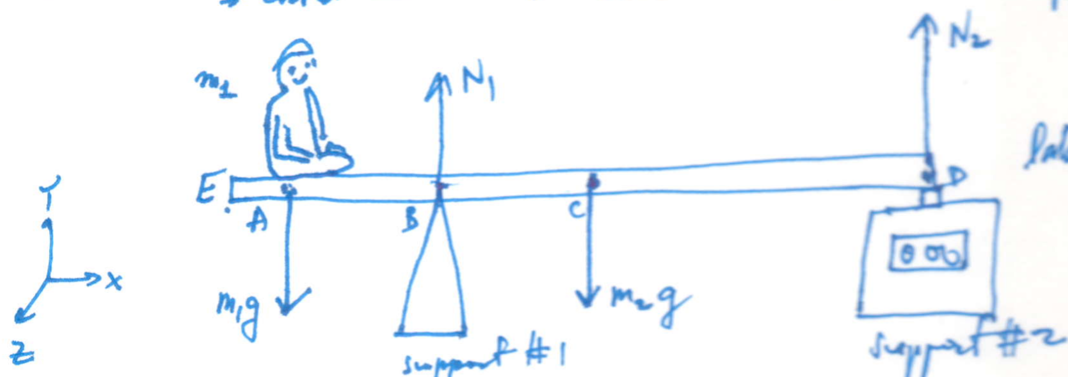
Ch12: Static Equilibrium

Application of force & torque balance:

Static equilibrium $\begin{cases} 1) \ \sum_i \vec{F}_i = 0 & \text{(No net force on object)} \\ 2) \ \sum_i \vec{\tau}_i = 0 & \text{(No net torque on object)} \end{cases}$

→ if it will not move neither rotate

12.22 → Scale: 3 parts: beam & 2 supports.
→ child → where does he sit so support #2 "feels"
a) 100N
b) 300N



Data:
- Child: $m_1 = 40kg$
- Beam: $m_2 = 60kg$
- $ED = L = 2.4m$
- S1 is 0.8m from left end.
- $EB = 0.8m$.
- $EC = 1.2m = CD$

→ Focus on beam: which should be in static equilibrium when a reading is taken.

↳ 1) $\sum_i \vec{F}_i = 0$ (on beam!)
2) $\sum_i \vec{\tau}_i = 0$ (on beam!)

Forces on beam: child's weight $m_1 g$; beam's weight $m_2 g$.
+ normal forces by supports on beam: N_1 & N_2

↳ $\sum_i \vec{F}_i = 0$: $N_1 + N_2 - m_1 g - m_2 g = 0$

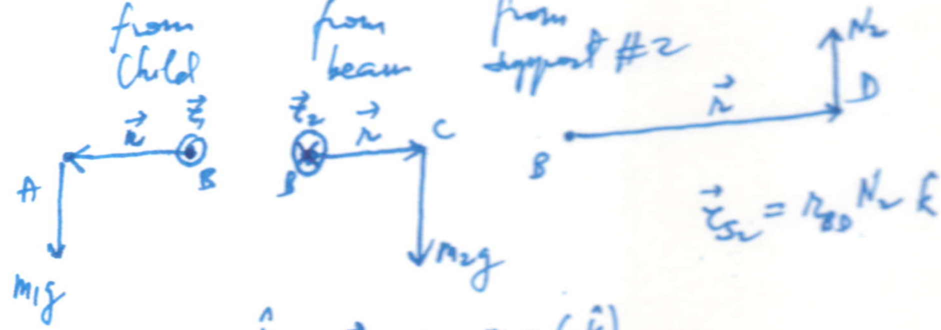
Torques on beam: torque definition requires a pivot, what is it? For the calculations: any force application point (there are 4) will do →
If there is a force we don't know or don't care about → pivot at its application point so then the torque due to that force is 0!

In this problem we don't want to place the pivot at A neither D. ⇒ B or C, since we know m_2, but not N_1 → B.

Any force applying on a pivot contributes no torque!

→ 3 torques:

$$\sum_i \vec{\tau}_i = 0 \rightarrow \underset{\text{from child}}{\vec{\tau}_1} + \underset{\text{from beam}}{\vec{\tau}_2} + \underset{\text{from support \#2}}{\vec{\tau}_{S2}} = 0$$



$$\vec{\tau}_1 = r_{BA} m_1 g \hat{k} \qquad \vec{\tau}_2 = r_{BC} m_2 g (-\hat{k}) \qquad \vec{\tau}_{S2} = r_{BD} N_2 \hat{k}$$

$$\underbrace{\left(r_{BA} m_1 g - r_{BC} m_2 g + r_{BD} N_2\right)}_{0} \hat{k} = 0$$

$$r_{BA} = \frac{r_{BC} m_2 g - r_{BD} N_2}{m_1 g}$$

we know from data:
- $ED = 2.4\,m$.
- $EC = CD = 1.2\,m$.
- $EB = 0.8\,m$.

⇒ $r_{BC} = 0.4\,m$

⇒ $r_{BD} = 1.6\,m$

$N_2 = 100\,N$ or $300\,N$ by action & reaction (3rd Newton's law)

c) $r_{BA} = \dfrac{0.4 \times 60 \times 9.81 - 1.6 \times 100}{40 \times 9.81} = 0.19\,m$

Child sits @ 0.19m from pivot point B.

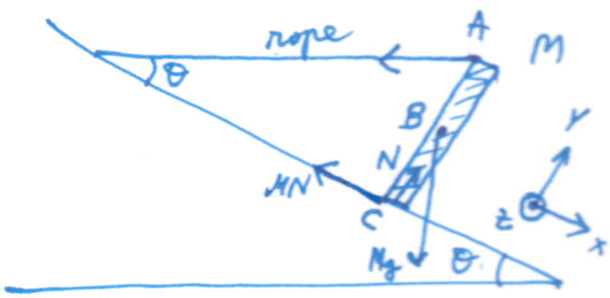
or 0.61m from left end E

5) $r_{BA} = \dfrac{0.4 \times 60 \times 9.81 - 1.6 \times 300}{40 \times 9.81} = \ominus 0.62\,m$

child sits @ 0.62 m
<u>to the right of pivot B!</u>

or $0.8 + 0.62 = 1.42\,m$
from left end E !

12.57



Min coef of friction b/w pole & slope for pole in static equilibrium

1) $\sum_i \vec{F}_i = 0$ (on pole)
2) $\sum_i \vec{\tau}_i = 0$ (on pole)

→ Forces acting on pole:
- Mg (applying on CM of pole or B)
- N (applying @ contact point w/ slope or C)
- T (applying @ rope or point A)
- μN (applying @ contact w/ slope or C, uphill since if there was no friction, bottom of pole would tend to slide downhill in this example)

Components of these forces along coordinate axes x & y:

Mg → 

$Mg\sin\theta\,\hat{\imath} - Mg\cos\theta\,\hat{\jmath}$

N → 

$N\hat{\jmath}$

T 

$-T\cos\theta\,\hat{\imath} - T\sin\theta\,\hat{\jmath}$

$-\mu N\,\hat{i}$

1) $\sum_i \vec{F}_i = 0$ (net force on pole is zero)

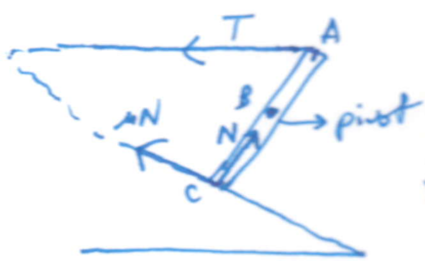
$\hookrightarrow \begin{cases} x: & \underbrace{(Mg\sin\theta - T\cos\theta - \mu N)}_{0 \ \ 1}\,\hat{i} = 0 \\ y: & \underbrace{(-Mg\cos\theta + N - T\sin\theta)}_{0 \ \ 2}\,\hat{j} = 0 \end{cases}$

2) $\sum_i \vec{\tau}_i = 0$ (net torque on pole is zero).

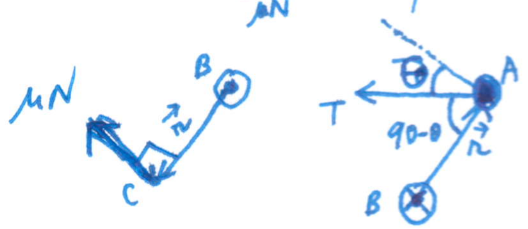
→ Pivot: CM or point B (not C since we need friction to stay in the equation; not A since tension is related to θ, a known parameter)

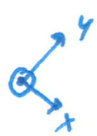
↳ @B → Mg contributes no torque! → 3 torques, effectively 2 since $\vec{\tau}_N = 0$ b/c $\vec{N} \parallel \vec{r}_N$ ($\theta = 180 \to \sin\theta = 0$)

↓ $\vec{\tau}_{\mu N} + \vec{\tau}_T = 0$



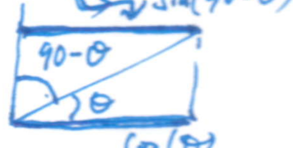
$\vec{\tau}_N = \vec{r}_{BC} \times \vec{N} = 0$





$\vec{\tau}_T = r_{BA} T \underbrace{\sin(90-\theta)}\,\hat{k} = r_{BA} T \cos\theta\,\hat{k}$

$\hookrightarrow \sin(90-\theta)$



$\vec{\tau}_{\mu N} = r_{BC}\,\mu N(-\hat{k})$

$\sum_i \vec{\tau}_i = 0 \to \underbrace{(-r_{BC}\,\mu N + r_{BA} T\cos\theta)}_{0}\,\hat{k} = 0$ $\quad$ 3 $\boxed{-\mu N + T\cos\theta = 0}$

Pivot point B is CM of pole $\to r_{BC} = r_{BA}$

From force & torque balance for pole:

$$\begin{cases} Mg\sin\theta - T\cos\theta - \mu N = 0 & (1) \\ -Mg\cos\theta + N - T\sin\theta = 0 & (2) \\ -\mu N + T\cos\theta = 0 & (3) \end{cases}$$

(3) in (1) $\rightarrow$ $Mg\sin\theta - 2T\cos\theta = 0$

$$T = \frac{Mg\sin\theta}{2\cos\theta} = \frac{Mg}{2}\tan\theta$$

Use this in (2) : $N = Mg\cos\theta + T\sin\theta$

$$= Mg\cos\theta + \frac{Mg}{2}\tan\theta\sin\theta$$

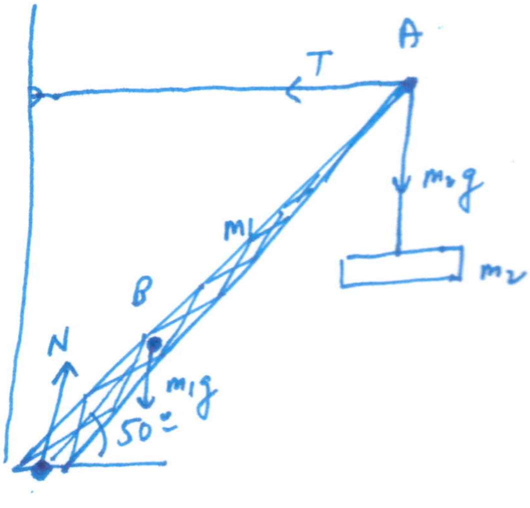
$$= Mg\left[\cos\theta + \frac{\tan\theta\sin\theta}{2}\right]$$

(3) $$\mu = \frac{T\cos\theta}{N} = \frac{\frac{\cancel{Mg}}{2}\tan\theta\cos\theta}{\cancel{Mg}\left(\cos\theta + \frac{\tan\theta\sin\theta}{2}\right)} \quad \frac{\times\frac{2}{\cos\theta}}{\times\frac{2}{\cos\theta}}$$

$$\mu = \frac{\tan\theta}{(2+\tan^2\theta)} \quad (\text{min})$$

$$\rightarrow \boxed{\mu_s \geq \frac{\tan\theta}{2+\tan^2\theta}}$$

12-40



Data: $CA = 15\,m$, $CB = 5\,m$

B @ CM of boom (mass m_1)

$m_1 = 860\,kg$

$m_2 = 2500\,kg$

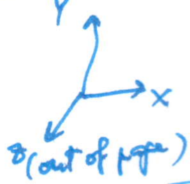
Static equilibrium for m_1:

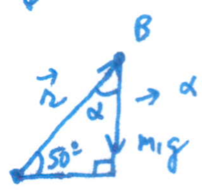
$\sum_i \vec{F}_i = 0$ → net force on boom is zero

$\boxed{\sum_i \vec{\tau}_i = 0}$ → pivot: A B C

we don't know N & its angle

$\vec{\tau}_{m_1 g} + \vec{\tau}_{m_2 g} + \vec{\tau}_T = 0$

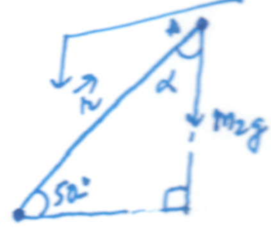




$\alpha + 50 + 90 = 180$

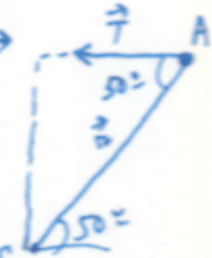
$\alpha = 40°$

$\vec{\tau}_{m_1 g} = \underbrace{r_{CB}}_{5m} m_1 g \sin 40° (-\hat{k})$



$\alpha = 40°$

$\vec{\tau}_{m_2 g} = \underbrace{r_{CA}}_{15m} m_2 g \sin 40° (-\hat{k})$



$\vec{\tau}_T = \underbrace{r_{CA}}_{15m} T \sin 50° (\hat{k})$

$$-5 \times 860 \times 9.81 \sin 40° - 15 \times 2500 \times 9.81 \sin 40° + 15\,T \sin 50° = 0$$

$$T = \frac{5 \times 860 \times 9.81 \sin 40° + 15 \times 2500 \times 9.81 \sin 40°}{15 \sin 50°}\ N$$

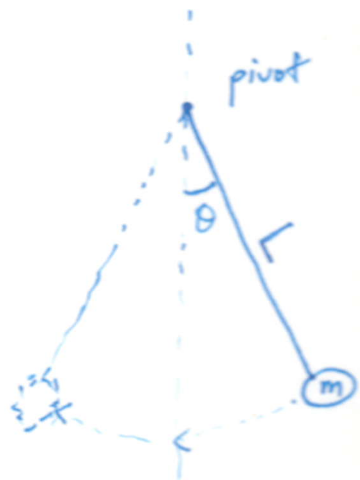
$$T = 22900\ N$$

Ch 13 Oscillatory Motion

A different type of motion:

Example.

1) Pendulum



Bob attached to a string, which is attached to a fixed point or pivot. System swinging about pivot point.

Better described with an angle θ (closer to a rotation than a linear motion)

$$\tau = I\alpha \rightarrow \alpha = \frac{\tau}{I} \rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Very typical approximation is θ small or "small angle approximation"

$$\sin\theta \simeq \theta \rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

2nd Linear differential eq.

$$\hookrightarrow \theta = \theta_M \cos(\omega t)$$

θ_M = amplitude

$\boxed{\omega = \text{angular freq.}}$

ω: # oscillations per second. What is it in terms of the length L and/or the mass of the bob m?

Plug solution back into the equation:

$$\frac{d}{dt}\Big[\underbrace{\frac{d}{dt}\theta_M\cos(\omega t)}_{-\theta_M\omega\sin(\omega t)}\Big] = -\frac{g}{L}\theta_M\cos(\omega t)$$

$$\cancel{+\theta_M}\,\omega^2\cancel{\cos\omega t} = +\frac{g}{L}\cancel{\theta_M}\cancel{\cos\omega t} \rightarrow \omega^2 = \frac{g}{L} \rightarrow \boxed{\omega = \sqrt{\frac{g}{L}}}$$

does not depend on time...

For a pendulum (where we can apply the "small angle approx."), the number of oscillations per second or angular frequency depends on g and L (but not on m)

$$\omega = \sqrt{\frac{g}{L}}$$

less ω for longer pendulum
more osc. per second for shorter pendulum.

2) Torsional pendulum:

Not a swinging motion, more of a twisting back & forth

$\tau = I\alpha$

Twisting: $\tau = -\kappa\theta$ → twisting angle

"kappa": torsional constant (Analog of Hooke's Law for twisting motion)

$$-\kappa\theta = I\frac{d^2\theta}{dt^2}$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta}$$

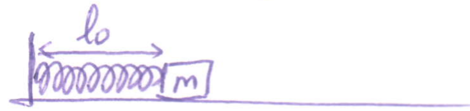
2nd order linear differential equation

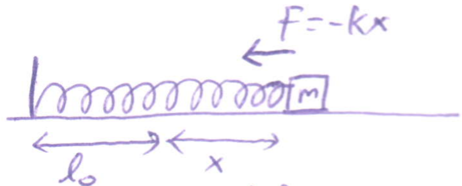
$\theta = \theta_m \cos\omega t$; $\boxed{\omega = \sqrt{\frac{\kappa}{I}}}$

θ_m: amplitude of oscillation

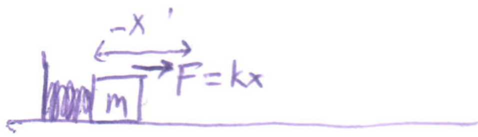
3) Spring & bob $\left\{ \begin{array}{l} k = \text{spring constant} \\ m = \text{mass of bob} \end{array} \right.$

natural length





stretch



compression

Spring always oppose the motion of m
→ m undergoes oscillatory motion.

Newton's 2nd Law:

$$F = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

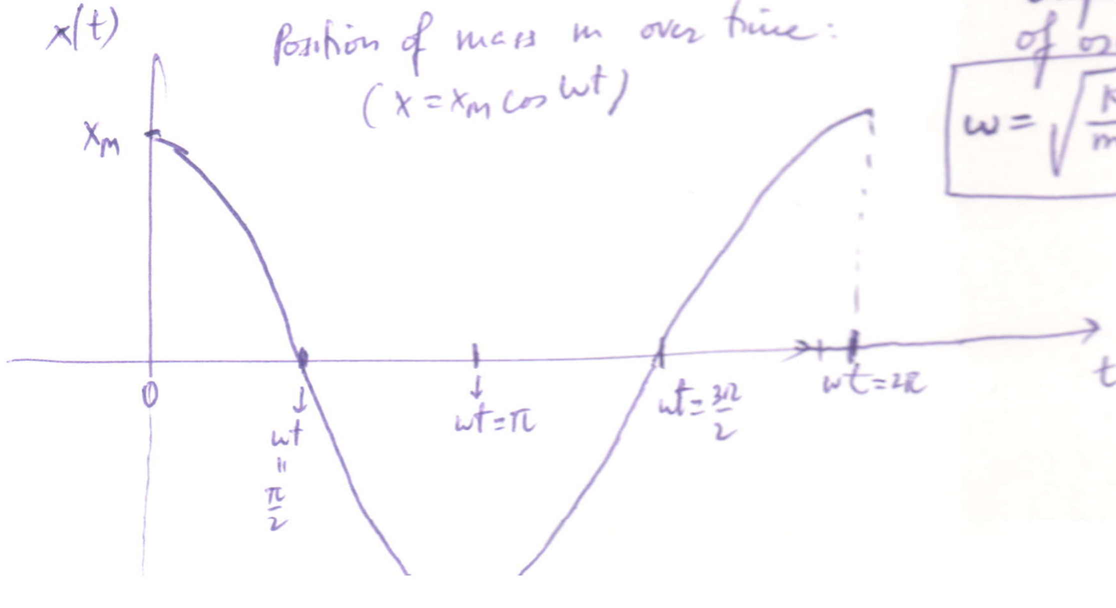
2nd order linear differential eq.

$x = x_m \cos(\omega t)$

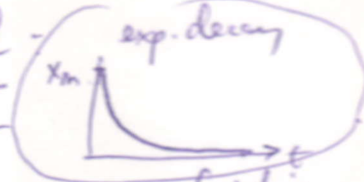
x_m → amplitude of oscillation

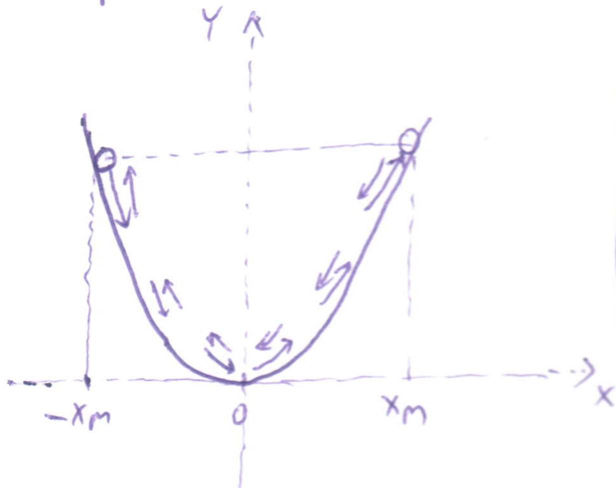
$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

Position of mass m over time:
($x = x_m \cos \omega t$)



4) A particle trapped in a potential well -





→ Assume there is no friction
$x(t) = x_m \cos(\omega t)$ → Simple Harmonic Motion SHM

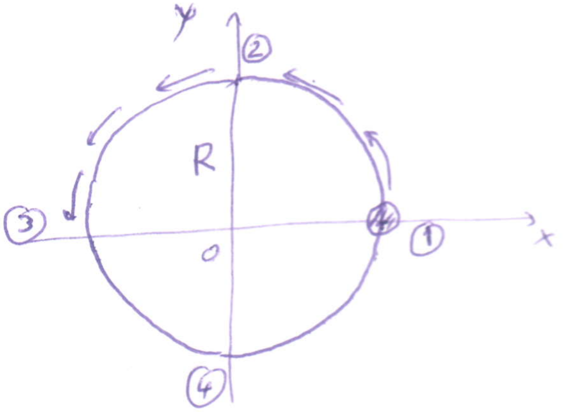
→ If there is friction : damped oscillatory motion :

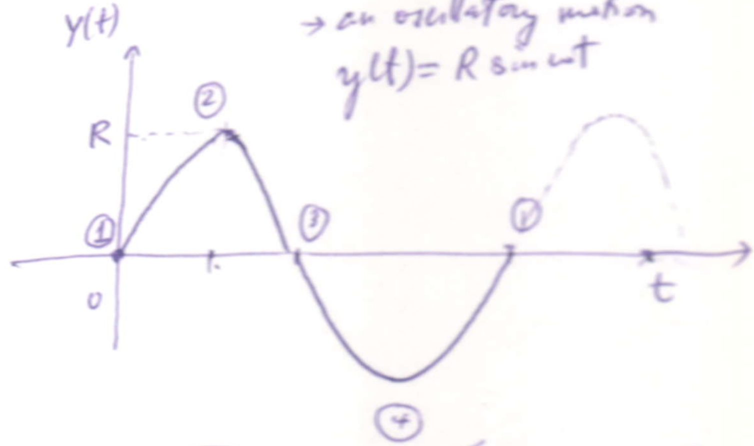
$$x(t) = \underbrace{x_m e^{-\frac{bt}{2m}}}_{\text{amplitude}} \cos(\omega t + \phi)$$

phi

amplitude decaying exponentially

5) Coordinates x & y of a particle undergoing a Uniform Circular Motion or UCM :

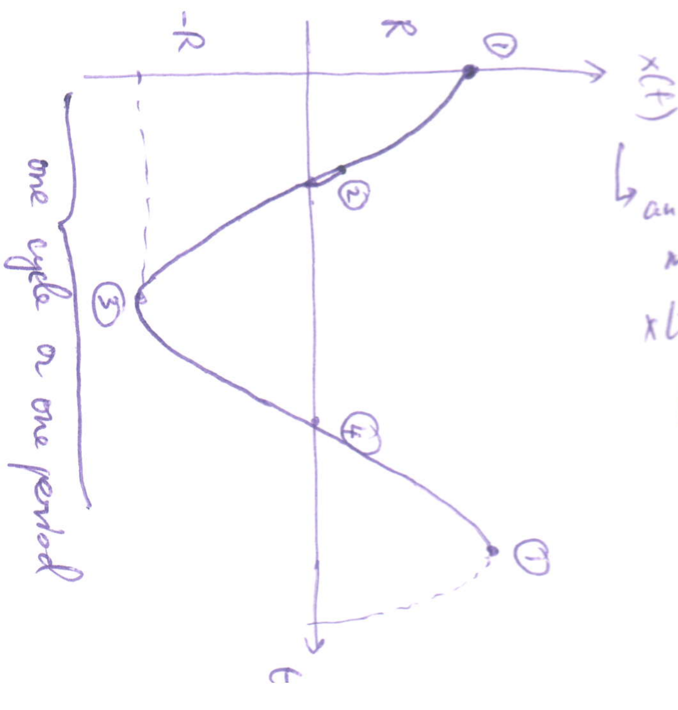




→ an oscillatory motion
$y(t) = R \sin \omega t$

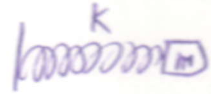
one cycle or one period T

$$\boxed{T = \frac{2\pi}{\omega}}$$



↳ an oscillatory motion

$x(t) = R \cos(\omega t)$ (x & y as coords of a particle in UCM) → are shifted by a phase of 90° or $\frac{\pi}{2}$



Summary: For a spring & bob:

SHM: $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ (x: position of mass m)

↳ solution is $x(t) = x_m \cos \omega t$; $\omega = \sqrt{\frac{k}{m}}$

Damped SHM: $\frac{d^2x}{dt^2} = -\frac{k}{m}x \underbrace{- \frac{b}{m}\frac{dx}{dt}}_{\text{damping term}}$

↳ solution is $x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega t + \phi)$ phi

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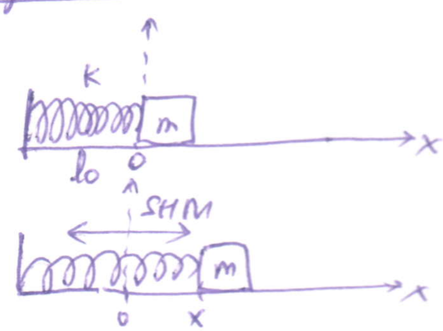
Energy in SHM: For a spring & bob



SHM: $x(t) = x_m \cos \omega t$

↳ displacement w.r.t. equilibrium length

$\omega = \sqrt{\frac{k}{m}}$

Total energy: since system laying on a horizontal surface
→ same grav. potential energy at all times
→ can ignore.

↳ $E = KE + U_{elastic} = \underbrace{\frac{1}{2}mv^2}_{\text{mass } m} + \underbrace{\frac{1}{2}kx^2}_{\text{spring } k}$

SHM: $\left\{ \begin{array}{l} x(t) = x_m \cos \omega t \\ v(t) = \frac{dx}{dt} = -x_m \omega \sin \omega t \end{array} \right\} \to E = \frac{1}{2} m x_m^2 \omega^2 \sin^2 \omega t + \frac{1}{2} k x_m^2 \cos^2 \omega t$

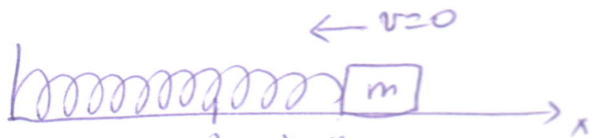
$\omega = \sqrt{\frac{k}{m}} \rightarrow \omega^2 = \frac{k}{m}$

$E = \frac{1}{2} \cancel{m} x_m^2 \frac{k}{\cancel{m}} \sin^2 \omega t + \frac{1}{2} k x_m^2 \cos^2 \omega t$

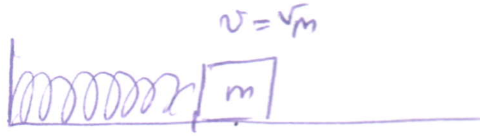
$= \frac{1}{2} k x_m^2 \underbrace{[\sin^2 \omega t + \cos^2 \omega t]}_{1}$

Total energy in a mass + spring system in SHM is constant over time! equal to $\frac{1}{2} k x_m^2$.

$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$



$E = \frac{1}{2} m \, 0^2 + \frac{1}{2} k x_m^2 = \frac{1}{2} k x_m^2$



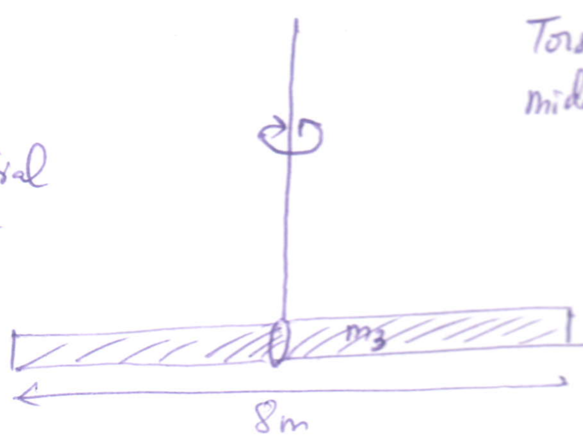
$E = \frac{1}{2} m v_m^2 + \frac{1}{2} k \, 0^2 = \frac{1}{2} m x_m^2 \omega^2 = \frac{1}{2} \cancel{m} x_m^2 \frac{k}{\cancel{m}} = \frac{1}{2} k x_m^2$

- - -

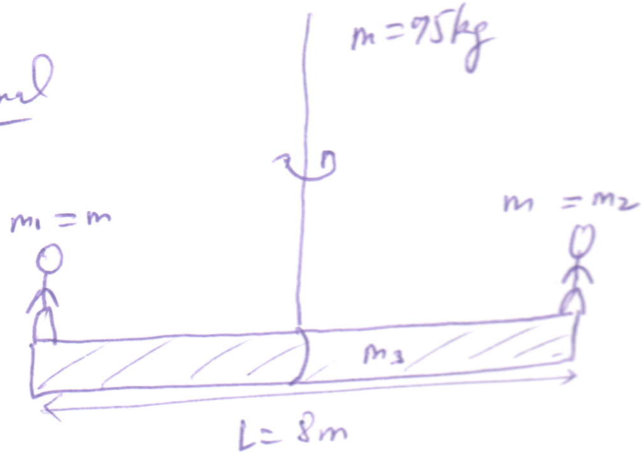
13.50

Torsional oscillations about middle axis. $\rightarrow \omega_i = \sqrt{\frac{K}{I_i}}$

initial



final



$$\omega_f = \sqrt{\frac{K}{I_f}}$$

K : material of string
torsional constant
$\rightarrow K_i = K_f = K$

$$I_f = I_i + 2m\left(\frac{L}{2}\right)^2$$

$$I_f > I_i \rightarrow \omega_f < \omega_i$$

$$\frac{\omega_f}{\omega_i} = 0.8 = \frac{\sqrt{\frac{K}{I_f}}}{\sqrt{\frac{K}{I_i}}} = \sqrt{\frac{I_i}{I_f}}$$

ω_f diminishes by 20% $\quad \omega_f = \omega_i - 0.2\,\omega_i = 0.8\,\omega_i$

When there is a square-root $\rightarrow$ square up both sides

$$0.8^2 = \frac{I_i}{I_f} = \frac{I_i}{I_i + m\frac{L^2}{2}}$$

$$0.8^2 I_i + mL^2 \frac{0.8^2}{2} = I_i$$

$$\boxed{I_i = \frac{mL^2 \frac{0.8^2}{2}}{1 - 0.8^2}}$$

Also: I_i = moment of inertia of a beam $\nearrow$ L, m_3 w.r.t. center axis: $I_i = \frac{1}{12} m_3 L^2$

$$\rightarrow \frac{1}{12} m_3 \cancel{L^2} = \frac{m\cancel{L^2}}{2} \frac{0.8^2}{1 - 0.8^2}$$

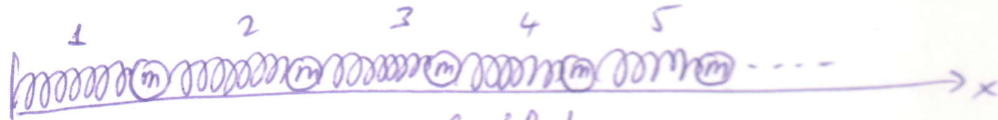
$$m_3 = 6m \frac{0.8^2}{1 - 0.8^2} = 450 \frac{0.8^2}{1 - 0.8^2}$$

$$\boxed{m_3 = 800 \text{ kg}}$$

Ch14 Wave Motion

Wave ≠ oscillation

<u>Wave</u>: a propagation of a local <u>oscillation</u> (or disturbance)
(Note: not a propagation of matter!)



Masses of spring are negligible!
If an oscillation is started on spring #1, at that moment spring #5 is not yet affected: it takes some time for an oscillation or disturbance to propagate a certain distance

Observations:

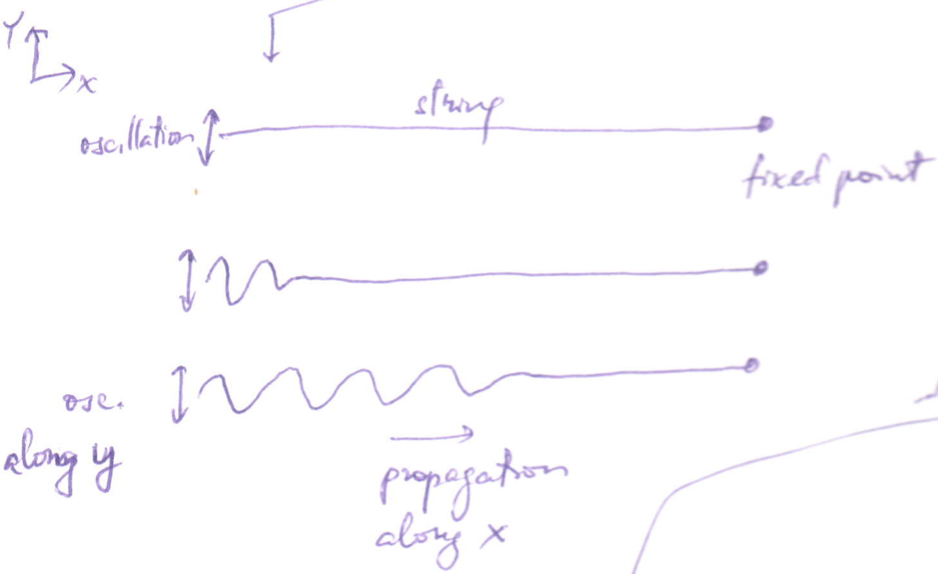
1) Propagation is not instantaneous: propagation speed v is $v = \frac{\lambda}{T}$ (λ = lambda = wavelength → space sep. b/w 2 consecutive peaks
T: period: time separation b/w 2 consecutive peaks)

2) ~~Oscillations~~ Masses & springs are local, only the oscillation is propagated

3) Propagation is over a distance (in space)
Oscillation is over a time (in time) } both are SHM → in a wave we have SHM both in time & in space

Waves → 2 types: { <u>Longitudinal waves</u> : oscillations in same direction as propagation (e.g. springs & masses above)

<u>Transverse waves</u> : oscillations are perpendicular to propagation. (e.g. waves on a string)



} $y(t)$

wave: $\boxed{y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) = A\sin(kx - \omega t)}$

$k = \frac{2\pi}{\lambda}$ (wave number) (m^{-1})

$\omega = \frac{2\pi}{T}$ (ang. frequency) (s^{-1})

SHM in both space x & time t.

13.45   <u>Pendulum</u> → SHM → $\omega = \sqrt{\frac{g}{L}}$, L: pendulum length

↓ in a rocket

a) Rocket @ rest → $T_a = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}}$

↓ $\vec{a} = 0$

b) Rocket accelerating upward $a = \frac{g}{2}$ → $\vec{a} = \frac{g}{2}\hat{j}$, $\vec{g} = -g\hat{j}$

$T_b = \frac{2\pi}{\sqrt{\frac{|\vec{a} - \vec{g}|}{L}}} = \frac{2\pi}{\sqrt{\frac{|+\frac{3g}{2}\hat{j}|}{L}}} = \frac{2\pi}{\sqrt{\frac{\frac{3g}{2}}{L}}} = \frac{2\pi}{\sqrt{\frac{3g}{2L}}}$

* Recall person in elevator: when elevator accelerating upward, person feels heavier

$\rightarrow \boxed{\vec{a} - \vec{g}}$

$$T_b = \sqrt{\frac{2}{3}} \cdot \underbrace{\frac{2\pi}{\sqrt{\frac{g}{L}}}}_{T_a} = \sqrt{\frac{2}{3}}\, T_a$$

(shorter period)

(Period = time for one full swing

)

c) Rocket acc. downward = $a = \frac{g}{2} \rightarrow \left.\begin{array}{l} \vec{a} = -\frac{g}{2}\hat{j} \\ \vec{g} = -g\hat{j} \end{array}\right\}$

Y, X

$$\hookrightarrow T_c = \frac{2\pi}{\sqrt{\frac{|\vec{a}-\vec{g}|}{L}}} = \frac{2\pi}{\sqrt{\frac{|[-\frac{g}{2} - (-g)]\hat{j}|}{L}}} = \frac{2\pi}{\sqrt{\frac{|\frac{g}{2}\hat{j}|}{L}}}$$

$$= \frac{2\pi}{\sqrt{\frac{g}{2L}}} = \sqrt{2}\,\frac{2\pi}{\sqrt{\frac{g}{L}}} = \sqrt{2}\, T_a \text{ (longer period)}$$

d) Rocket in free fall (downward) $a = g \rightarrow \left.\begin{array}{l} \vec{a} = -g\hat{j} \\ \vec{g} = -g\hat{j} \end{array}\right\}$

$$\hookrightarrow T_d = \frac{2\pi}{\sqrt{\frac{|\vec{a}-\vec{g}|}{L}}} = \frac{2\pi}{\sqrt{\frac{|[-g-(-g)]\hat{j}|}{L}}} = \frac{2\pi}{0} = \infty$$

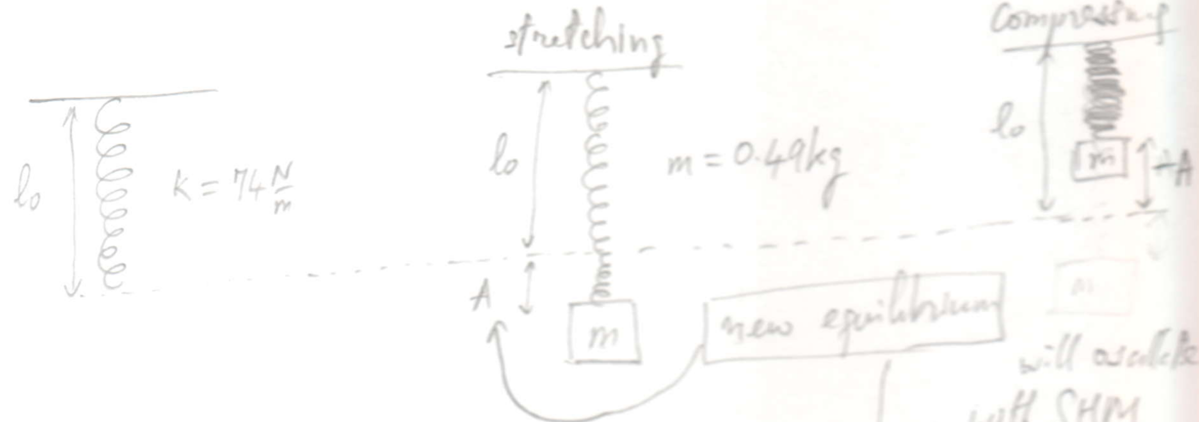
$\downarrow$

(infinite period)

$\downarrow$

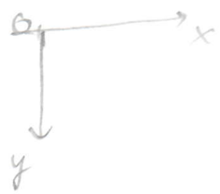
pendulum stops.

13.64



will oscillate with SHM of amplitude A

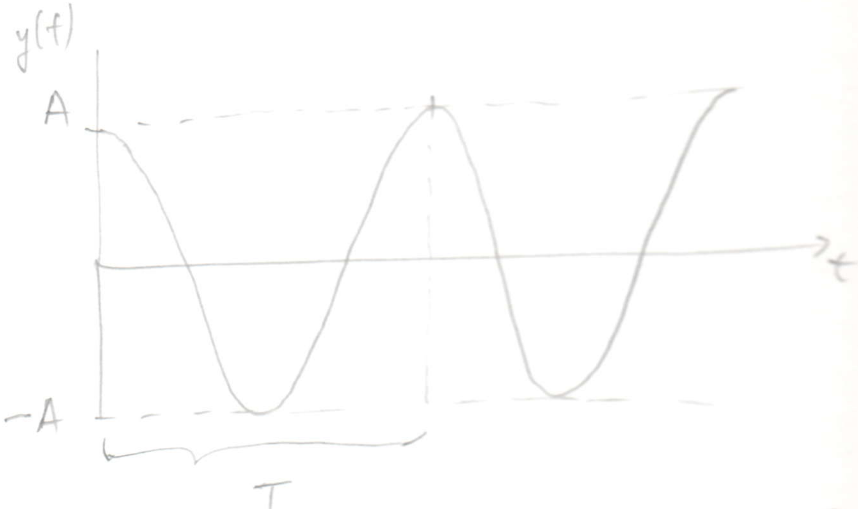
a)



spring will pull with force $F = \boxed{kA = mg}$

$A = \frac{mg}{k} = \frac{0.49 \times 9.81}{74}$

$= 0.065m$

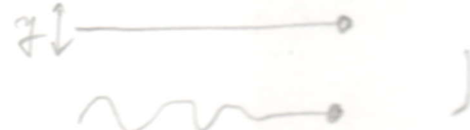


b) Period of SHM for the spring & mass?

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.49}{74}} = 0.5s$$

Ch 14 Wave Motion (Cont.)

Transverse wave: (eg. wave along a string

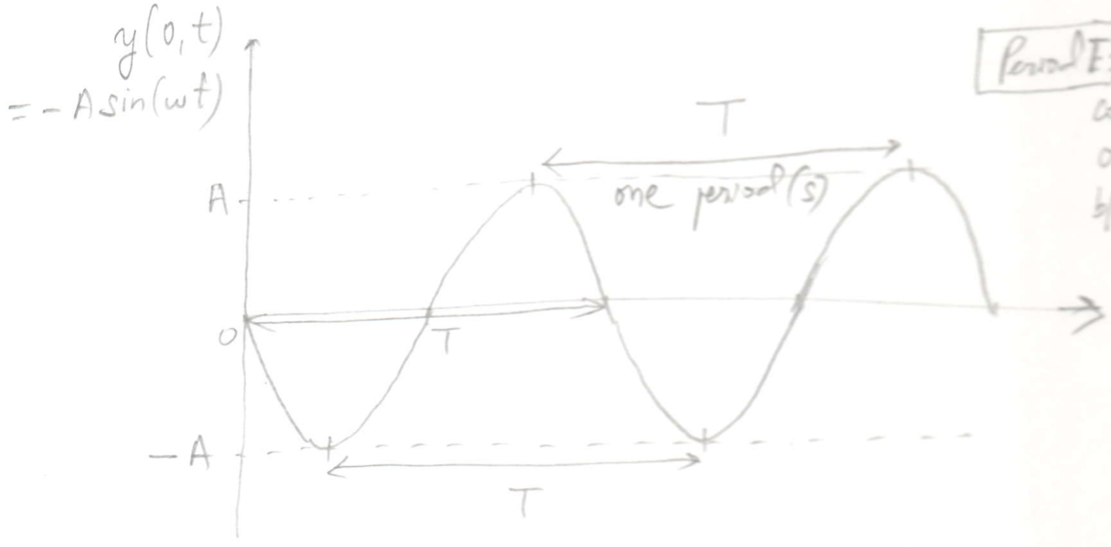


y: direction of oscillation
x: direction of propagation

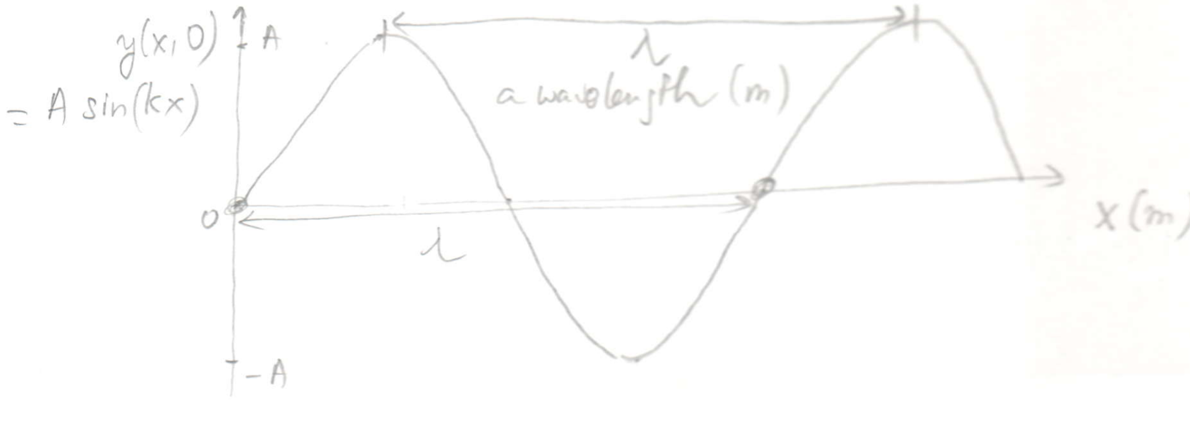
$$y(x,t) = A \sin(kx - \omega t)$$

k = wave number $= \frac{2\pi}{\lambda}$ (m^{-1})
λ = wave length (m)
ω = angular frequency (s^{-1})
$= \frac{2\pi}{T}$
T = period (s)
A = amplitude (m)

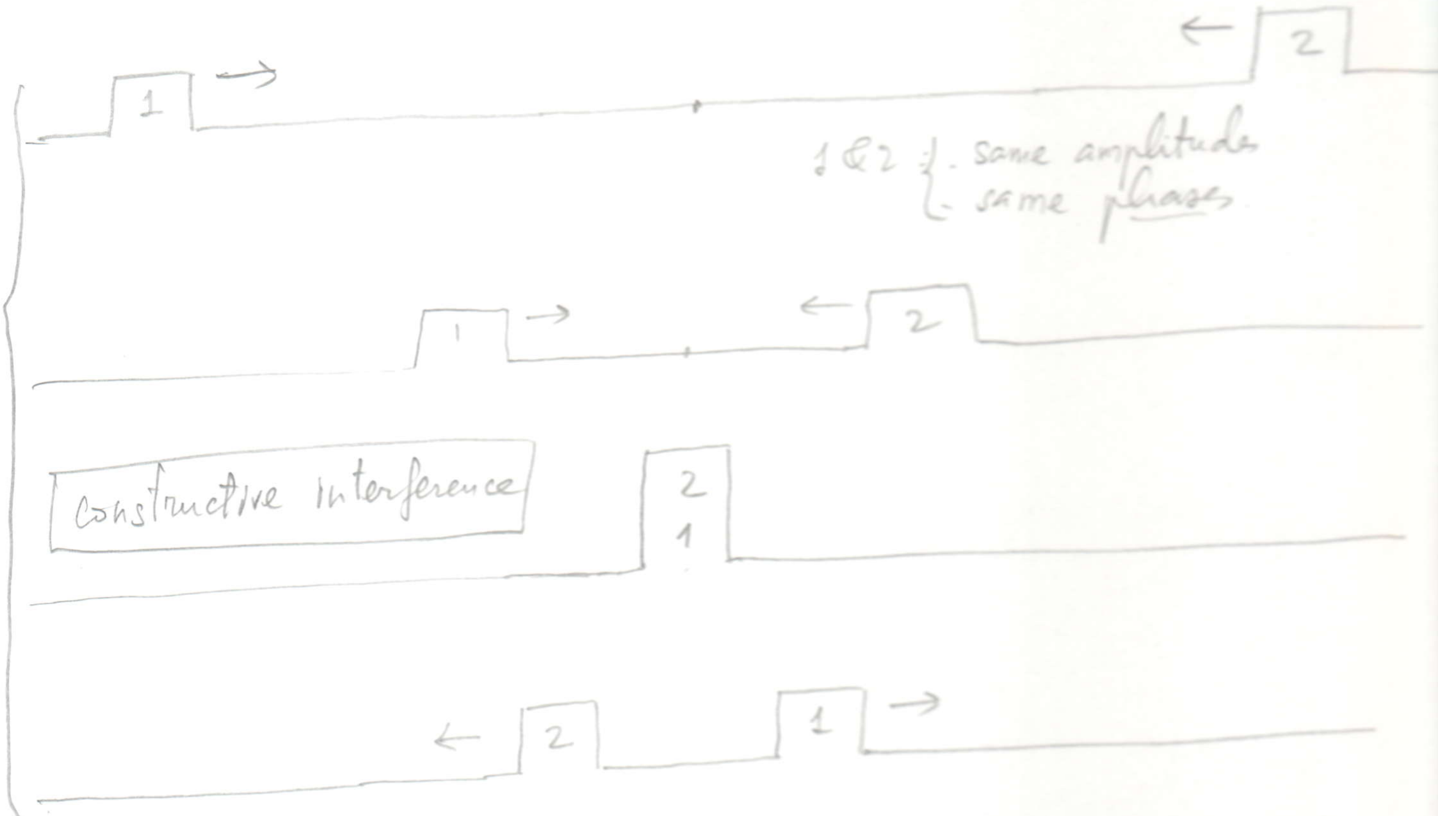
$\sin(-\omega t) = -\sin(\omega t)$

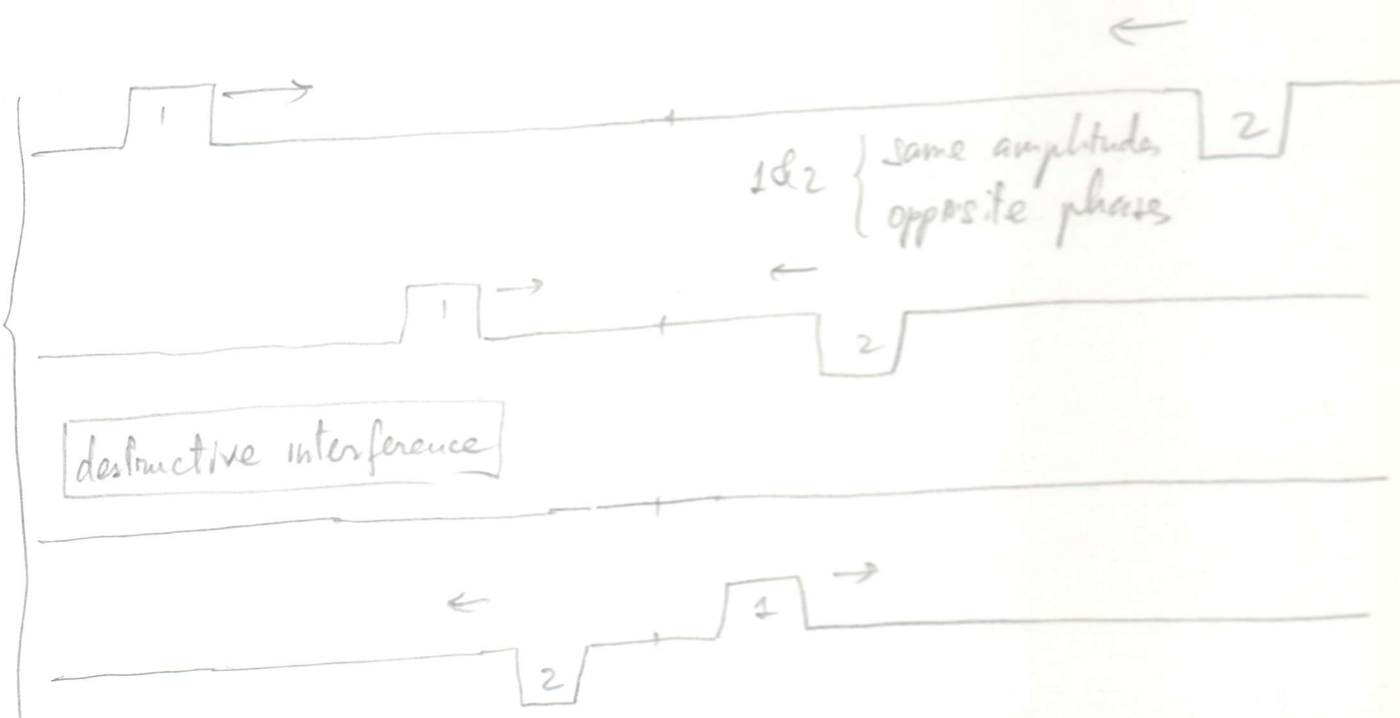


$\boxed{\text{Period } T}$: time separation b/w consecutive peaks or troughs or b/w two similar zeros (function increasing or decreasing)



Wave Superposition: (1 + 1 is NOT always 2)





How about a quantitative expression for a wave superposition or interference?

Transverse wave $= \quad y(x,t) = A\sin(kx - \omega t)$

Superposition or interference of 2 waves coinciding at $x=0$ (e.g. midpoint in earlier visualization). To be more interesting the two waves will have different frequencies:

$$\omega_1, \omega_2 \longrightarrow \begin{cases} y_1(0,t) = A\sin(-\omega_1 t) \\ y_2(0,t) = A\sin(-\omega_2 t) \end{cases}$$

$$y_1(0,t) + y_2(0,t) = -A\left[\sin\omega_1 t + \sin\omega_2 t\right]$$

Trig. identity: $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$$= -2A\left[\sin\left(\frac{\omega_1+\omega_2}{2}t\right)\cdot\cos\left(\frac{\omega_1-\omega_2}{2}t\right)\right]$$

allows us to describe a real situation:

When ω_1 & ω_2 are close ($\omega_1 = 1000\ \frac{rad}{s}$ & $\omega_2 = 1004\ \frac{rad}{s}$)

- Average ~ either one
- Difference ~ much smaller than either frequency
  - This imposes a very slow modulation on the amplitude of the combined wave

$$y_1(0,t) + y_2(0,t) = \underbrace{-2A\cos\left(\frac{\omega_1-\omega_2}{2}t\right)}_{\text{slowly modulated}}\ \underbrace{\sin\left(\frac{\omega_1+\omega_2}{2}t\right)}_{\text{similar to either}}$$

Practical applications of this slowly modulated amplitude

or BEAT PHENOMENON :

1) Tuning of string instrument

2) Maintenance of turbo (helix) aircrafts.

(one helix on each wing = these engines supposed to generate exact same frequencies.

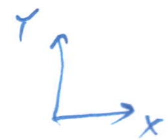
High rpm → high frequency sound → we hear flat noise from each engine.

When both engines are on, we hear oscillations, which are actually the beats b/w the two sounds.

14.54

Wave in a wire : $y(x,t) = 1.5 \sin(0.1x - 560t)$

$\downarrow$ oscillation along y $\qquad$ $\downarrow$ propagation along x



Transverse wave :

$y(x,t) = A \sin(kx - \omega t)$

a) Amplitude $A = 1.5$ cm

b) Wave number $k = 0.1\,cm^{-1} = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{0.1\,cm^{-1}} = 62.8$ cm.

c) Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{560}\,s = 11.2 \times 10^{-3}\,s = 11.2$ ms

d) Wave speed : $v = \frac{\lambda}{T} = \frac{62.8 \times 10^{-2}\,m}{11.2 \times 10^{-3}\,s} = 56$ m/s

e) Power carried by the wave : $\boxed{\bar{P} = \frac{1}{2}\mu\omega^2 A^2 v}$

- μ : linear density of the wire
- ω : ang freq. of the wave
- A : amplitude of wave
- v : wave speed

μ is not given, but the tension in the wire is $T = 28N$

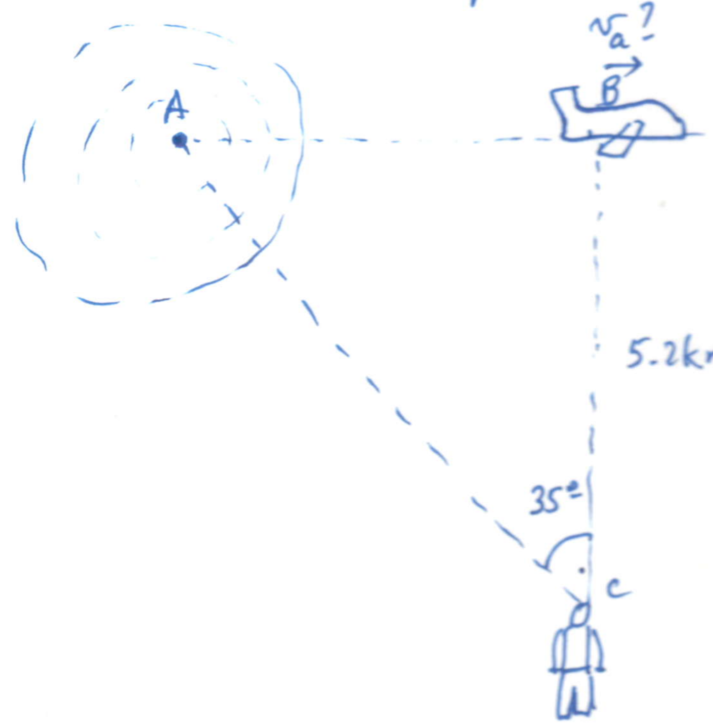
Transverse wave in a wire = $\boxed{v = \sqrt{\frac{T}{\mu}}} \rightarrow \boxed{\mu = \frac{T}{v^2}}$

$\bar{P} = \frac{1}{2}\frac{T}{v^{\cancel{2}}}\omega^2 A^2 \cancel{v} = \frac{1}{2}\frac{T}{v}\omega^2 A^2 = \frac{1}{2}\frac{28 \times 560^2}{56} 0.015^2$ W

$\bar{P} = \cancel{35028}$ W $= \frac{1}{2} 28 \times 5600 \times 0.015^2 = \cancel{14 \times 0.15^2}$

$\bar{P} = \cancel{0.315}$ W 17.4 W

4.61 Sound wave takes some time to reach you, in the same time the airplane has continued along its path:



$v_{sound} = 330$ m/s (in water)

Not saying the contrary: → Airplane is traveling at constant speed

$$\begin{cases} v_a = \dfrac{AB}{t} \; ; \; t: \text{time for sound wave to propagate from A to C} \\[2ex] t = \dfrac{AC}{v_{sound}} \end{cases}$$

$$\hookrightarrow v_a = \frac{AB}{\frac{AC}{v_{sound}}} = \underbrace{\frac{AB}{AC}} v_{sound}$$

$$= \sin 35° \, v_{sound}$$

$$= 330 \sin 35° = 189 \text{ m/s}$$

14.27

Transverse wave : $\begin{cases} A = 3\,cm = 0.03\,m \\ \lambda = 0.75\,m \\ v = 6.7\,m/s \end{cases}$

↳ on a stretched spring : $\mu = 0.17\,kg/m$ → linear mass density

Find spring tension T:

For a transverse wave in a string : $v = \sqrt{\frac{T}{\mu}} \rightarrow T = \mu v^2$

↓ wave speed

$= 0.17 \times 6.7^2$

$\boxed{T = 7.63\,N}$

→ The other given data (A, λ) would be useful to calculate ω, $\overline{P}$

Ch 14 Wave Motion (Cont.)

Standing waves:

- string along x, at length L it is attached to a fixed point.

- perturb free end at origin by moving it up & down.
  → this generates a wave that propagates in +x

- @ L the wave gets reflected b/c of the fixed point

- if you keep sending in waves by perturbing the free end, the incoming wave gets combined with the reflected wave. → formation of standing waves: its particular shape depends on how fast you perturb the free end.

Math description:

$$\underbrace{y_T(x,t)}_{\text{combined wave}} = \overbrace{y_1(x,t)}^{\text{incoming wave}} + \overbrace{y_2(x,t)}^{\text{reflected wave}}$$

$$= \underbrace{A\cos(kx - \omega t)}_{\text{propagation in } +x} - \underbrace{A\cos(kx + \omega t)}_{\text{propagation in } -x}$$

$$y_T(x,t) = A\left[\cos(\underbrace{kx-\omega t}_{\alpha}) - \cos(\underbrace{kx+\omega t}_{\beta})\right]$$

Trig. identity: $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$$y_T(x,t) = -2A \sin\left(\frac{kx - \cancel{\omega t} + kx + \cancel{\omega t}}{2}\right)\sin\left(\frac{\cancel{kx} - \omega t - \cancel{kx} - \omega t}{2}\right)$$

$$\boxed{y_T(x,t) = 2A \sin kx \sin \omega t}$$ (incoming + reflected wave.)

We need to incorporate the fact that $\boxed{y_T(x=L,t)=0}$ (fixed point @ $x=L$)

$$\underbrace{2A}_{\neq 0}\sin kL \underbrace{\sin\omega t}_{\neq 0} = 0 \quad \text{at all } t \Rightarrow \sin kL = 0$$

$\Rightarrow$ what $\underbrace{\text{non zero}}$ values for kL would give me $\sin kL = 0$

$kL = n\pi$ (n: 1, 2, 3, ...)

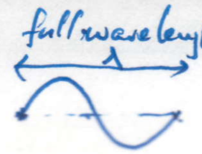
$$\frac{2\pi}{\lambda}L = n\pi$$

$$\rightarrow \boxed{\lambda = \frac{2L}{n} \quad (n = 1,2,3,4, \text{etc}...)}$$

Possible wavelengths for standing waves in a string of length L:

$\lambda_1 = 2L$; $\lambda_2 = L$; $\lambda_3 = \frac{2L}{3}$; $\lambda_4 = \frac{L}{2}$;

etc..



What do standing waves look like?

$\lambda_1 = 2L \rightarrow$ in L there is half a wave length

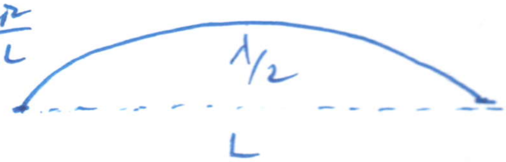
$$v = \frac{\lambda}{T} = \frac{\frac{2\pi}{k}}{\frac{2\pi}{\omega}} = \frac{\omega}{k}$$

$$\cancel{v = \frac{\lambda}{T} = \frac{\lambda}{\frac{2\pi}{\omega}} \Rightarrow}$$

$$\boxed{\omega = vk = v\frac{2\pi}{\lambda}}$$

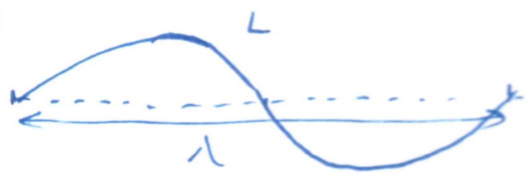
$\hookrightarrow \omega_1 = v\frac{2\pi}{2L}$

$$\boxed{\omega_1 = v\frac{\pi}{L}}$$



$\lambda_2 = L$

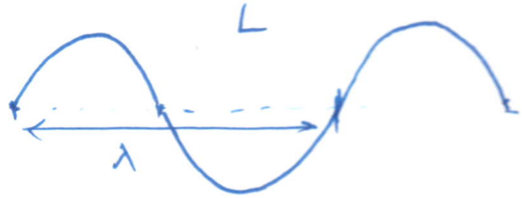
$$\boxed{\begin{aligned}\omega_2 &= v\frac{2\pi}{L}\\ &= 2\omega_1\end{aligned}}$$



$\lambda_3 = \frac{2L}{3}$

$\omega_3 = v\frac{2\pi}{\frac{2L}{3}}$

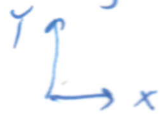
$$\boxed{\omega_3 = 3\omega_1}$$

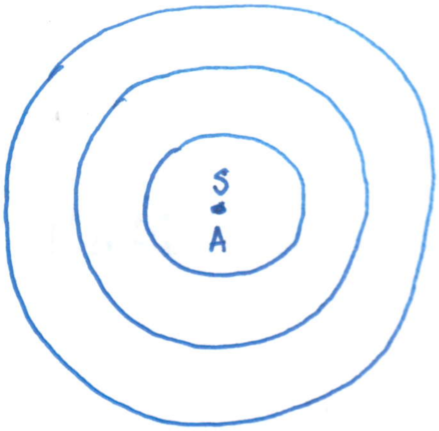


. . .

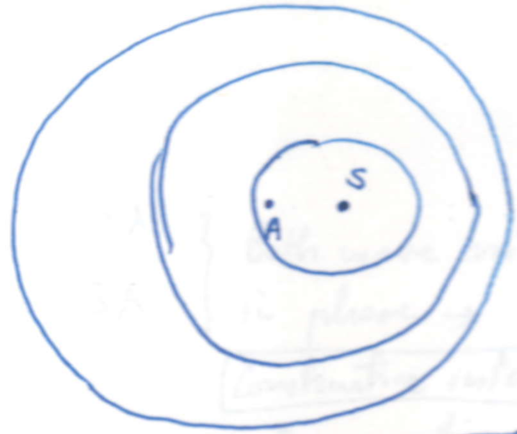
→ Showed standing waves movies.

Doppler Effect : moving source





source @ rest



wavefronts are further from each other

↓

sounds from a leaving train have a longer effective wavelength

$$\boxed{\lambda' = \lambda + uT}$$

receding

$$\boxed{f'_{\text{receding source}} = \frac{f}{1 + \frac{u}{v}}}$$

wave fronts are closer together

↓

sounds from a coming train will have an effective shorter wavelengths

$$\boxed{\lambda'_{\text{approaching}} = \underbrace{\lambda}_{\text{wavelengths when source was static.}} - \underbrace{u}_{\text{source speed}}\underbrace{T}_{\text{Period of sound wave}}}$$

$$\boxed{f'_{\text{approach}} = \frac{f}{1 - \frac{u}{v}}}$$

u = source speed

v = wave speed

f = original frequency of sound when source was static.