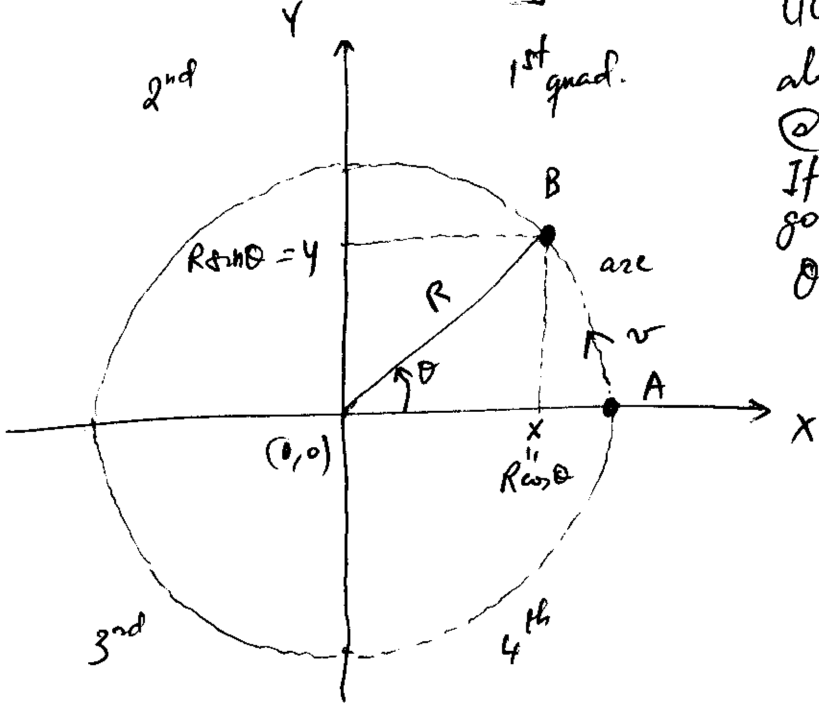
↳ Constant speed along the circular motion
(velocity can change)

$\underbrace{\vec{v}}_{\text{velocity}} = (\underbrace{v}_{\text{speed}}, \theta)$

polar coords.
(magnitude & direction)

speed is constant in UCM
since θ is changing in a circular motion, the velocity is changing.

Since $\vec{v}$ is changing $\rightarrow$ $\boxed{\vec{a} = \dfrac{d\vec{v}}{dt}}$:



UCM → object going along circular path @ constant speed v. If it takes time t to go from A to B

$$\theta = \frac{arc}{R} = \frac{v \cdot t}{R}$$

$$\vec{r} = x\hat{i} + y\hat{j} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

(position vector)

$$\vec{r} = R\left[\cos\frac{v \cdot t}{R}\,\hat{i} + \sin\frac{v \cdot t}{R}\,\hat{j}\right]$$

(Position of the object @ any time t during its U.C.M.)

$$|\vec{r}| = \sqrt{R^2\underbrace{\left[\cos^2\frac{v \cdot t}{R} + \sin^2\frac{v \cdot t}{R}\right]}_{1}} = R \quad \text{(makes sense)}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \cancel{R}\left[-\frac{v}{\cancel{R}}\sin\frac{v \cdot t}{R}\,\hat{i} + \frac{v}{\cancel{R}}\cos\frac{v \cdot t}{R}\,\hat{j}\right]$$

$$\vec{v} = v\left[-\sin\frac{v \cdot t}{R}\,\hat{i} + \cos\frac{v \cdot t}{R}\,\hat{j}\right]$$

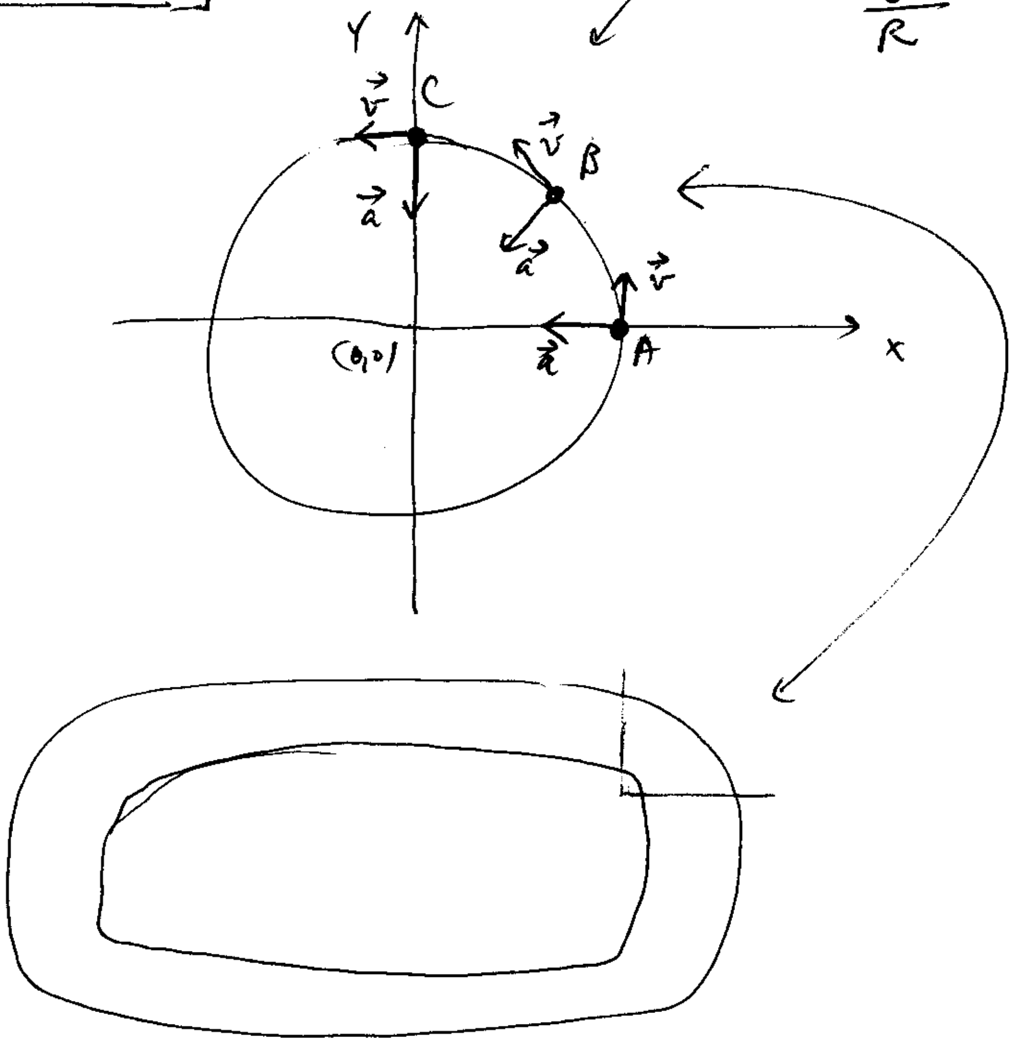
$$|\vec{v}| = \sqrt{v^2\underbrace{\left[\sin^2\frac{v \cdot t}{R} + \cos^2\frac{v \cdot t}{R}\right]}_{1}} = v \quad \text{(makes sense!)}$$

$$\vec{a} \overset{\text{UCM}}{\downarrow}{=} \frac{d\vec{v}}{dt} = v\left[-\frac{v}{R}\cos\frac{v.t}{R}\,\hat{i} - \frac{v}{R}\sin\frac{v.t}{R}\,\hat{j} \right]$$

$$\vec{a} = -\frac{v^2}{R}\Bigg[\underbrace{\cos\frac{v.t}{R}\,\hat{i} + \sin\frac{v.t}{R}\,\hat{j}}_{\text{Magnitude } = 1} \Bigg]$$

$\longrightarrow$ Acceleration vector is changing direction but its magnitude is constant & equal to $\frac{v^2}{R}$

$$\boxed{|\vec{a}| = \frac{v^2}{R}}$$

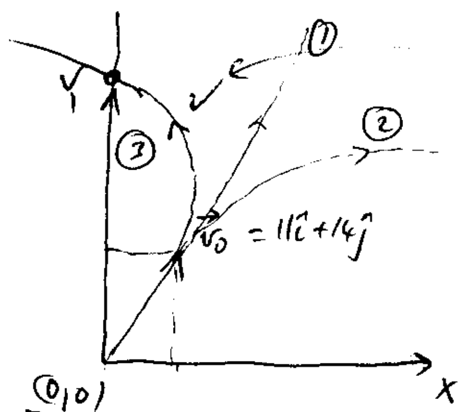


3.60

Facts: $\begin{cases} \vec{v}_0 = 11\hat{i} + 14\hat{j} \ \frac{m}{s} & @ \ (x,y) = (0,0) \\ \vec{a} = -1.2\hat{i} + 0.26\hat{j} \ \frac{m}{s^2} & \text{(const. acceleration)} \end{cases}$

a) When does the particle cross the y-axis?

Qualitative check if this question makes sense given the data:



$\vec{a} = \begin{cases} \text{negative in } x \\ \text{positive in } y \end{cases}$

→ OK will find t from our 2D KE equations:

1) $\vec{v} = \vec{v}_0 + \vec{a}\cdot t \quad \begin{cases} v_x = v_{0x} + a_x \cdot t \\ v_y = v_{0y} + a_y \cdot t \end{cases}$

2) $\vec{r} = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{1}{2}\vec{a}\cdot t^2 \quad \begin{cases} x = x_0 + v_{0x}\cdot t + \frac{1}{2}a_x t^2 \\ y = y_0 + v_{0y}\cdot t + \frac{1}{2}a_y t^2 \end{cases}$

$(x_0 = 0 = y_0)$

Crossing y-axis $\left\{ \boxed{x = 0} \right. \rightarrow$

$0 = 11 \cdot t + \frac{1}{2}(-1.2)\cdot t^2$

$0 = 11 - 0.6t \rightarrow \boxed{t = \frac{11}{0.6} = 18.3\,s}$

b) y coord. as it crosses the y-axis:

$y = 14 \cdot 18.3 + \frac{1}{2}\, 0.26 \cdot 18.3^2 = 300\,m.$

c) how fast & in what direction (final velocity information in polar coords.) as it crosses the y-axis?

KE#1 $\begin{cases} v_x = 11 - 1.2 \cdot 18.3 = -10.96 \ \frac{m}{s} \\ v_y = 14 + 0.26 \cdot 18.3 = 18.8 \ \frac{m}{s} \end{cases} \rightarrow \boxed{2^{nd} \text{ quadrant}}$

Polar coords: $\begin{cases} v = \sqrt{10.96^2 + 18.8^2} = 21.7 \ \frac{m}{s} \\ \theta_v = \tan^{-1} \dfrac{18.8}{-10.96} = -60^\circ \end{cases}$

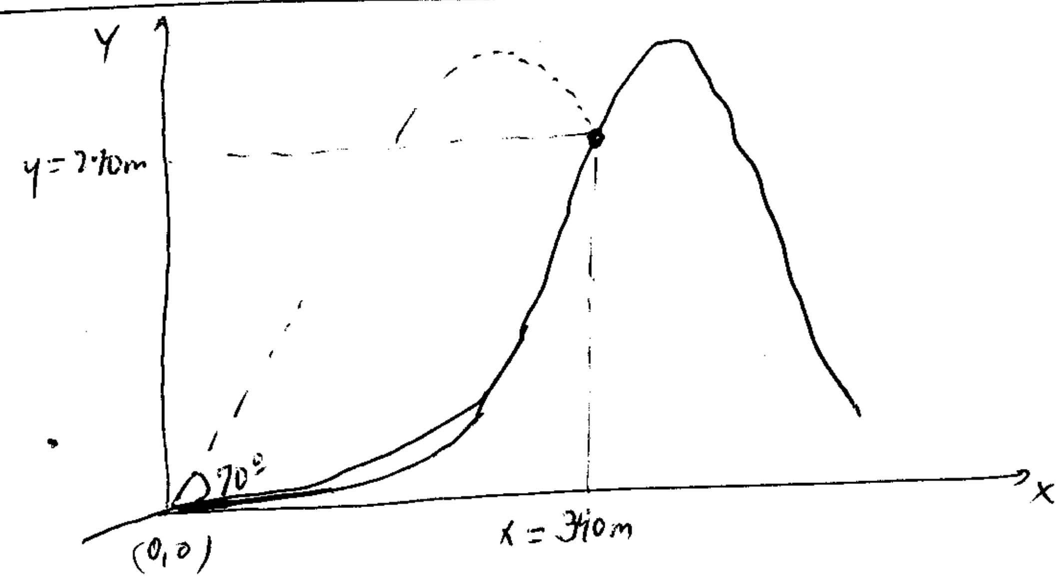
$\downarrow$

4^{th} quad.

$\downarrow$

$-60^\circ \boxed{+180^\circ} = \boxed{120^\circ}$

3.73



Reasoning: $(x, y) = (390m, 270m)$ has to belong to the parabolic trajectory or projectile motion, in another words they have to satisfy the trajectory equation.

$$y = x \cdot \tan\theta - \frac{g}{2} \frac{x^2}{v_0^2 \cdot \cos^2\theta} \quad \begin{cases} x = 390m; \ y = 270m \\ \theta = 70^\circ \\ \rightarrow v_0 ? \end{cases}$$

$$x\tan\theta - y = \frac{g}{2}\frac{x^2}{v_0^2\cos^2\theta}$$

$$v_0^2 = \frac{g}{2}\frac{x^2}{(x\tan\theta - y)\cos^2\theta}$$

$$\boxed{v_0 = \sqrt{\frac{9.81}{2}\frac{390^2}{(390\tan 70^\circ - 270)\cos^2 70^\circ}} = 89.8\ \frac{m}{s}}$$

3.52 $\vec{r} = 12t\,\hat{i} + (15t - 5t^2)\,\hat{j}$ (m)

a) $\vec{r}(t=2s)$? $= 24\,\hat{i} + 10\,\hat{j}$ (m)

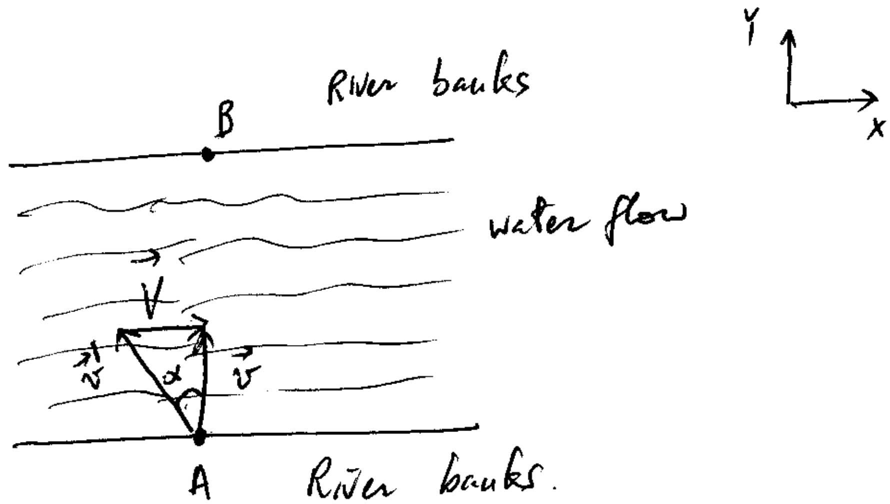
b) $$\overline{\vec{v}} = \frac{\vec{v}(t=2s) + \vec{v}(t=0s)}{2} = \frac{12\hat{i} - 5\hat{j} + 12\hat{i} + 15\hat{j}}{2} = \frac{24\hat{i} + 10\hat{j}}{2} = \boxed{12\hat{i} + 5\hat{j}\ \frac{m}{s}}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 12\,\hat{i} + (15 - 10t)\,\hat{j} \quad \left(\frac{m}{s}\right) \quad \begin{cases} \vec{v}(t=2s) = 12\hat{i} - 5\hat{j} \\ \vec{v}(t=0s) = 12\hat{i} + 15\hat{j} \end{cases}$$

c) Instantaneous velocity @ t = 2s:

$$\vec{v}(t=2s) = 12\,\hat{i} + (15 - 10\cdot 2)\,\hat{j} = 12\,\hat{i} - 5\,\hat{j}\ \frac{m}{s}$$

3.34



Identify the 3 velocities:

1) $\vec{v}'$ = boat wrt water ($v' = 1.3\,\frac{m}{s}$)

2) $\vec{V} = 0.57\,\frac{m}{s}\,\hat{i}$

3) $\vec{v} = v\hat{j}$ boat wrt ground (fixed)

$$\vec{v} = \vec{v}' + \vec{V} \rightarrow \boxed{\vec{v}' = \vec{v} - \vec{V} = v\hat{j} - 0.57\hat{i}}$$

c) Question: direction of $\vec{v}'$: $\quad v' = 1.3\,\frac{m}{s} = \sqrt{0.57^2 + v^2}$

$$1.3^2 = 0.57^2 + v^2$$

$$v = \sqrt{1.3^2 - 0.57^2} = 1.17\,\frac{m}{s}$$

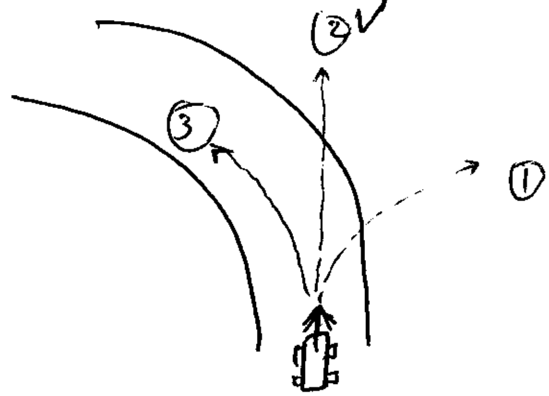
$$\alpha = \tan^{-1}\left(\frac{0.57}{1.17}\right) = 26^\circ$$

b) How long to cross $AB = 63\,m$: is in the y-direction $\rightarrow$ use $\vec{v} = 1.17\hat{j}$ in that same direction (straight across the river) $\rightarrow$ $t = \frac{63}{1.17}\,s = \boxed{53.9\,s}$

Ch 4 $\boxed{\text{Force}}$ & Motion

Motion → $\vec{r}, \vec{v}, \vec{a}, t$

Visual experiment: driving in curved road, downhill and icy



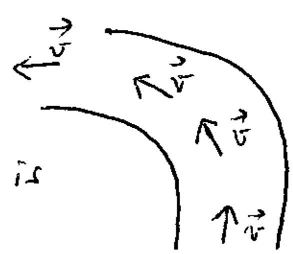
Friction force is missing $\boxed{\text{b/w tires \& road}}$

Our vehicle would follow path #2: the agent or force that would normally pull the vehicle towards the center of curvature is absent: the $\boxed{\text{friction force}}$ is absent.

Conclusion: vehicle entering the curve in forward direction will continue to do so if there is no force or agent to change ~~#~~ its motion. A force is needed to change the motion.

- There can be several forces acting on an object, the net force is the one we need to look at.

- For example in a UCM: $\vec{v}$ is changing direction continuously, a force is needed to makes this happens: by intuition it has to be perpendicular to the velocity & pointing towards the center of curvature (radial direction towards center). Also $\vec{a}$ ($a = \frac{v^2}{R}$) points in the same direction



• In previous observation: force has a direction as an acceleration → should be described as a <u>vector</u>!

<u>1st Newton's Law</u>: a body in uniform motion will stay in uniform motion, a body at rest will stay at rest, unless there is a net force acting on the body.
(Law of inertia)

<u>2nd Newton's Law</u>: $\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$ $\begin{cases} \vec{p} : \text{linear momentum} \\ \vec{p} \equiv m \cdot \vec{v} \end{cases}$

$$\vec{F}_{net} = \frac{d(m\cdot\vec{v})}{dt} = \frac{dm}{dt}\cdot\vec{v} + m\cdot\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}$$

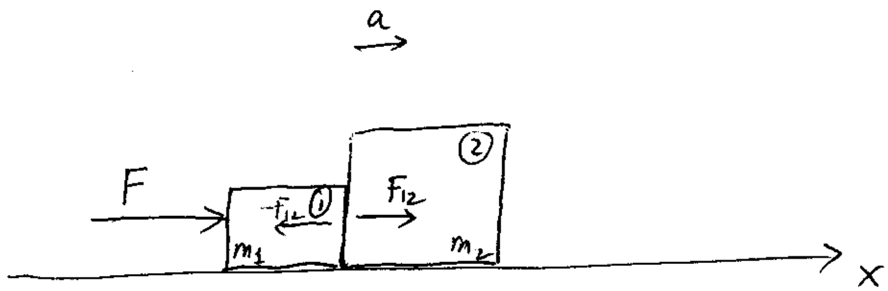
If $m = \text{constant} \rightarrow \dfrac{dm}{dt} = 0 \rightarrow \vec{F}_{net} = m\cdot\vec{a}$

Dimension of $\vec{F}$: $[F] = \dfrac{[p]}{[t]} = \dfrac{[m]\cdot[v]}{[t]} = \dfrac{M\cdot\frac{L}{T}}{T} = M\dfrac{L}{T^2}$

Unit in SI: $kg\,\dfrac{m}{s^2} \equiv N$ (Newton).

<u>3rd Newton's Law</u>: Law of action & reaction
If A exerts a force on B, B exerts an equal and opposite force on A.

Two boxes on a horizontal surface: (no friction)



F is applied on box 1, moving both boxes to the right $(+x)$ with acceleration a

→ What force is applied to box 2 ? $F_{net\,2}$

- Can't be F : $F = (m_1 + m_2) \cdot a$

$$\boxed{F_{net_2} = m_2 \cdot a < F}$$

- Who applies F_{net_2} ? Box 1 is applying this force F_{12} (Force by 1 on 2)

$$\vec{F}_{net_2} = F_{12}\,\hat{\imath}$$

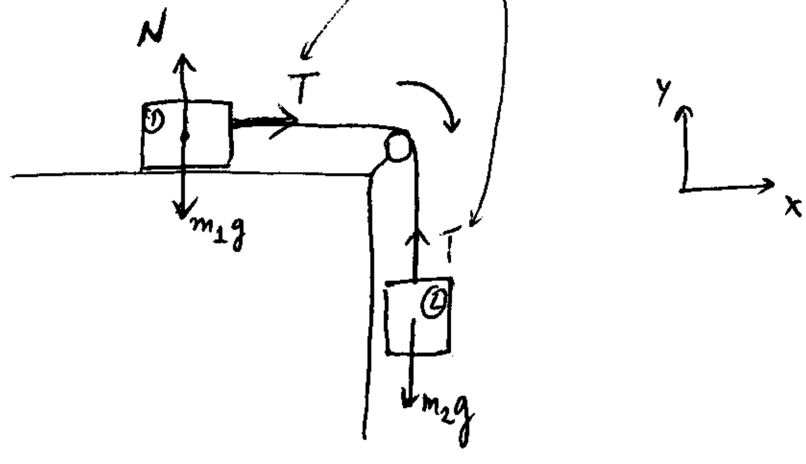
→ By 3rd Newton's Law: box 2 applies $-F_{12}$ on box 1

- Net force on box 1 : $\vec{F}_{net_1} = \underbrace{(F - F_{12})}_{\boxed{F_{net_1} < F}}\,\hat{\imath}$

* Net force on system of boxes 1 & 2 : F
(F_{12} & $-F_{12}$ are internal forces between components and they cancel by pairs).
However for each component, action and reaction forces need to be considered!

Two boxes connected via a massless string, no friction.

Boxes 1 & 2 moving at acceleration a



Let's analyze each component:

1 $\vec{F}_{net_1} = T\hat{\imath} + \underbrace{(N - m_1 g)}_{0}\hat{\jmath}$

2nd Newton's Law:

$\vec{F}_{net_1} = m_1 \cdot \vec{a}$

$T\hat{\imath} = m_1 \cdot a\,\hat{\imath}$

$\boxed{T = m_1 \cdot a}$

2 $\vec{F}_{net_2} = (T - m_2 g)\hat{\jmath}$

2nd Newton's Law:

$\vec{F}_{net_2} = m_2 \cdot \vec{a}$

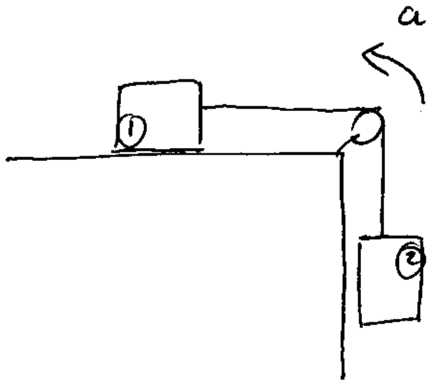
$(T - m_2 g)\hat{\jmath} = -m_2 a\,\hat{\jmath}$

$\boxed{T - m_2 g = -m_2 a}$

$m_1 a - m_2 g = -m_2 a$

$(m_1 + m_2) a = m_2 g$

$$\boxed{a = \frac{m_2}{m_1 + m_2} g}$$



$a_1 = -a\hat{i}$

$a_2 = a\hat{j}$

1 $\quad T = -m_1 a$

2 $\quad T - m_2 g = m_2 a$

$$-m_1 a - m_2 g = m_2 a$$

$$-(m_1 + m_2) a = m_2 g$$

$$a = -\frac{m_2}{m_1 + m_2} g$$

The actual acceleration is opposite to what we assumed here (with tension alone, motion has to be CW at pulley)

- if $m_2 \to 2m_2 \;\to\; a \longrightarrow \dfrac{2m_2}{m_1 + 2m_2} g$ (less than doubled)
- if $m_2 \to 2m_2$, $m_1 \to 2m_1 \;\to\; a \longrightarrow \dfrac{\cancel{2}m_2}{\cancel{2}m_1 + \cancel{2}m_2} g$ (same acceleration)

Measuring forces:

Spring scale:

Hooke's Law:

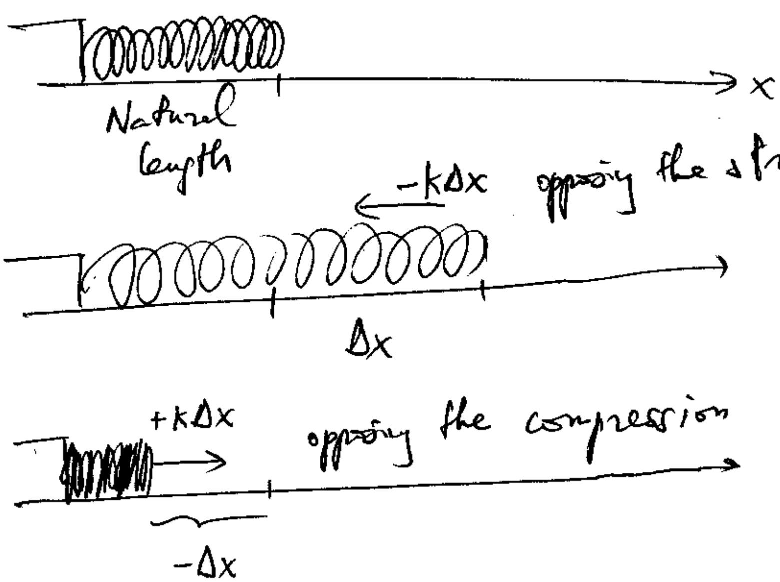
$$F_{\text{by spring}} = -k \cdot \Delta x$$

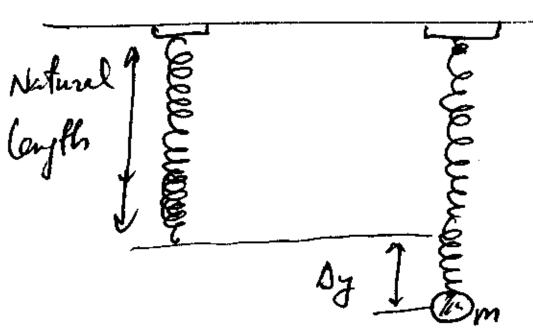
Δx: change of length (elongation) from the natural length ℓ

k: spring constant, SI $\left(\frac{N}{m}\right)$

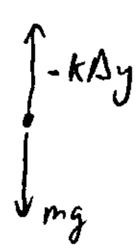
$-$: Opposing any change in length.

- During the linear regime of the spring.





New equilibrium ($a=0$) w/ a mass m attached.



Free-body diagram (no spring, & no mass)

$$F_{net} = mg - k\Delta y = m \cdot a = 0$$

$$mg = k\Delta y \rightarrow \boxed{\Delta y = \frac{mg}{k}}$$

Frictional forces: are present when an object is in contact with a surface

Static friction : $F_S = \mu_S N$ (N: normal force exerted by surface on object)

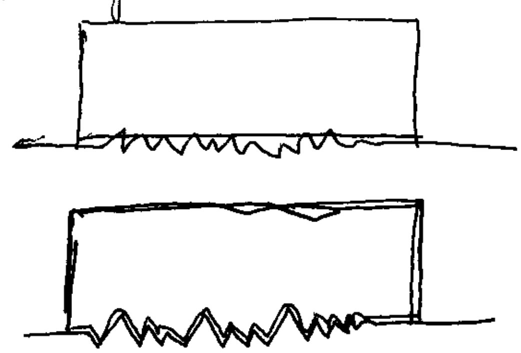
μ_S ↓ mu sub s is the coeff. of static friction
depends: texture of contacts, materials, roughness, etc.
Threshold to push or pull a sitting object.

Kinetic friction : happens when object is already in motion ~~but~~ in contact with a surface.

$F_K = \mu_K N$

μ_K ↓ coeff. of kinetic friction $< \mu_S$

Box on a floor:
- recently or already moving
- sitting on the floor for a while

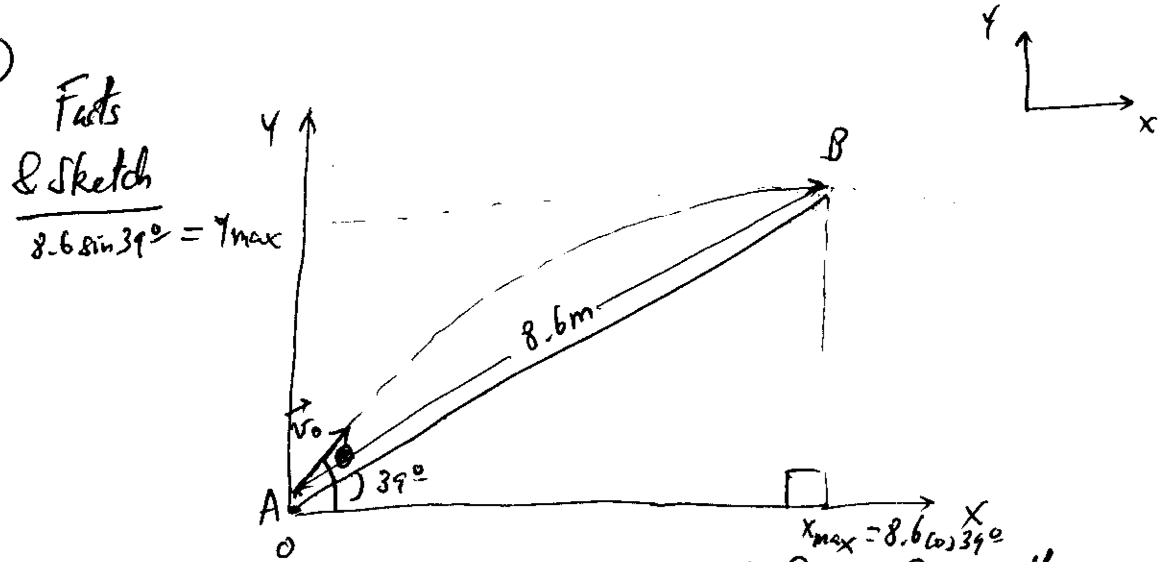


Observation: $\mu_K < \mu_S$

When you try to push a heavy box, you would apply increasing force until you reach the threshold to overcome the static friction (between box & floor). Then it starts moving kind of fast: this is because you are still applying the threshold static friction force F_S while in motion the kinetic friction F_K is lower: leaving a net force on the box $F_S - F_K$ allowing to accelerate forward. ($F_S - F_K = m \cdot a$)

(Go to page 51)

3.66



- Velocity at B is horizontal ⇒ B is the max. altitude point of the projectile motion of the chocolate bar.

Solve for $\vec{v}_0$ (v_0 & θ or v_{0x} & v_{0y}) (Note θ is not 39°)

Method #1:

$$\left\{\begin{array}{l} x_{max} = \dfrac{v_0^2 \sin 2\theta}{2g} = 8.6\cos 39° \\ y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g} = 8.6\sin 39° \end{array}\right\} \text{ algebra is involved.}$$

Method #2: Note that the trajectory eq & eqs for the max. altitude point were derived from the kinematic equations for constant acceleration!

1) $\vec{v} = \vec{v}_0 + \vec{a}\cdot t$ $\left\{\begin{array}{ll} v_x = v_{0x} & (1a) \\ v_y = v_{0y} - g\cdot t & (1b) \end{array}\right.$ Projectile motion.

2) $\vec{r} = \vec{r}_0 + \vec{v}_0\cdot t + \frac{1}{2}\vec{a}\cdot t^2$ $\left\{\begin{array}{ll} x - x_0 = v_{0x}\cdot t & (2a) \\ y - y_0 = v_{0y}\cdot t - \frac{1}{2}g t^2 & (2b) \end{array}\right.$

We have $\left\{\begin{array}{l} x - x_0 = 8.6\cos 39° \\ y - y_0 = 8.6\sin 39° \end{array}\right.$ → can obtain v_{0x} & v_{0y} by eliminating t out of eq. 2)

Since $\vec{v}$ is horizontal @ $\vec{B}$ $\Rightarrow$ $v_y = 0$

(1b) $0 = v_{oy} - gt \rightarrow t = \frac{v_{oy}}{g}$

(2b) $8.6 \sin 39° = \frac{v_{oy}^2}{g} - \frac{1}{2} \cancel{g} \frac{v_{oy}^2}{g^{\cancel{2}}} = \frac{1}{2} \frac{v_{oy}^2}{g}$

$$v_{oy} = \sqrt{2g \cdot 8.6 \sin 39°} = \sqrt{2 \cdot 9.81 \cdot 8.6 \cdot \sin 39°} = 10.3 \frac{m}{s}$$

(2a) $8.6 \cos 39° = v_{ox} \cdot \frac{v_{oy}}{g} \rightarrow v_{ox} = \frac{8.6 \cos 39° \cdot 9.81}{10.3}$

$$= 6.36 \frac{m}{s}$$

$$\vec{v}_0 = \underbrace{6.36\hat{i} + 10.3\hat{j}}_{1^{st}\text{ quad.}} \frac{m}{s} \rightarrow \begin{cases} v_0 = \sqrt{6.36^2 + 10.3^2} = 12.1 \frac{m}{s} \\ \theta_{v_0} = \tan^{-1} \frac{10.3}{6.36} = 58.3° \end{cases}$$

No need to add 180°!

<u>Method # 3</u>: Since there is no time information, use <u>KE of # 3</u>

$$\begin{cases} 3a)\ \frac{v_x^2 - v_{ox}^2}{x - x_0} = 2a_x \\ 3b)\ \frac{v_y^2 - v_{oy}^2}{y - y_0} = 2a_y \end{cases} \text{Projectile Motion} \begin{cases} a_x = 0 \\ a_y = -g. \end{cases}$$

3b) $v_y = 0$: $-v_{oy}^2 = (y - y_0) 2a_y$

$v_{oy}^2 = (y - y_0) 2g \rightarrow v_{oy} = \sqrt{8.6 \sin 39° \cdot 2 \cdot 9.81}$

$= 10.3\, m/s.$

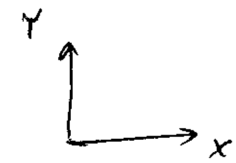
How do I find v_{ox}? $v_{ox} = v_x = \frac{x - x_0}{t} = \frac{8.6 \cos 39°}{\frac{10.3}{9.81}} = 6.36\, m/s$

uniform motion

$\underset{0 \text{ at } B}{v_y} = v_{oy} - gt \rightarrow t = \frac{v_{oy}}{g}$

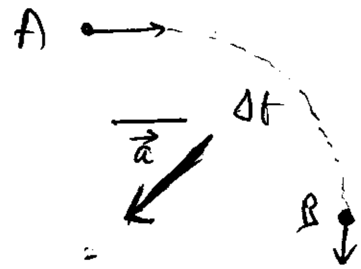
$\rightarrow \vec{v}_0 = 6.36\hat{i} + 10.3\hat{j} = (12.1, \theta = 58.3°)$

3.29



Eastward.

$\vec{v}_0 = v_0 \hat{i}$



"Speedometer remains constant"

$\vec{v} = v_0 (-\hat{j})$ Southward

Average acceleration vector : $\overline{\vec{a}} = \dfrac{\vec{v} - \vec{v}_0}{\Delta t}$ (Δt: time to make the turn from A to B)

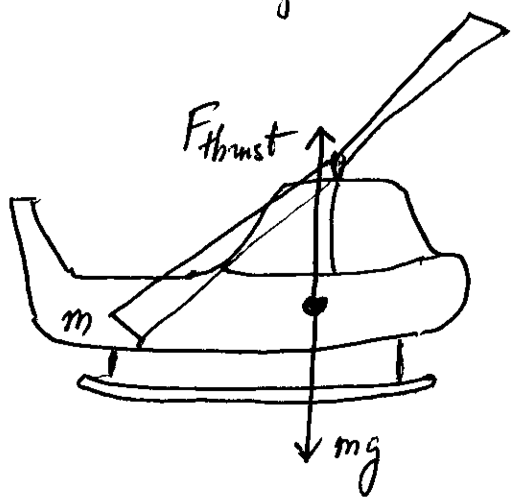
$\overline{\vec{a}} = \dfrac{-v_0 \hat{j} - v_0 \hat{i}}{\Delta t}$ (3^{rd} quadrant)

Direction: $\theta_a = \tan^{-1}\left(\dfrac{a_y}{a_x}\right) = \tan^{-1}\left(\dfrac{-\frac{v_0}{\Delta t}}{-\frac{v_0}{\Delta t}}\right) = \tan^{-1}(1) = 45^{\circ}$

calculator

Correct angle: $\theta_a = 45^{\circ} + 180^{\circ} = 225^{\circ}$.

4.53



$m = 4300\,kg$

$\underbrace{\underset{(downward)}{F_{on\,air}} = - \underset{(upward)}{F_{thrust}}}_{3^{rd}\ \text{Newton's Law}}$

a) Hovering @ constant altitude : $a = 0$. What is Force exerted on air by blades? $F_{on\,air}$?

2^{nd} Newton's Law : $\underset{\text{on helicopter}}{F_{net}} = m \cdot a$

$F_{thrust} - mg = 0 \rightarrow F_{thrust} = mg = 4300 \cdot 9.81$
$= 42 \times 10^{3} N = 42\,kN$ (upward)

$F_{on\,air} = 42\,kN$ (downward)

b) What is $F_{on\,air}$ when helicopter is dropping @ $21\frac{m}{s}$ with speed decreasing @ $3.2\frac{m}{s^2}$:
deceleration (downward) or acceleration-upward

a downward deceleration is the same as an upward acceleration.

2nd Newton's law: $\vec{F}_{net} = m \cdot \vec{a}$

$$\underbrace{F_{thrust}}_{upward} - mg = \underbrace{m \cdot a}_{upward}$$

$$F_{thrust} = m\,(a+g) = 4300\,(3.2 + 9.81)$$
$$= 55.9\ kN \quad (upward).$$

$\Rightarrow F_{on\,air} = 55.9\,kN$ (downward)

c) $F_{on\,air}$? when rising @ $17\frac{m}{s}$ with speed increasing @ $3.2\,m/s^2$
upward-acceleration
same as in b)!

2nd Newton's Law: $F_{thrust} = m\,(a+g) = 55.9\ kN$ (upward)

$\rightarrow F_{on\,air} = 55.9\,kN$ (downward)

d) $F_{on\,air}$! rising @ steady $15\frac{m}{s}$ → upward acceleration $a=0$

2nd Newton's Law: $F_{thrust} - mg = m \cdot 0$

$F_{thrust} = mg = 42\,kN$ (upward)

$\rightarrow F_{on\,air} = 42\,kN$ (downward)

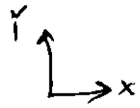
e) $F_{on\,air}$? rising @ $15\frac{m}{s}$ with speed decreasing at $3.2\,m/s^2$
→ upward deceleration $a = -3.2\ m/s^2$

2nd Newton's Law: $F_{thrust} - mg = ma \rightarrow F_{thrust} = m(g+a)$
$= 4300\,(9.81 - 3.2) = 28.4\,kN$

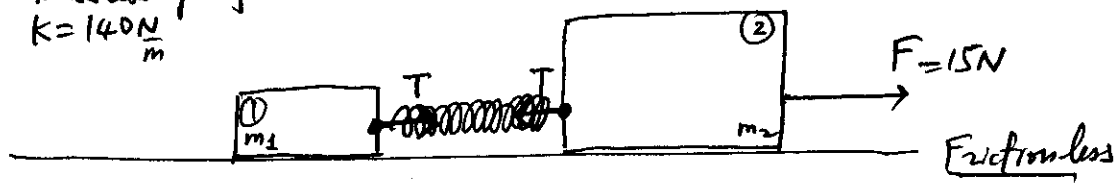
$\boxed{F_{on\,air} = 28.4\,kN\ (downward)}$

4.51

$m_1 = 2kg$
$m_2 = 3kg$
massless spring
$k = 140 \frac{N}{m}$



① & ② are connected → same acceleration a for both!



Let's look at forces & motions along x.

→ Let's focus on object #①:

$$F_{net_{①}} = m_1 \cdot a \quad (2^{nd} \text{ Newton's Law})$$

$$1) \quad T = m_1 a$$

→ Let's focus on object #②:

$$F_{net_{②}} = m_2 \cdot a$$

$$2) \quad F - T = m_2 \cdot a$$

To find Δx for the spring I need T (because by 3rd Newton's Law: if spring is pulling the object with tension T, object is pulling on the spring with same force in opposite direction!)

Solve for our system of 2 equations and 2 unknowns: (T & a)

$$1) \quad T = m_1 a$$

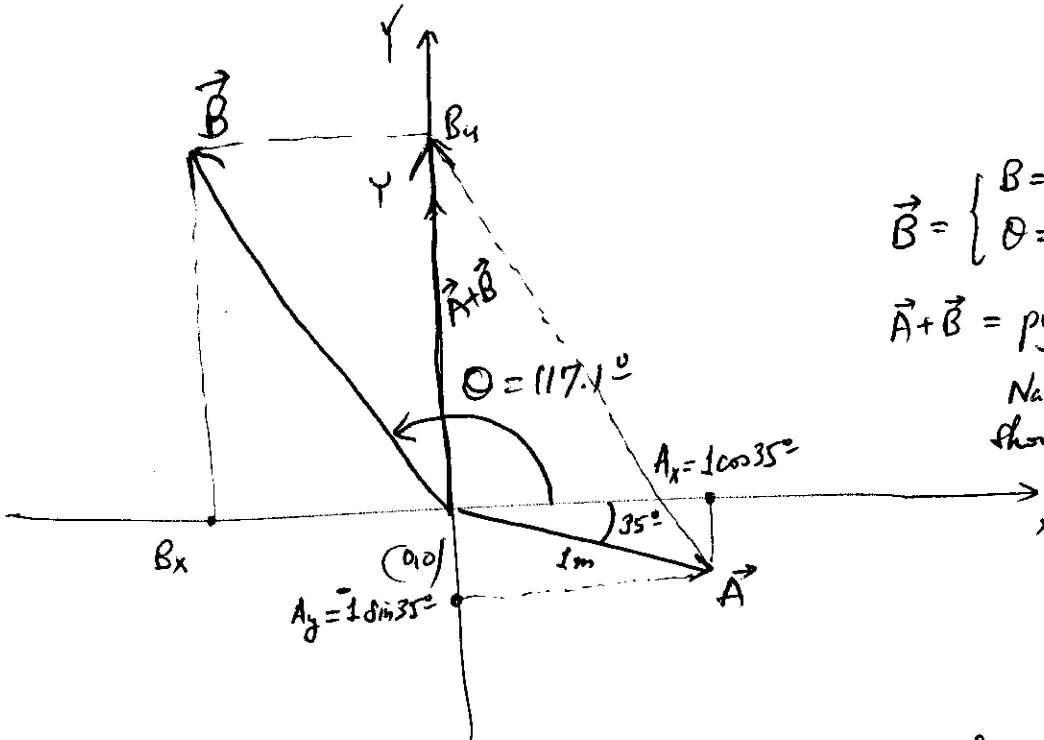
$$2) \quad F - T = m_2 a$$

Eliminate a: $\underset{1)}{a = \frac{T}{m_1}} \underset{2)}{\rightarrow} F - T = \frac{m_2}{m_1} T$.

$$T = \frac{F}{1 + \frac{m_2}{m_1}} = \frac{15}{1 + \frac{3}{2}} = 6N$$

$$\boxed{\Delta x = \frac{T}{k} = \frac{6N}{140 \frac{N}{m}} = 0.0429 m}$$

3.49



$$\vec{B} = \begin{cases} B = 1.8m \\ \theta = ? \text{ (From x-axis)} \end{cases}$$

$\vec{A}+\vec{B}$ = purely vertical

Natural description should the Cartesian Conducts.

$$\left.\begin{aligned} \vec{B} &= \underbrace{1.8\cos\theta}_{B_x}\,\hat{i} + \underbrace{1.8\sin\theta}_{B_y}\,\hat{j} \\ \vec{A} &= 1\cos 35°\,\hat{i} - 1\sin 35°\,\hat{j} \end{aligned}\right\}$$

~~$B_x A_x A_x$~~

→ Since $\vec{A}+\vec{B}$ is purely along y (no x-component)

$$B_x + A_x = 0$$

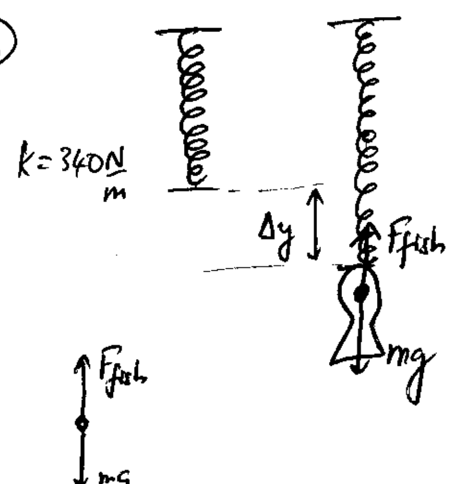
$$1.8\cos\theta + 1\cos 35° = 0$$

$$\cos\theta = -\frac{\cos 35°}{1.8}$$

$$\theta = \cos^{-1}\left(-\frac{\cos 35°}{1.8}\right)$$

$$\boxed{\theta = 117.1°}$$

4.38



Spring is stretched by Δx? with a fish of mass $m = 6.7kg$.

→ Can determined $\Delta x = (-)\frac{F_s}{k}$ where F_s is force on spring

(−) ↓ opposing any change of length.

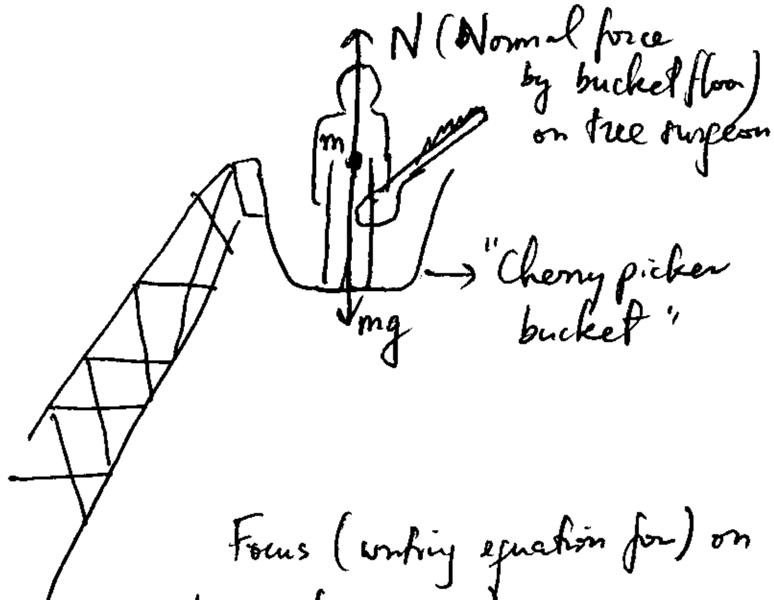
→ Can't find F_s directly by we can find F_{fish} by the spring. (by action & reaction $F_s = -F_{fish}$)

When fish is hanging : $F_{fish} - mg = m.a = m.0 = 0 \Rightarrow F_{fish} = mg$

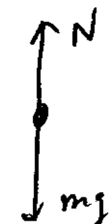
$= 6.7 \times 9.81\,N$

$$\boxed{\Delta x = -\frac{F_s}{k} = -\frac{(-F_{fish})}{k} = \frac{mg}{k} = \frac{6.7\times 9.81\ N}{340\ \frac{N}{m}} = 0.193\,m}$$ stretch.

4.41



$m = 74\,kg$



Free-body diagram.

Focus (writing equation for) on the tree surgeon (we know his mass)

2nd Newton's Eq for tree surgeon: $F_{net} = m\cdot a$
(Net force on tree surgeon is his mass times his acceleration)

$$N - mg = m\cdot a$$

a) At rest: $a = 0 \rightarrow$ $N - mg = 0$ $\Rightarrow N = mg = 74\times 9.81$

$\boxed{N = 725\,N}$

b) Moving up @ steady $2.4\,\frac{m}{s}$ $\rightarrow a = 0 \rightarrow$ $\boxed{N = 725\,N}$

c) Moving down @ steady $2.4\,\frac{m}{s}$ $\rightarrow a = 0 \rightarrow$ $\boxed{N = 725\,N}$

d) Accelerating upward @ $1.7\,\frac{m}{s^2}$ $\rightarrow$ $a = +1.7\,\frac{m}{s^2}$

$N - mg = m\cdot a \rightarrow N = m(g+a) = 74\cdot(9.81+1.7)$

$\boxed{N = 851\,N}$

Yes (we "feel" heavier when an elevator accelerate upward: more normal force)

e) Accelerating downward @ $1.7\,m/s^2$ $\rightarrow a = -1.7\,\frac{m}{s^2}$

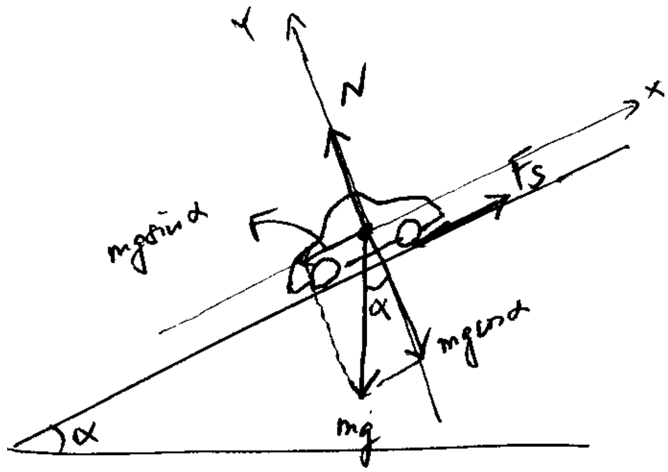
$N - mg = ma \rightarrow$ $N = m(g+a) = 74(9.81 - 1.7)$

$\boxed{N = 599\,N}$

Yes (we "feel" lighter when an elevator accelerate downward)

Free-body diagram including static friction:

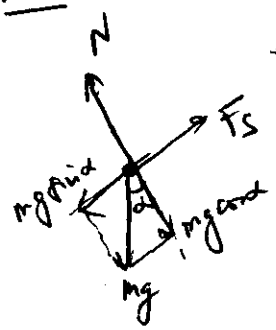
Car parked on a slope of angle α



Forces on the car:

1) weight mg (vertically downward)
2) Normal force by slope on the car
3) Static friction F_S pointing against possible downhill motion → up hill in $+x$ direction. $F_S = \mu_S N$

FBD



Note about friction force: it does not have a pre-defined direction: it's just against the direction of (possible) motion. As opposed to the gravity force which has a predefined downward & vertical direction.

Note about net force: we are involving force into the description of motion; → easier to use components in direction of motion.

→ In this case, use Cartesian components.

$\boxed{\vec{F}_{\text{net on car}} = m\cdot\vec{a}}$ 2nd Newton's Law

x - component : 1) $F_S - mg\sin\alpha = m\cdot a_x = m\cdot 0$ ↳ car is parked.

y - component : 2) $N - mg\cos\alpha = m\cdot 0$ ↳ car is not jumping up & down.

$$1)\ \mu_S N - mg\sin\alpha = 0$$
$$2)\ N - mg\cos\alpha = 0$$

$$\mu_S \cancel{mg}\cos\alpha - \cancel{mg}\sin\alpha = 0$$
$$\mu_S = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

$\boxed{\text{For car to park: } \mu_S \geq \tan\alpha}$