Wave motion:

→ Wave superposition → Beat Phenomenon

→ Standing waves : different modes (not all wavelength can exist, only $\lambda_n = \frac{2L}{n}$ $n = 1, 2, 3, \ldots$ in a string)

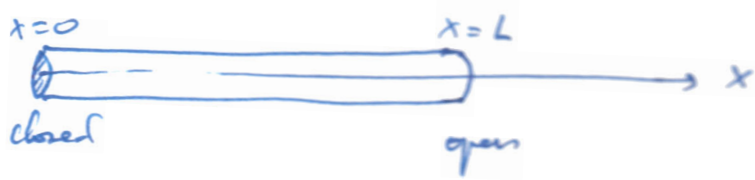
→ Constructive (loud spots when combining ~~water~~ sound waves, bright spots when combining light waves, etc.) & destructive interference (quiet spots, dark spots, etc.).

→

→ Doppler Effect : when source is moving

14.72

Pipe with one end open : $L = 1.5\,m$



→ Similar to a string with a fixed end!

→ Only certain wavelengths can exist as standing waves:

$$\omega = v k$$
$$2\pi f = v \cdot \frac{2\pi}{\lambda} \rightarrow \boxed{f = \frac{v}{\lambda}}$$

(ω : angular frequency; f : linear frequency)

v : wave speed

k : wave number $= \frac{2\pi}{\lambda}$

→ Only certain frequencies can exist as standing waves in the pipe: $f_n = 225\,Hz$; $f_{n+1} = 375\,Hz$.

→ Lowest or fundamental freq : f_0 (λ_0 : longest wavelength).

What is a standing wave in a pipe with one end open @ $x=L$?

$\downarrow$

combination of in-coming & reflected waves

$$A\cos(kx - \omega t) - A\cos(kx + \omega t)$$

$$= 2A \sin kx \sin \omega t$$

Our pipe:

closed @ $x=0 \rightarrow y(x=0,t)=0$ ✓

open @ $x=L \rightarrow y(x=L,t)=$ max

$\rightarrow \sin kL = \pm 1$

$kL = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

(odd multiples of $\frac{\pi}{2}$)

$$kL = (2n+1)\frac{\pi}{2} \quad (n=0,1,2,\ldots)$$

$$\frac{2\cancel{\pi}}{\lambda}L = (2n+1)\frac{\cancel{\pi}}{2}$$

$$\boxed{\frac{1}{\lambda} = \frac{2n+1}{4L} \quad (n=0,1,2,\ldots)}$$

$$\rightarrow f = \frac{v}{\lambda} = \frac{(2n+1)v}{4L} \quad (n=0,1,2,\ldots)$$

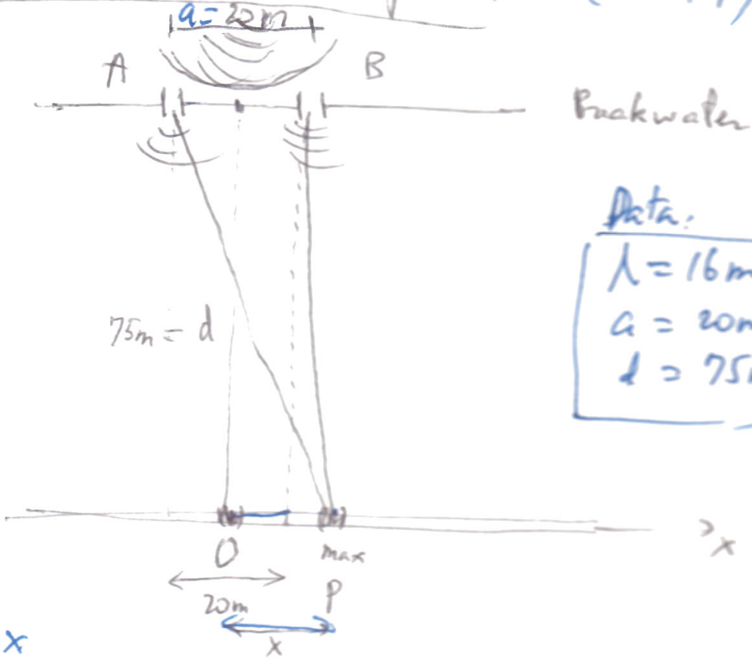
Different frequencies that can exist in the pipe:

mode n $\quad f_n = \left(\frac{2n+1}{4L}\right)v \quad (n=0,1,2,\ldots) \quad (1)$

next mode $n \rightarrow n+1$ $\quad f_{n+1} = \left(\frac{2(n+1)+1}{4L}\right)v = \frac{2n+3}{4L}v$

$$\rightarrow \frac{f_{n+1}}{f_n} = \frac{\frac{2n+3}{\cancel{4L}}}{\frac{2n+1}{\cancel{4L}}}\frac{\cancel{v}}{\cancel{v}} = \frac{2n+3}{2n+1} = \frac{375}{225} = \frac{125\times3}{75\times3} = \frac{5\times3\times5}{3\times3\times5}$$

Constructive & destructive interference: (Ch. 14)



Data:

$$\boxed{\begin{aligned}\lambda &= 16\,m\\ a &= 20\,m\\ d &= 75\,m\end{aligned}}$$

P, 1st max: @ x

$AP - BP = \lambda \longrightarrow$ of water wave

$$\sqrt{d^2+(x+10)^2} - \sqrt{d^2+(x-10)^2} = \lambda \rightarrow d^2+(x+10)^2 + d^2+(x-10)^2 - 2\sqrt{d^2+(x+10)^2}\sqrt{d^2+(x-10)^2} = \lambda^2$$

$$2d^2 + 2x^2 + 200 - \lambda^2 = 2\sqrt{d^2+(x+10)^2}\sqrt{d^2+(x-10)^2}$$

$\boxed{\lambda = 16\,m}$ data

$$\cancel{11250}\ [11194 + 2x^2]^2 = 4(5725 + x^2 + 20x)(5725 + x^2 - 20x)$$

$$11194^2 + 4\times 11194 x^2 + 4x^4 = 4\times 5725^2 + 4\times 2\times 5725 x^2 - 4\times 20^2 x^2 + 4x^4$$

simplify: $y = x^2 \rightarrow$ quadratic eq in $y \rightarrow$ solve for y

$\hookrightarrow x = \pm\sqrt{y} \rightarrow \boxed{x = \pm 33\,m}$

Find 1st min $\longrightarrow$ replace λ by $\frac{\lambda}{2}$ in previous eq

Find 2nd max $\longrightarrow$ replace λ by 2λ

Find 2nd min: " λ " $\frac{3\lambda}{2}$ . . .

Ch 15 Fluid Motion

- Gas : ρ can be variable (gas is compressible)
- Liquid : ρ is constant (liquid is incompressible)

ρ : density

→ Density $\rho \equiv \frac{\text{mass}}{\text{volume}} = \frac{dm}{dV}$ (S.I: $\frac{kg}{m^3}$)

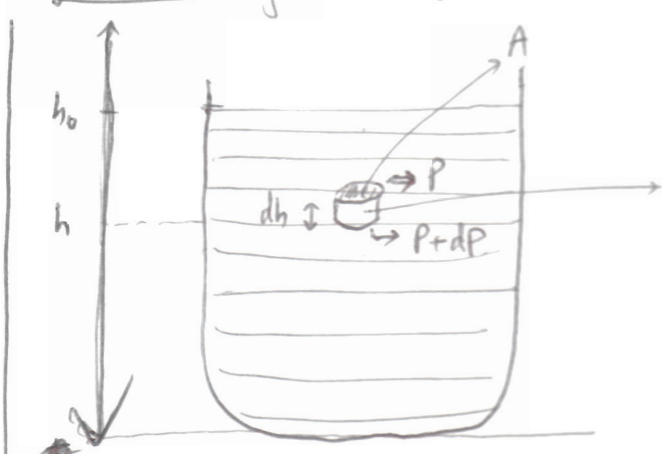
→ Pressure P = $\boxed{\text{normal}}$ force per unit area (SI. $\frac{N}{m^2}$ = Pa or Pascal)

$P = \frac{F}{A}$ or $\frac{dF}{dA}$ → direction is not relevant; P is not a vector

→ Alternative unit Atm (Atmosphere)

$$\boxed{1\,\text{Atm} = 1.013 \times 10^5\ \text{Pa}}$$

Hydrostatic equilibrium



→ Vase filled with water

Let's focus on this infinitesimal piece of water of mass dm and cross-sectional area A

dP : infinitesimal increase in pressure from top to bottom of the tiny cylinder of water

→ piece of water in equilibrium:

$$\underbrace{(P+dP)A}_{\text{upward force}} - \underbrace{PA}_{\text{downward force}} = g\,dm \rightarrow A\,dP = g\,dm = g\rho \underbrace{dV}_{A\,dh}$$

volume of piece of water

$$\boxed{\frac{dP}{dh} = g\rho}$$

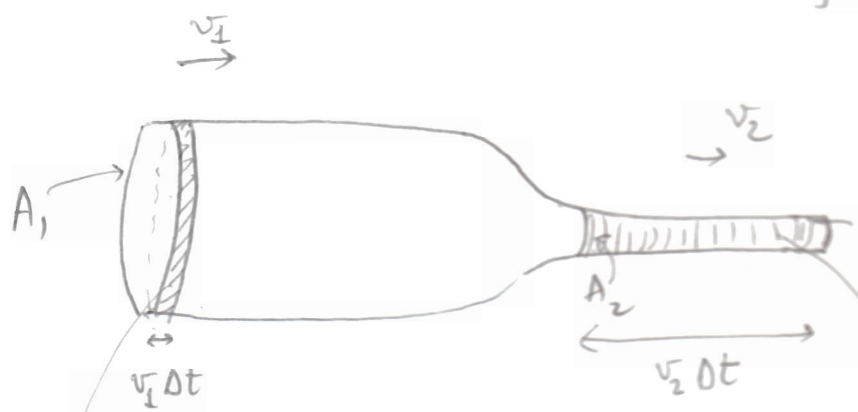
$$dP = g\rho\, dh \rightarrow P = \int_{h_0}^{h} g\rho\, dh = g\rho\,(h-h_0) \quad (h > h_0)$$

$$\int_{h_0}^{h} dh = h\Big]_{h_0}^{h} = h - h_0$$

If $h_0 = 0 \rightarrow \boxed{P = g\rho h}$

~~Boyancy~~ Bouyancy: $\rightarrow F_{\text{bouyant}} = P.A = g\rho \underbrace{hA}_{\text{vol.}}$

<u>Conservation of mass</u>: (no leaking or loss of fluid molecules)



a disk of fluid of mass m_1

same disk of fluid of mass $m_2 = m_1$

$m_1 = \rho_1 V_1$ $m_2 = \rho_2 V_2$ $\rightarrow$ same fluid

$\rho_1 = \rho_2$ (incompressible fluid)

Conservation of mass $\rightarrow$

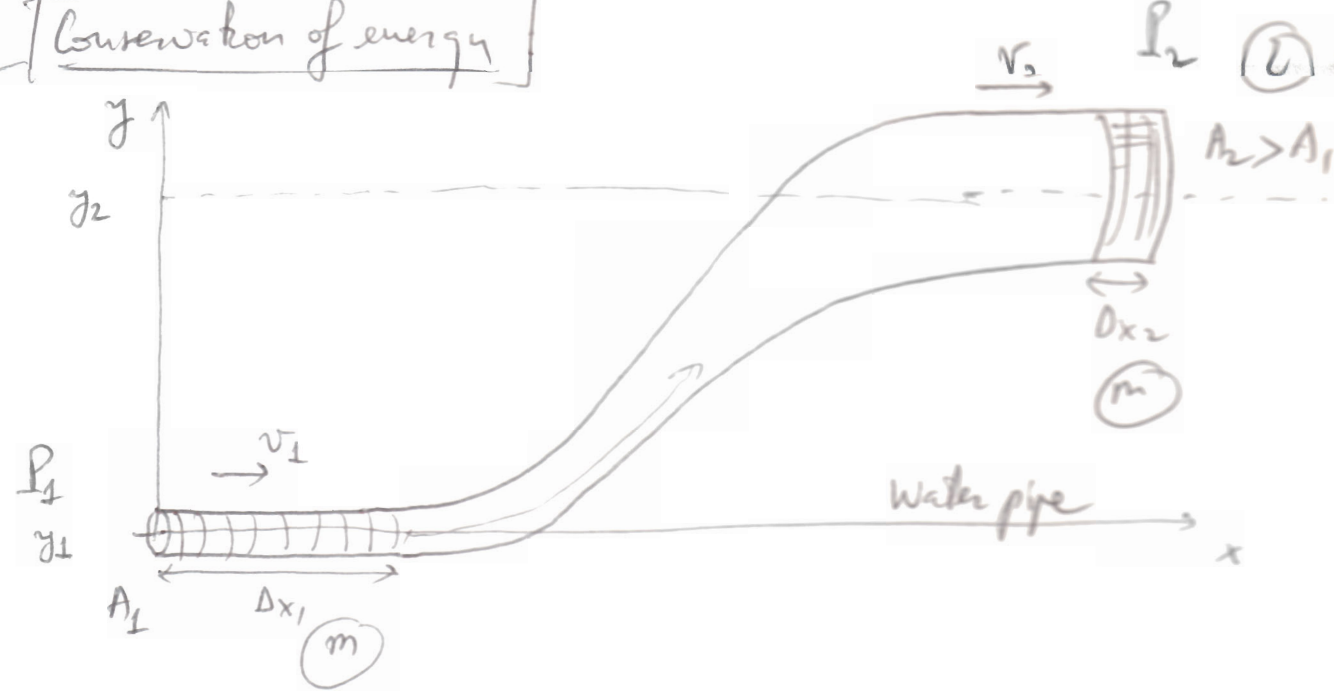
$m_1 = m_2 \rightarrow \rho V_1 = \rho V_2 \rightarrow V_1 = V_2$

$\downarrow$

$A_1 v_1 \cancel{\Delta t} = A_2 v_2 \cancel{\Delta t}$

$\rightarrow \boxed{vA \text{ is constant}} \rightarrow$ { larger cross-sectional area $\rightarrow$ lower speed.

Conservation of energy



1

Diffrence in pressure from 1 to 2 : water went from 1 up to 2 → Work done by pressure is

$$\Delta W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

$$\boxed{\Delta W = \Delta KE + \Delta P.E.}$$

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

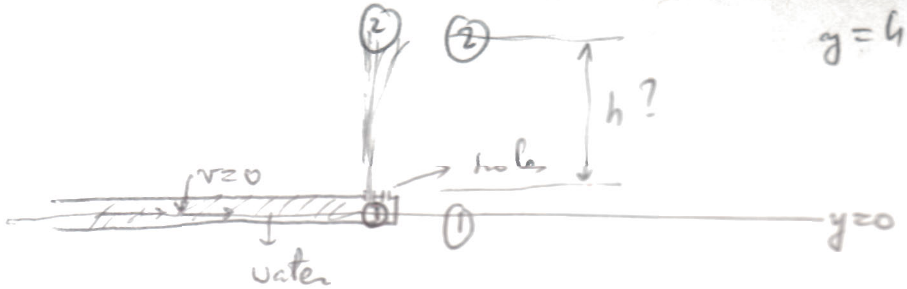
$$\rightarrow \boxed{\frac{1}{2} m v^2 + mgy + PA\Delta x = \text{constant}}$$

Rearrange: dividing by $V = A\Delta x$

$$\frac{1}{2}\frac{m}{V} v^2 + \frac{m}{V} gy + P = \text{const}$$

$$\boxed{\frac{1}{2}\rho v^2 + \rho g y + P = \text{const}} \rightarrow \text{Bernoulli's equation.}$$

15.57



→ Conservation of energy or Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$$P_1 + \frac{1}{2}\rho \cancel{0^2} + \rho g 0 = P_2 + \frac{1}{2}\rho 0^2 + mgh$$

$$\rightarrow h = \frac{P_1 - P_2}{\rho g}$$

$$= \frac{140\,\text{kPa} + \cancel{P_{atm}} - \cancel{P_{atm}}}{10^3 \frac{kg}{m^3}\, 9.81\, m/s^2}$$

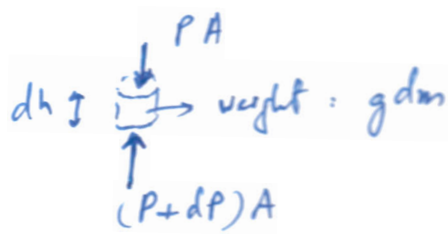
$$\boxed{\rho_{water} = 1000\, kg/m^3}$$

$$\boxed{h = \frac{140}{9.81}\, m = 14.3\, m}$$

Ch15 Fluid Motion:

- **Hydrostatic equilibrium**: or fluid in static equilibrium.

  The difference in pressure above (less) and below (more) on an element of fluid exactly compensates the weight of that element of fluid.



$$\rightarrow \quad A\,dP = g\,dm = g\rho\,dV = g\rho A\,dh$$

$$\boxed{\frac{dP}{dh} = g\rho}$$

Bouyancy: $F_b = P.A = \underbrace{g\rho h}_{P} A$

The bouyant force of a liquid on a cylinder or rectangular box of cross-sectional area A, and height h is $g\rho hA$, where ρ is the density of the fluid $\left(\rho = \frac{M_{fluid}}{V_{fluid}}\right)$

- **Conservation of mass**: (assuming no leaking)

  $\hookrightarrow \quad vA = \text{constant}$

  v: speed of fluid; A: cross sectional area of pipe

$v_2 > v_1$

$A_1 v_1 = A_2 v_2$ (since $\frac{A_1}{A_2} > 1 \rightarrow v_2 > v_1$)

- Conservation of energy:
  - Work done by the fluid pressure equals the change in its mechanical energy (KE+PE)
    - $$\underbrace{\frac{1}{2}mv^2 + mgh}_{\text{M.E.}} + \underbrace{PA\Delta x}_{\text{Work done by pressure}} = \text{constant}$$
      - Rearranging:

$$\boxed{\frac{1}{2}\rho v^2 + \rho g h + P = \text{constant}}$$

Bernoulli's equation.

ρ: fluid density; v = fluid speed; P : fluid pressure; h = change in fluid height.

Last exam:

Ch 10: Rotational Motion:

- Torque (select pivot point, $\vec{\tau} = \vec{r} \times \vec{F}$
  $\vec{r}$: from pivot to force application point)
- Analog of 2nd Newton's law: $\vec{\tau} = I\vec{\alpha}$ (I : moment of inertia, $\vec{\alpha}$: angular acceleration)

Ch 11: Angular momentum:

- $\vec{L} = \vec{r} \otimes \vec{p}$ ($\vec{p} = m\vec{v}$)
  - cross-product
    - $\vec{L} \perp \vec{r}$ & $\vec{L} \perp \vec{p}$
    - Direction of $\vec{L}$ given by R.H.R.
    - Magnitude $L = rp\sin\theta$ (θ angle b/w $\vec{r}$ & $\vec{p}$)
  - (round object: $L = I\omega$)
- Conservation of angular momentum:
  $$\vec{\tau}_{\substack{net\\external}} = \frac{d\vec{L}}{dt} \Rightarrow \vec{\tau}_{\substack{net\\ext}} = 0 \rightarrow \vec{L}_i = \vec{L}_f$$

Ch 12: Static equilibrium:

- 1) $\sum_i \vec{F}_i = 0$ (Net force on system is 0)
  2) $\sum_i \vec{\tau}_i = 0$ (net torque on system is 0)
- Use geometry to find angles.
- Define pivot point to eliminate unknown forces from the torque balance equation $\sum_i \vec{\tau}_i = 0$

→ Ch 13: Oscillatory Motion

↳ SHM :

- Pendulum : $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \rightarrow \theta = \theta_m \cos\omega t$
  $\omega = \sqrt{\frac{g}{L}}$
- Torsional pendulum :
  $\frac{d^2\theta}{dt^2} = -\frac{K}{I}\theta \rightarrow \theta = \theta_m \cos\omega t$
  $\omega = \sqrt{\frac{K}{I}}$
- Spring & bob :
  $\frac{d^2x}{dt^2} = -\frac{k}{m}x \rightarrow x = x_m \cos\omega t$
  $\omega = \sqrt{\frac{k}{m}}$

↳ Total energy stays constant;
$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}k x_m^2$

→ Ch 14: Wave motion :

→ Types : → longitudinal : osc. parallel to propagation
↳ transverse : osc. perpendicular to propagation

↳ wave in a string : $y(x,t) = A\sin(kx - \omega t)$

↳ prop. in $+x$ → $A\sin(kx - \omega t)$
prop in $-x$ → $A\sin(kx + \omega t)$

↳ $v = \sqrt{\frac{T}{\mu}}$ (v: wave speed, T tension in the string, μ: its linear mass density).

→ Doppler Effect : $f' = \frac{f_0}{1 \pm \frac{u}{v}}$ → receding, → approach.

→ Superposition of waves :

↳ Beat Phenomenon : $y_1(0,t) + y_2(0,t) = \underbrace{-2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)}_{\text{slowly modulated amplitude (beats) when } \omega_1 \text{ is close to } \omega_2}\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)$

↓

Intensity change @ $(\omega_1 - \omega_2)$

→ Ch 15: Fluid Motion

→ Hydrostatic Equilibrium: $\frac{dP}{dh} = g\rho$

→ Bouyancy: $F_b = g\rho h . A$

Bouyant force of a liquid on a submerged cylinder or rectangular box of cross-sectional area A and height h is $g\rho h A$ (ρ: fluid density)

→ Conservation of mass: $vA = \text{constant}$

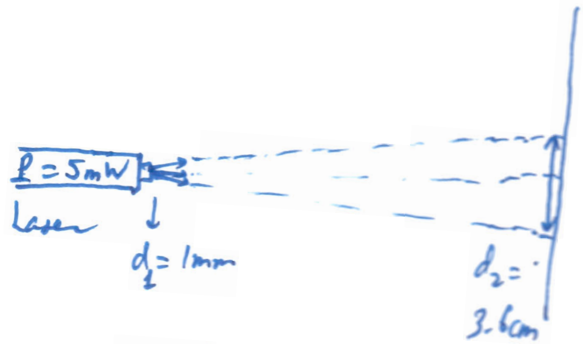
speed of fluid; its cross-sectional area.

→ Conservation of energy: $\frac{1}{2}\rho v^2 + \rho g h + P = \text{const.}$

(Pressure $P = \frac{F}{A}$; unit SI: $\frac{N}{m^2} = Pa$ (Pascal)

$1\,Atm = 1.013 \times 10^5\, Pa$

14.52 Intensity of laser beam: $I = \frac{P}{A}$



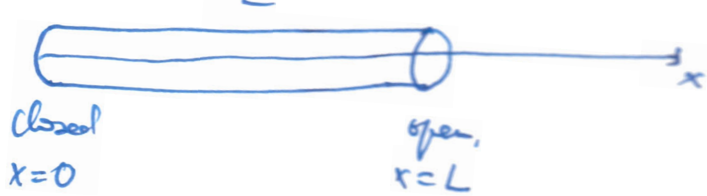
$I_1 = ?$

$$I_1 = \frac{5\times10^{-3}}{\pi(0.5\times10^{-3})^2} = \frac{5\times10^{3}}{\pi\times0.5^2} = 6.37\ \frac{kW}{m^2}$$

$I_2 = ?$

$$I_2 = \frac{5\times10^{-3}}{\pi(1.8\times10^{-2})^2} = \frac{50}{\pi\times1.8^2} = 4.91\ \frac{W}{m^2}$$

14.41 Human vocal tract = pipe closed @ one end. x=0
open @ the other end x=L



Standing waves: → $y_T(x,t) = 2A \sin kx \sin \omega t$

Max amplitude for sound waves @ open end x=L:

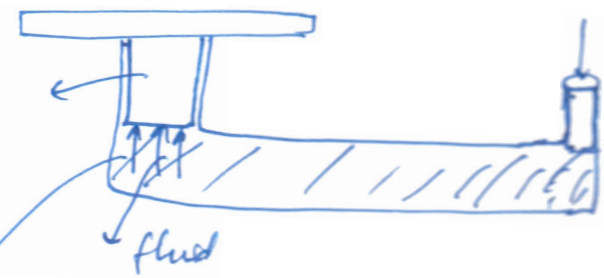
$y_T(L,t) = \max$ → $\sin kL = \pm1$

kL = odd multiple of $\frac{\pi}{2}$

$kL = (2n+1)\frac{\pi}{2}$ $(n = 0, 1, 2 \ldots)$

$\frac{2\pi}{\lambda}L = (2n+1)\frac{\pi}{2}$ → $\frac{1}{\lambda} = \frac{2n+1}{4L}$ → $\boxed{\lambda = \frac{4L}{2n+1}}$ (n=0,1, e.t.)

15.47



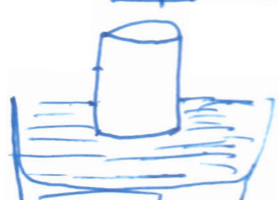
$$\boxed{F_{up} = P \cdot A} = 500 \times 10^3 \times \pi \left(\frac{0.45}{2}\right)^2 = 79.5 \times 10^3\ N$$

$$mg = F_{up} \rightarrow m = \frac{F_{up}}{g} = \frac{79.5 \times 10^3}{9.81} = 8112\,kg$$

14.54 done

15.50

Empty 1



Max. load (2)

completely submerged, about to sink.

Beaker: floating with $\frac{h}{3}$ submerged in water

$h = 10\,cm$

$d = 4\,cm$

m_1

floating with h submerged in water. with n rocks inside

$m_2 = 0.015\,kg$ (mass per rock)

$$\boxed{F_{b_1} = m_1 g \qquad \rho_w g \frac{h}{3} A}$$

$$\boxed{F_{b_2} = m_1 g + n m_2 g \qquad \rho_w g h A \qquad n?}$$

$$\boxed{\rho_w = 1000\,kg/m^3 \quad \text{data}}$$

$$3 m_1 \cancel{g} = m_1 \cancel{g} + n m_2 \cancel{g}$$

$$2 m_1 = n m_2$$

$$n = \frac{2 m_1}{m_2} = \frac{2 \rho_w \frac{h}{3} A}{m_2}$$

$$m_1 = \rho_w \frac{h}{3} A$$

$$\hookrightarrow n = \frac{2 \times 1000 \times \frac{0.1}{3} \times \pi (0.02)^2}{\ } = 5.59 \rightarrow \boxed{n = 5}$$ (it will sink with $n = 6$)