Ch 1 Doing Physics

Dimensional Analysis: (help remember formulas)

Speed: $[v] = \frac{[\Delta s]}{[\Delta t]} = \frac{L}{T}$ (Length) / (Time)

dimension of speed

Δs = increment of space or distance ; $[\Delta s] = L$
(Δ : delta)

Δt = increment of time ; $[\Delta t] = T$

Acceleration: $[a] = \frac{[\Delta v]}{[\Delta t]} = \frac{\frac{L}{T}}{T} = \frac{L}{T^2}$

Kinetic Energy: $[K.E.] = \left[\frac{1}{2}mv^2\right] = \underbrace{\left[\tfrac{1}{2}\right]}_{1}\cdot[m]\cdot[v]^2 = \frac{ML^2}{T^2}$

$1 \cdot M \cdot \frac{L^2}{T^2}$

How to apply dimensional analysis: e.g. which of the following formulas is right for a speed?

$v = \frac{1}{2}gh^2 : [g]\cdot[h]^2 = \frac{L}{T^2}\cdot L^2 = \frac{L^3}{T^2}$

$v = \sqrt{gh} : \sqrt{[g]\cdot[h]} = \sqrt{\frac{L}{T^2}\cdot L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T}$ ✓

g : acceleration of gravity
h : height or vertical position (length)

Limitation: will not fix the constant. To get the correct constant you may need to use curve-fitting or data modeling.

Units: S.I. or system of international units

Dimension or quantity	S.I.
L	m (meter)
T	s (second)
M	kg (kilogram)
Area	m^2
Volume	m^3
Energy	$\frac{kg\, m^2}{s^2} = J$ (Joule)

Unit conversion:

Other	m	1nm	1µm	1mm	1cm	1km	1 light-year	1mi	1ft
S.I.	m	$10^{-9}m$	$10^{-6}m$	$10^{-3}m$	$10^{-2}m$	$10^{3}m$	$9.46\times10^{15}m$	1609m	0.3048m

other	in
S.I.	0.0254m or 2.54cm

1 lb = 0.454 kg

1 hr = 3600 s ; 1 day = 86400 s , etc.

$1\,km^2 = (10^3 m)^2 = 10^6\, m^2$; $1\,mm^2 = (10^{-3} m)^2 = 10^{-6}\, m^2$

$1\,cm^2 = (10^{-2} m)^2 = 10^{-4}\, m^2$

$1\,km^3 = (10^3 m)^3 = 10^9\, m^3$; $1\,mm^3 = (10^{-3} m)^3 = 10^{-9}\, m^3$, etc.

Accuracy & Significant Figures

- Scientific Notation: uses powers of 10 to simplify a number

$$\Delta s = 6\,176\,000 \text{ m} = \underbrace{6.176}_{\text{coefficient}} \times \underbrace{10^6}_{\text{power of 10}} \text{ m} = 6.176\text{E}6 \text{ (calculator)}$$

coefficient ↓ less than 10

↳ helps in multiplication & division

$$\Delta t = 3000 \text{ s} = 3 \times 10^3 \text{ s}$$

$$\text{speed} = \frac{\Delta s}{\Delta t} = \frac{6.176 \times 10^6}{3 \times 10^3} = 2.059 \times 10^{(6-3)} = 2.059 \times 10^3 \frac{\text{m}}{\text{s}}$$

- Accuracy: (in addition & subtraction)

$$\pi - 1.14 = 3.1416 - 1.14 \begin{cases} 2.0016 \\ 2.00 \end{cases}$$

2.00 ← since 1.14 only has two decimal digits of accuracy.
↓
Keep same accuracy as the least accurate term in the equation.

Assume: $\pi = 3.1416$

- Significant Figures: (in multiplication & division)

$\underbrace{6\,370\,000}_{3 \text{ s.f's}}$ m

(zeros at end do not count)

$\underbrace{6\,370\,001}_{7 \text{ s.f's}}$ m

(middle zeros do count)

in × & ÷ keep the smallest number of s.f.'s except for numeric constants :

Circumference of Earth : $2\pi R_E = \underbrace{2 \times 3.1416}_{\text{not counting numeric constants.}} \times \underbrace{6.37 \times 10^6}_{R_E}$

R_E has 3 s.f.'s
↓
Least # s.f.'s

$= 4.002398 \times 10^7$ m

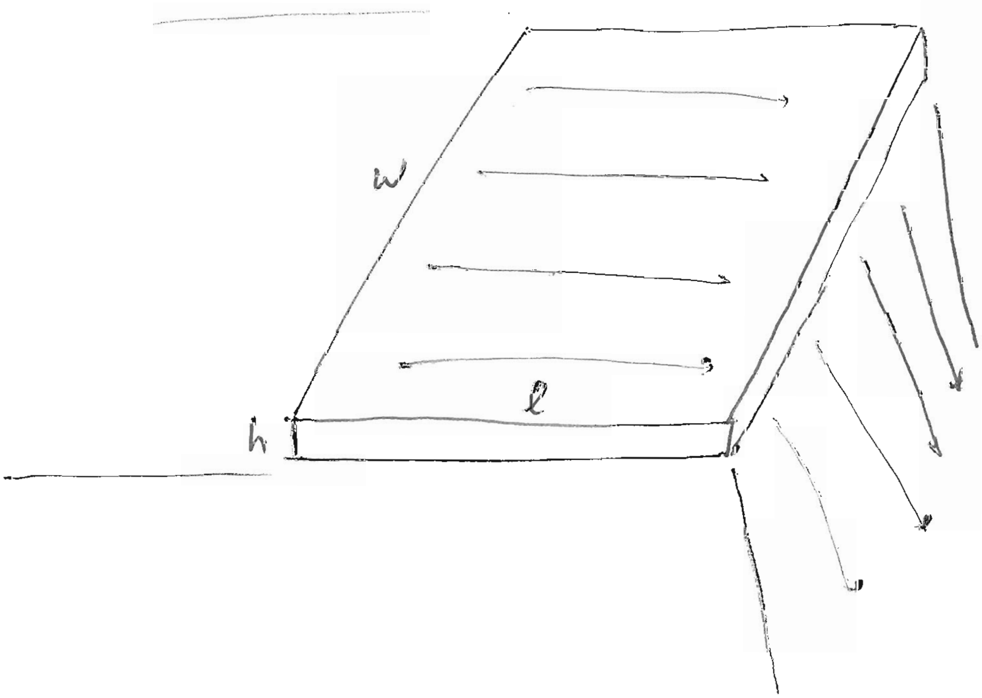
$= \underbrace{4.00}_{3 \text{ s.f.'s.}} \times 10^7$ m

1.40

a) Estimate volume of water flowing over Niagara Falls in 1 s.

Guesses: $10^3 \frac{m^3}{s}$;

Estimation: use some geometrical shape for the slab of water: $h \times w \times l$ (top of Falls)



Volume of water per second = flow rate (per unit time) $\rightarrow \frac{hwl}{t}$

Three different ways to do $\frac{hwl}{t}$

1) $\frac{h}{t} wl$
2) $h \frac{w}{t} l$
3) $hw \frac{l}{t}$ ✓

involves speed of water going over the Falls.

Flow rate numeric estimate: $h\,w\,\underset{\sim}{\frac{l}{t}} = 1m \cdot 1000m \cdot 10\frac{m}{s} = 10^4 \frac{m^3}{s}$

water speed towards edge of Falls. (visible)

$$\begin{cases} h: & 1m, \; 10m, \; \cancel{100m} \\ w: & 100m, \; 1000m, \; 10000m \\ \frac{l}{t}: & 1\frac{m}{s}, \; 10\frac{m}{s}, \; 100\frac{m}{s} \end{cases}$$

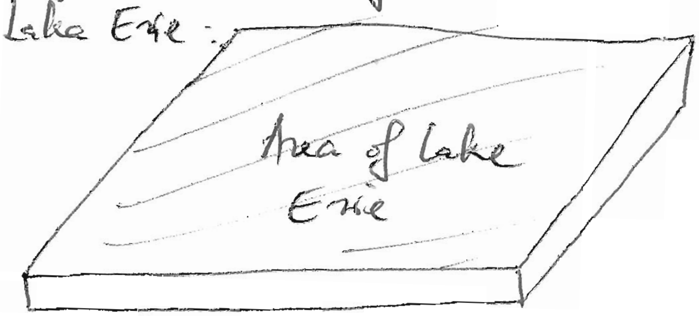
Car speed in $\frac{m}{s}$: $65\frac{mi}{h} \cdot \frac{1609m}{1mi} \cdot \frac{1h}{3600s} = 29\frac{m}{s}$

5) Niagara Falls provide outlet for Lake Erie. If Niagara Falls are shut-off: water level at Lake Erie will rise: how long will it rise one meter.

Need 2 important information:

1) Area of Lake Erie (smaller → takes less time)
2) Flow rate of outlet (Falls)

Estimation: geometrical shape for the water rise @ Lake Erie: → Find volume of water in the rise



1m

Area of Lake Erie: $75\,km \times 375\,km$

→ Volume of water $= 75 \times 10^3 \times 375 \times 10^3 \times 1\,m = 75 \times 375 \times 10^6\,m^3$

→ How long for this much water to accumulate if Falls are shut-off?

$$\text{Time} = \frac{\text{Volume of water}}{\text{Flow rate of water}} = \frac{75 \times 375 \times \cancel{10^6}^{\,2}\,\cancel{m^3}}{\cancel{10^4}\,\frac{\cancel{m^3}}{s}}$$

$$= 75 \times 375 \times 10^2\,s = 28125 \times 10^2\,\cancel{s}\,\frac{1\,\text{day}}{86400\,\cancel{s}} = 32.6\,\text{days}$$

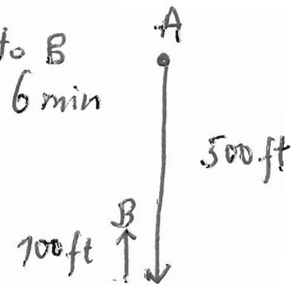
Ch2. Motion in a Straight Line

Average Motion: average velocity & average acceleration

$\boxed{\text{Speed}}$ ⟵ same dimension ⟶ $\boxed{\text{Velocity}}$

$= \dfrac{\text{distance}}{\text{time}}$ $\qquad\qquad$ $= \dfrac{\text{displacement}}{\text{time}}$

A to B in 6 min



$= \dfrac{600\,ft}{6\,min} = 100\,ft/min$ $\qquad\qquad$ $= \dfrac{400\,ft}{6\,min} = 66.67\,ft/min$

- velocity is lower than speed because direction of motion is taken into consideration

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Average velocity: $\bar{v} = \dfrac{\Delta x}{\Delta t}$

(Unit in SI is $\frac{m}{s}$)

Δx : increment of position or displacement

Δt : increment of time or time

Instantaneous velocity: $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} \underset{\text{Calculus}}{=} \dfrac{dx}{dt}$ ("derivative of x wrt t")

(Unit in SI is $\frac{m}{s}$)

Example: $x = \underset{\text{constant}}{a} t^3 \longrightarrow v = \dfrac{dx}{dt} = a\dfrac{dt^3}{dt} = 3at^2$

$x = at^n \longrightarrow v = a\dfrac{dt^n}{dt} = nat^{n-1}$ $\left(\dfrac{dt^n}{dt} = nt^{n-1}\right)$

Acceleration: change of velocity over time

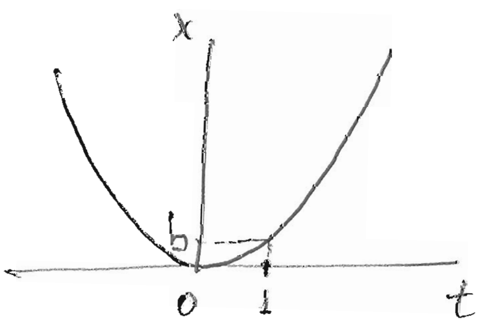
Average acceleration: $\bar{a} = \dfrac{\Delta v}{\Delta t}$
(unit in SI: m/s^2)

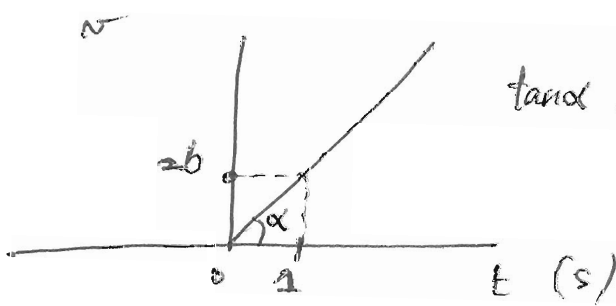
Δv = increment or change in velocity
Δt = increment of time or time

Instantaneous acceleration: $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$ ("derivative of v wrt t")
(unit in SI: m/s^2)

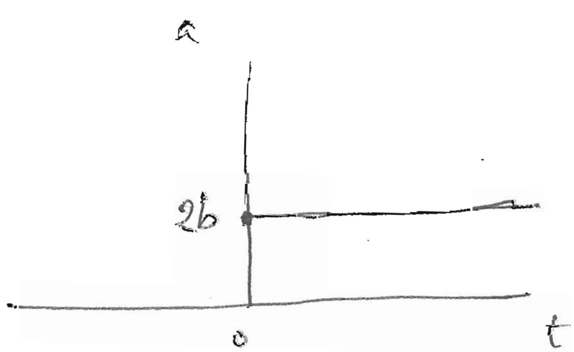
Example: 1) $x = at^3 \rightarrow v = 3at^2$ (instantaneous velocity)
$\rightarrow a = 6at$ (instantaneous acceleration)
↓
not constant

2) $x = bt^2 \rightarrow v = 2bt$ (linear in time)
$a = 2b$ (constant acceleration)





$\tan\alpha$ = slope = $2b$



instantaneous acceleration = average acceleration
$a = \bar{a}$

From these definitions we now derive our 2 equations of motions (kinematic equations) for constant acceleration in 1D.

Constant acceleration : $a = \bar{a}$

$$a = \frac{\Delta v}{\Delta t} = \frac{\overset{\text{current velocity}}{v} - \overset{\text{initial velocity}}{v_0}}{\underset{\text{current time}}{t} - \underset{\text{initial time}}{0}} \rightarrow v - v_0 = at$$

$$\boxed{v = v_0 + at} \quad (1)$$

Kinematic Equation #1

Now let's derive Kinematic eq. #2:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\overset{\text{current position}}{x} - \overset{\text{initial position}}{x_0}}{t - 0} \rightarrow x = x_0 + \bar{v}\cdot t \quad (A)$$

On the other hand the average velocity can be calculated using :

$$\bar{v} = \frac{1}{t-0}\int_0^t v\,dt$$

$$\overset{(1)}{=} \frac{1}{t}\int_0^t (v_0 + at)\,dt$$

$$= \frac{1}{t}\left[v_0 t + \frac{1}{2}at^2\right]_0^t = \frac{1}{t}\left(v_0 t + \frac{1}{2}at^2\right) = v_0 + \frac{1}{2}at$$

$$\left(\int t^n\,dt = \frac{t^{n+1}}{n+1}\right)$$

$$= \frac{1}{2}v_0 + \underbrace{\frac{1}{2}a.0}_{0} + \frac{1}{2}v_0 + \frac{1}{2}a.t$$

$$= \frac{1}{2}\left[(v_0 + a.0) + (v_0 + a.t)\right]$$

$$\bar{v} = \frac{1}{2}\left[v_0 + v \right] \quad (B)$$

Combining (B) into (A):

$$x = x_0 + \bar{v}.t = x_0 + \frac{1}{2}[v_0 + v].t$$

$$\overset{(1)}{=} x_0 + \frac{1}{2}[v_0 + v_0 + at].t$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \quad (2)$$

Kinematic eq. #2.

Summary: To describe a constant acceleration motion in 1D, we we will use these equations:

1) $v = v_0 + a.t$

2) $x = x_0 + v_0 t + \frac{1}{2}at^2$

3) $\dfrac{v^2 - v_0^2}{x - x_0} = 2a$ (can be derived from 1) & 2))

↳ No time variable → better option to start with in those problems where time is not given or not needed

x_0 = initial position ; x = current or final position

v_0 = initial velocity ; v = current or final velocity

a = constant acceleration ; t=time ($t_0 = 0$)

($a_0 = a$)

Application of Kinematic Equations (1D)

(2.35)

1) Facts

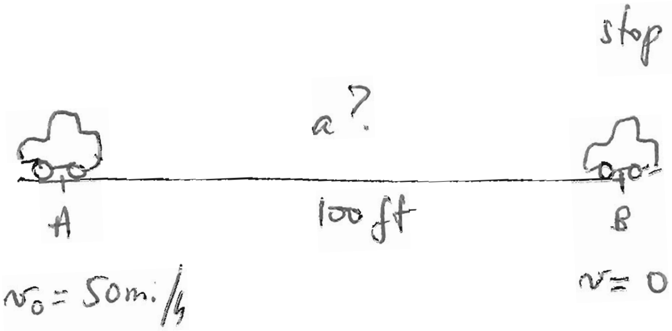
$$\underbrace{v_0 = 50\,mi/h}_{\text{write}} \leftarrow \underbrace{(\text{"a car moving initially at } 50\,mi/h\text{"})}_{\text{Don't write.}}$$

constant acceleration ⟷ ("decelerating at a constant rate")

v_0 to $v = 0$ in 100 ft ⟷ ("stop light in 100 ft")

Question: a ? (SI.)

2) Sketch:



3) Find appropriate equation to solve for our unknown (a)
(among the 3 there are)

Eq. 3 fits well to our data:

$$2a = \frac{v^2 - v_0^2}{x - x_0}$$

To get S.I. results → plug in SI data:

$$v_0 = 50\,\frac{\cancel{mi}}{\cancel{h}} \cdot \frac{1609\,m}{\cancel{mi}} \cdot \frac{1\cancel{h}}{3600\,s} = 22.35\,\frac{m}{s}$$

$$x - x_0 = 100\,\cancel{ft} \cdot \frac{0.3048\,m}{1\cancel{ft}} = 30.48\,m$$

$$\boxed{a = \frac{1}{2} \cdot \frac{0 - 22.35^2}{30.48} = -8.192\,\frac{m}{s^2}}$$

Negative since car needed to slow down

2-61

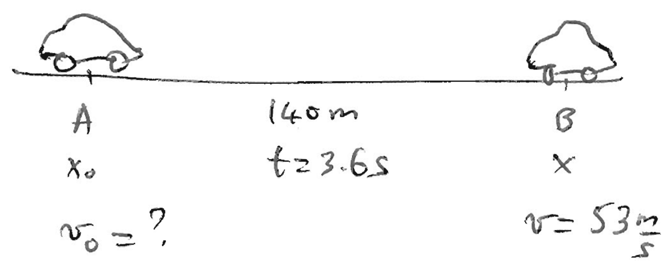
1) Facts (Data):

a constant ✓

$x - x_0 = 140\,m$

$t = 3.6\,s$

a)

2) 

3) Equations : time is there! $\rightarrow \begin{cases} 1)\ v = v_0 + at \\ 2)\ x - x_0 = v_0 t + \frac{1}{2} a t^2 \end{cases}$

Alternative #1: find a so to use eq. 1) to find v_0

$x - x_0 = v_0 t + \frac{1}{2} a t^2 \rightarrow$ since $v_0 = v - at$ (1)

$$\boxed{x - x_0 = (v - at) \cdot t + \frac{1}{2} a t^2 = vt \underbrace{- at^2 + \frac{1}{2} a t^2}_{-\frac{1}{2} a t^2} = vt - \frac{1}{2} a t^2}$$

$$a = -2 \frac{(x - x_0 - vt)}{t^2} = -2 \frac{(140 - 53 \cdot 3.6)}{3.6^2} = 7.83\,\frac{m}{s^2}$$

$$\rightarrow \boxed{v_0 = v - at = 53 - 7.83 \cdot 3.6 = 24.8\,\frac{m}{s}}$$

Alternative #2: to use eq. 2 to find v_0, I need to eliminate a:

Eq 1) $\rightarrow$ $v = v_0 + at \rightarrow a = \frac{v - v_0}{t}$

Eq 2) $\rightarrow$ $x - x_0 = v_0 t + \frac{1}{2} \cdot \left(\frac{v - v_0}{\cancel{t}}\right) \cdot t^{\cancel{2}}$

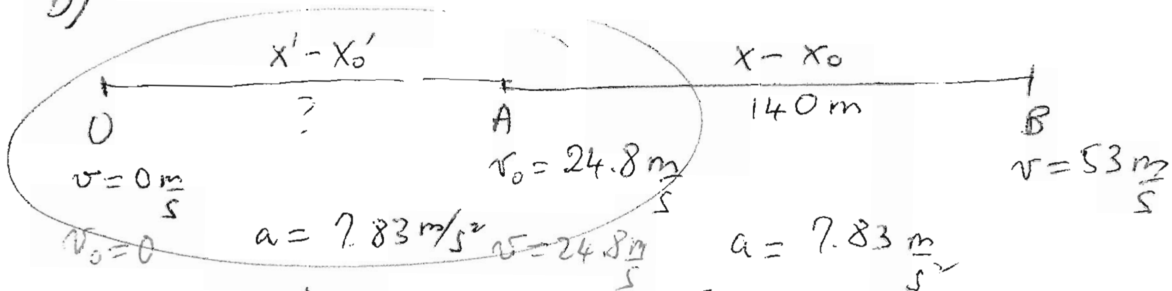
$$x - x_0 = v_0 t + \frac{1}{2} v t - \frac{1}{2} v_0 t = \frac{1}{2} v_0 t + \frac{1}{2} v t$$

$$2(x - x_0) = (v_0 + v) \cdot t \quad \rightarrow \quad v_0 = \frac{2(x - x_0)}{t} - v$$

$$= \frac{2(140\,m)}{3.6} - 53$$

$$\boxed{v_0 = 24.8\,m/s}$$

b)



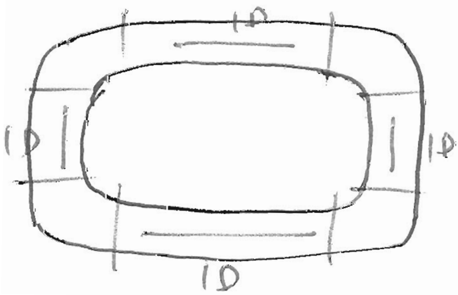
(if you used alternative #2 then $a = \dfrac{v - v_0}{t}$)

Use eq. 3: $2a = \dfrac{v^2 - v_0^2}{x' - x_0'}$

$$x' - x_0' = \frac{24.8^2 - 0}{2 * 7.83} = 39.4\,m$$

$\rightarrow$ $\boxed{OB = 179.4\,m}$

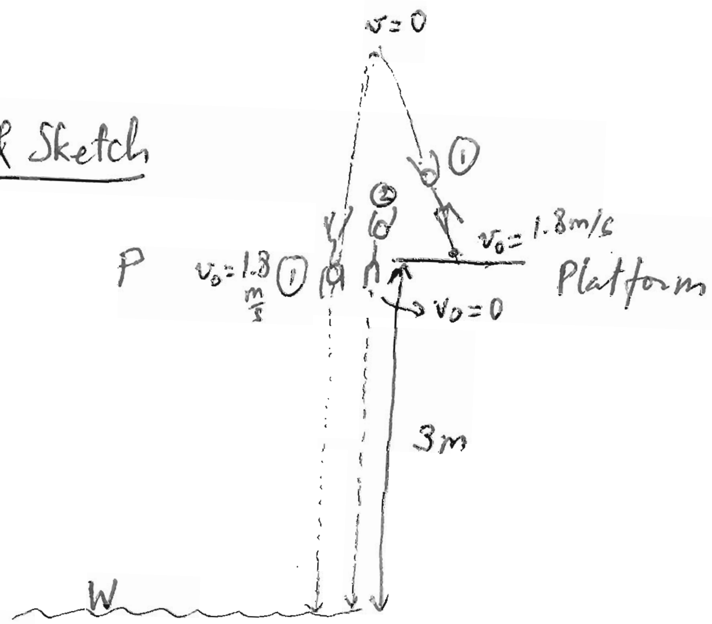
Race track



Tossing a coin up at an angle

2.69

Facts & Sketch



Questions:

a) speeds of divers as they hit water:

diver #1 : between P & W : $\begin{cases} v_0 = 1.8 \, m/s \\ x - x_0 = 3 \, m \end{cases}$

$a = g = 9.81 \, m/s^2$ (Free Fall)

For final speed at W: best candidate is eq. 3 (no time information)

$$\frac{v_W^2 - v_P^2}{x - x_0} = 2a \rightarrow v_W = \sqrt{2a(x - x_0) + v_P^2}$$

$$= \sqrt{2 \cdot 9.81 \cdot 3 + 1.8^2}$$

$$= 7.88 \, m/s$$

diver #2 : between P & W : $\begin{cases} v_0 = 0 \\ x - x_0 = 3 \\ a = g = 9.81 \, m/s^2 \end{cases}$

$$v_W = \sqrt{2 \cdot 9.81 \cdot 3 + 0} = 7.67 \, m/s$$

b) which diver enters water first? diver #1 since he had $v_0 = 1.8 m/s$ downward at P while diver #2 had $v_0 = 0$. By how much time? $t_2 - t_1$

t_2 : time needed by diver #2 to cover 3 m

t_1 : " " " " #1 " " "

→ Eq 1 (linear) or Eq 2 (quadratic in time)

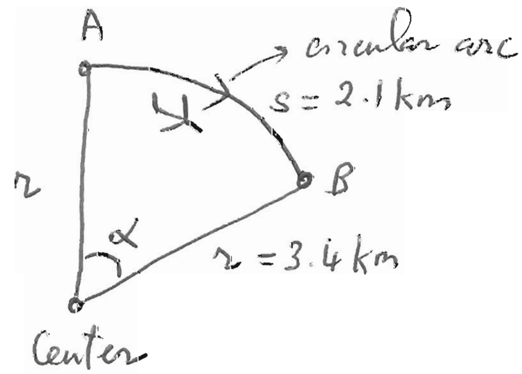
$V_W = V_P + g t$ ✓

$$t_1 = \frac{V_W - V_P}{g} = \frac{7.88 - 1.8}{9.81} = 0.62 s$$

$$t_2 = \frac{7.67 - 0}{9.81} = 0.782 s$$

$$t_2 - t_1 = \boxed{0.162 s}$$

1.17



$$\alpha = \text{angle} = \frac{\text{arc}}{\text{radius}} = \frac{2.1\,\cancel{km}}{3.4\,\cancel{km}} = 0.62\ \text{rad}$$

$\boxed{\text{SI unit : rad. ("no unit")}}$

$$\alpha = 0.62\,\cancel{rad}\ \frac{180^\circ}{\pi\,\cancel{rad}} = 35.4^\circ$$

Ch3: Motion in 2D and 3D

1D: (straight line): position of any object is determined by one variable (x)

2D: (in a plane): position of any object is determined by two variables (x, y)

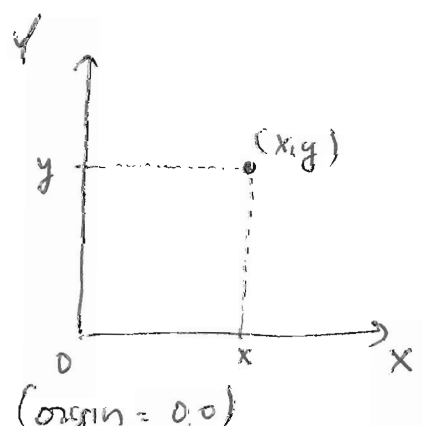
3D: (in space) → three variable (x, y, z)

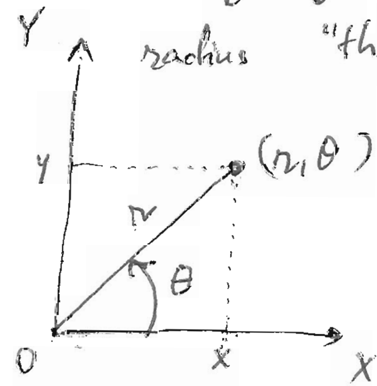
2D: Alternative descriptions of positions:

Cartesian Coordinats (x, y)

Polar Coordinats (r, θ)

r ↓ radius, θ ↓ "theta" → angle





r is measured from the origin

θ is measured from the x-axis

Cartesian → Polar:

length ← $r = \sqrt{x^2 + y^2}$ (Pythagoras Theorem)

angle ← $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\left(\tan\theta = \frac{y}{x}\right)$

Polar → Cartesian

$x = r\cos\theta$

$y = r\sin\theta$

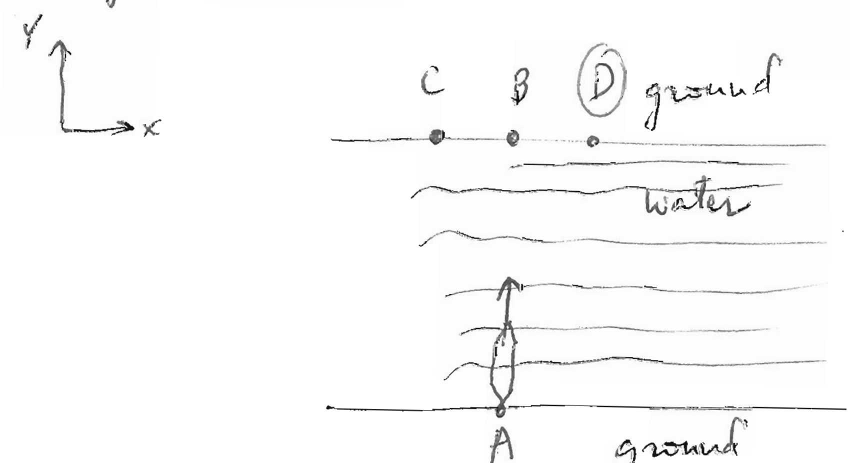
Alternative descriptions of motions in 2D:

Position vector $\vec{r} = (x, y) = (r, \theta)$

Velocity vector $\vec{v} = (v_x, v_y) = (v, \theta_v)$

Acceleration vector $\vec{a} = (a_x, a_y) = (a, \theta_a)$

This has been entirely math descriptions; a physics example of 2D motion: crossing a river in a row boat



river where water is flowing at velocity $\vec{V} = V\hat{i}$

unit vector in the x-direction

Our canoe is heading from A to B: it's velocity w.r.t. water is along y-direction: $\vec{v}' = v'\hat{j}$

unit vector in the y-direction.

The velocity of the canoe w.r.t. ground: $\boxed{\vec{v} = \vec{v}' + \vec{V}}$

Canoe ends up at D because of $\vec{V}$'s contribution

Handling of 2D vectors:

Unit vectors: have magnitude or length of one

2D:

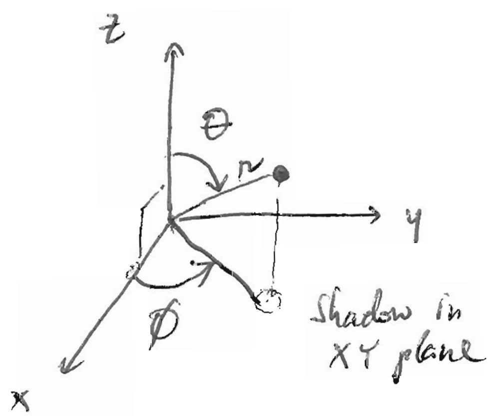
		Cartesian		Polar
along x-axis:	$\hat{i}$ =	$(1, 0)$ (x, y)	=	$(1, 0°)$ (r, θ)
along y-axis:	$\hat{j}$ =	$(0, 1)$ (x, y)	=	$(1, 90°)$ (r, θ)

3D:

		cartesian		Spherical (r, θ, ϕ)
x:	$\hat{i}$ =	$(1, 0, 0)$	=	$(1, 90°, 0)$
y:	$\hat{j}$ =	$(0, 1, 0)$	=	$(1, 90°, 90°)$
z:	$\hat{k}$ =	$(0, 0, 1)$	=	$(1, 0, 0)$

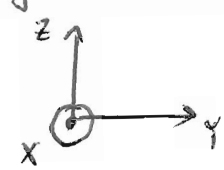
↓ Phi
Theta.

Spherical coordinates:



(View in perspective)

{ YZ are in plane of screen.
{ X is coming out of screen.



(Front view)

ϕ: angle of the shadow of r in the XY plane, from the x-axis.

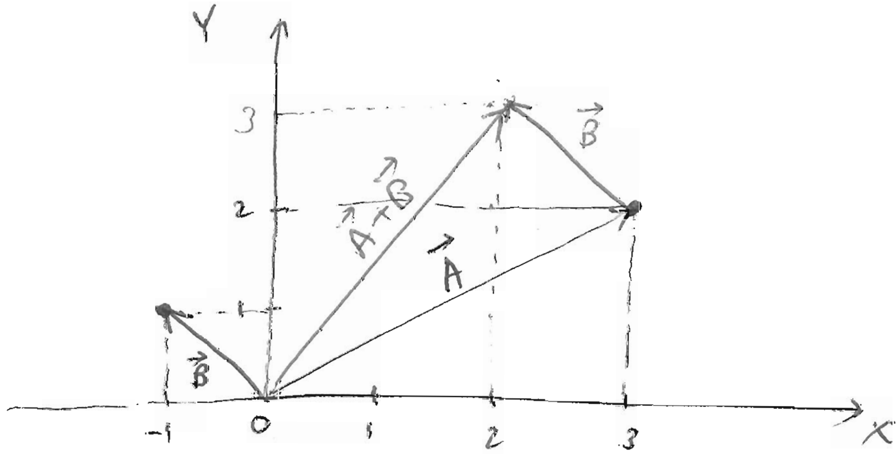
θ: angle of r (connecting the origin & the position or dot) from the z-axis.

r: length b/w origin & the position or dot.

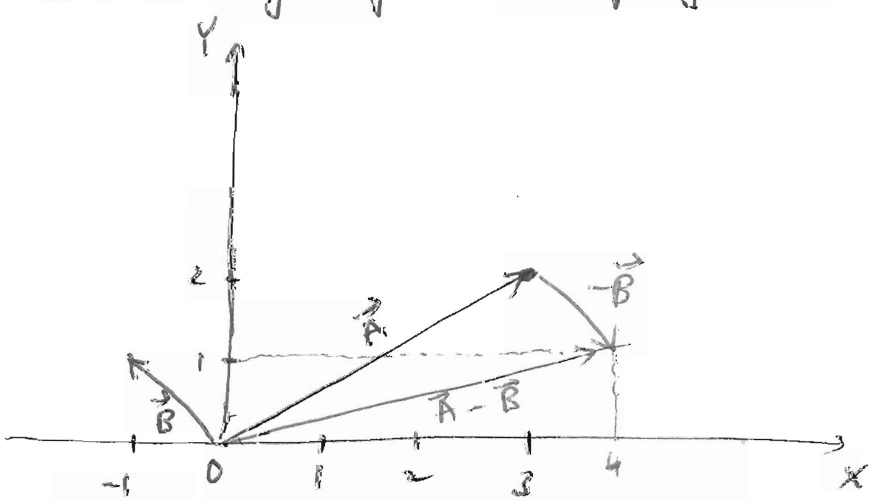
Using unit vectors in describing 2D motions :

$$\vec{r} = (x, y) = x\hat{i} + y\hat{j}$$
$$\vec{v} = (v_x, v_y) = v_x\hat{i} + v_y\hat{j}$$
$$\vec{a} = (a_x, a_y) = a_x\hat{i} + a_y\hat{j}$$

<u>Addition & Subtraction of vectors</u> : $\begin{cases} \text{Graphical Method} \\ \text{Math method (unit vectors)} \end{cases}$



How to draw $\vec{A}+\vec{B}$? draw a copy of $\vec{B}$ at tip of $\vec{A}$, then connect origin of $\vec{A}$ to tip of $\vec{B}$



How to draw $\vec{A}-\vec{B}$? draw a copy of $\vec{B}$, reverse direction, at the tip of $\vec{A}$, then connect origin of $\vec{A}$ to tip of $(-\vec{B})$

Math method using unit vector:

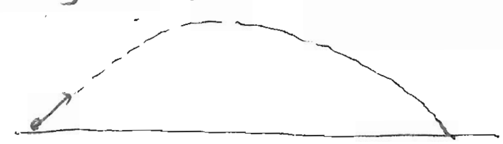
$$\vec{A} = 3\hat{i} + 2\hat{j}$$
$$\vec{B} = -\hat{i} + \hat{j}$$

$$\vec{A}+\vec{B} = (3-1)\hat{i} + (2+1)\hat{j} = 2\hat{i}+3\hat{j}$$
$$\vec{A}-\vec{B} = (3+1)\hat{i} + (2-1)\hat{j} = 4\hat{i}+\hat{j}$$

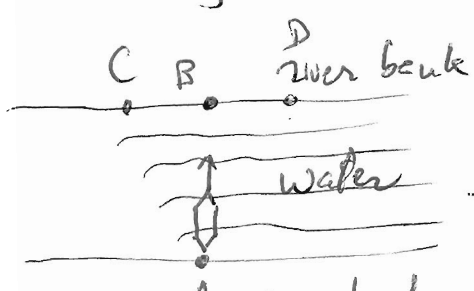
Length or magnitude of $\vec{A}+\vec{B}$: $|\vec{A}+\vec{B}| = \sqrt{2^2+3^2} = \sqrt{13}$

$\vec{A}-\vec{B} \to |\vec{A}-\vec{B}| = \sqrt{4^2+1^2} = \sqrt{17}$

2D motions
- → Crossing a river in a canoe
- → Car with a ball, rolling on frictionless track
- → Projectile motion:



More about crossing a river with a canoe:



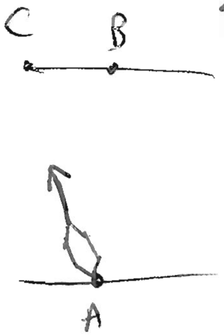
→ River with water flow along x-axis $\vec{V} = V\hat{i}$

A & B are fixed points wrt ground,

1) Canoe heading straight across A to B: velocity of canoe wrt. water $\vec{v}' = v'\hat{j}$

→ Velocity of canoe wrt. ground $\vec{v} = \vec{v}' + \vec{V} = v'\hat{j} + V\hat{i}$

→ canoe will end up at D.



2) Canoe heading A to C : $\vec{v}\,' = -V\hat{i} + v\hat{j}$

$$\rightarrow \quad \vec{v} = \vec{v}\,' + \vec{V}$$

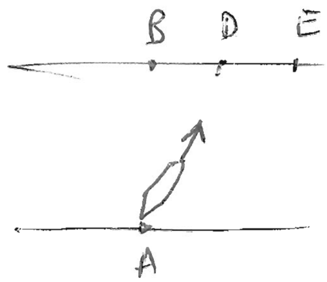
$$= \cancel{-V\hat{i}} + v\hat{j} + \cancel{V\hat{i}} = v\hat{j}$$

In this case canoe ends up at B!

3) Canoe heading A to D : $\vec{v}\,' = +V\hat{i} + v\hat{j}$

$$\rightarrow \quad \vec{v} = \vec{v}\,' + \vec{V}$$

$$= V\hat{i} + v\hat{j} + V\hat{i} = 2V\hat{i} + v\hat{j}$$



In this case canoe will end up at E!