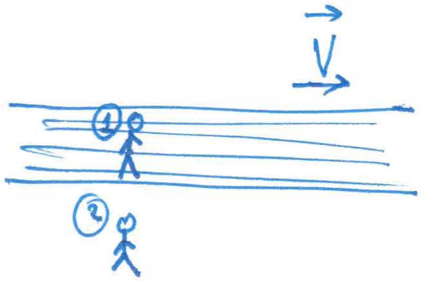
$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$
$$= (3 - (-1))\hat{i} + (2 - 1)\hat{j}$$
$$= 4\hat{i} + 1\hat{j}$$

Ch 3 : Motion in 2D or 3D (Cont.)

Relative motion:

1D:



automatic walkway moving at constant velocity $\vec{V}$

Passengers ① & ② walk at a same velocity $\vec{v}'$.

However: velocity of ① w.r.t floor is increased by that of the walkway: $\vec{v}_1 = \vec{v}' + \vec{V}$

velocity of ② w.r.t. floor is $\vec{v}_2 = \vec{v}'$

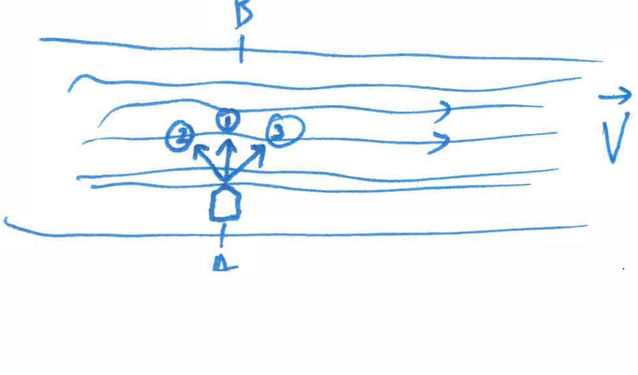
(Note: these equations include the situation where $\vec{v}'$ & $\vec{V}$ are in opposite directions)

2D : Crossing a river with a row boat

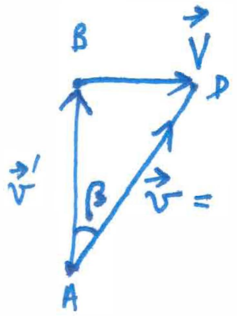
$\vec{v}'$: velocity of boat w.r.t water

$\vec{V}$: velocity of water

$\vec{v} = \vec{v}' + \vec{V}$: velocity of boat w.r.t ground (fixed)



(1)



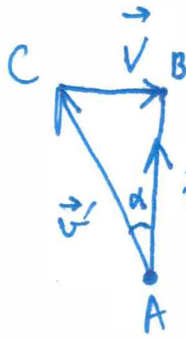
$\vec{v} = \vec{v}' + \vec{V}$

$\beta = \tan^{-1} \frac{V}{v'}$

or $\beta = \cos^{-1} \frac{v'}{v}$

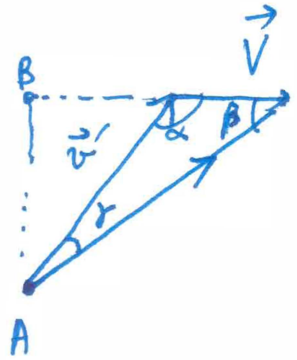
or $\beta = \sin^{-1} \frac{V}{v}$

(2)

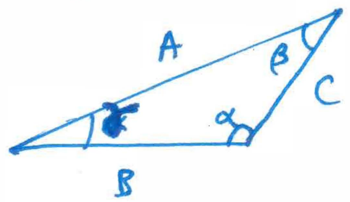


$\vec{v} = \vec{v}' + \vec{V}$

(3)



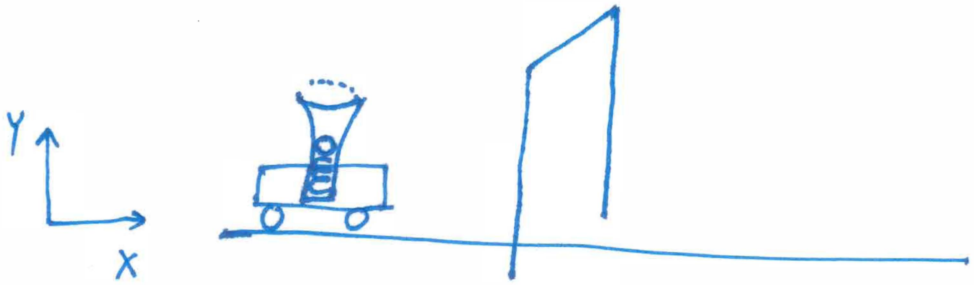
Since this is not a right triangle we need to use "Law of sines" Appendix A (Vol I)



$$\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$$

Equations of motion in 2D:

Important assumption: motions along perpendicular directions are independent (such as those along the cartesian coordinates x & y)



Cart rolling on a horizontal track (in the x-direction)
Has a funnel with a compressed spring at bottom, holding a ball, which can be ejected vertically when a button is pressed that release the spring.

The ball undergoes two motion
1) Horizontal, due to the cart (still there)
2) Vertical, due to the spring

If our assumption on independent motions along ⊥ directions is true:

1) Ball falls back behind the cart
2) Ball falls back in front of cart
3) Ball falls back into funnel ✓

↳ the horizontal motion of the ball (~~a~~ carried by the cart) is not eliminated when the spring ejects it up vertically. It still has the same velocity as the cart.

So when it falls back down it will meet the cart again: provided:

a) air resistance is negligible
b) friction b/w cart & track is negligible (so it maintains same speed as when ball left)

With this assumption in mind it is straight forward to write the kinematic equations for constant acceleration in 2D:

1D: $\begin{cases} v = v_0 + at & (1) \\ x = x_0 + v_0 t + \frac{1}{2} a t^2 & (2) \end{cases}$

$\downarrow$

2D: $\begin{cases} \left.\begin{aligned} v_x &= v_{0x} + a_x t \\ v_y &= v_{0y} + a_y t \end{aligned}\right\} (1) \\ \left.\begin{aligned} x &= x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\ y &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \end{aligned}\right\} (2) \end{cases}$

Note: v_x is affected by a_x, not a_y! x is affected v_{0x} and a_x, not v_{0y} & a_y!

Can use vector notation to write 2D equations in a shorter way, although for calculation we have to deal with a doubled number of equation (as compared to 1D)

$$\vec{r} = x\hat{i} + y\hat{j}$$
$$\vec{v} = v_x \hat{i} + v_y \hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$
$$\vec{a} = a_x \hat{i} + a_y \hat{j} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

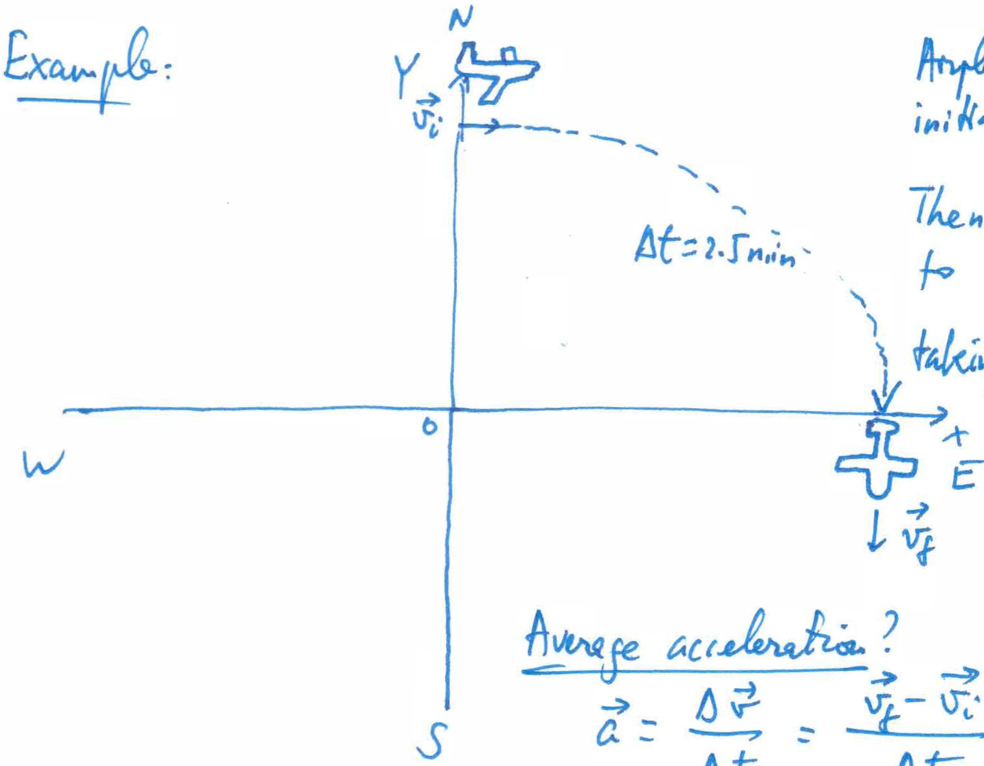
2D
$$\begin{cases} \vec{v} = \vec{v}_0 + \vec{a}t & (1) \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 & (2) \end{cases}$$

In 3D: consider a 3rd component z with unit vector $\hat{k}$.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$
$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

3D
$$\begin{cases} \vec{v} = \vec{v}_0 + \vec{a}t & (1) \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 & (2) \end{cases}$$

Example:



Airplane flying eastward initially @ $\vec{v}_i = 2100\,\frac{km}{h}\,\hat{i}$

Then it turns southward to $\vec{v}_f = -1800\,\frac{km}{h}\,\hat{j}$

taking 2.5 min

Average acceleration?

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{-500\hat{j} - 583.3\hat{i}}{150s}\,\frac{m}{s}$$

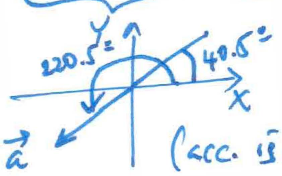
Conversion to SI units:

$$2100\,\frac{km}{h}\cdot\frac{1000\,m}{1\,km}\,\frac{1\,h}{3600\,s} = \frac{2100}{3.6}\,\frac{m}{s} = 583.3\,\frac{m}{s} \rightarrow \vec{v}_i = 583.3\,\frac{m}{s}\,\hat{i}$$

$$1800\ \frac{km}{h} = \frac{1800}{3.6}\ \frac{m}{s} = 500\ \frac{m}{s} \rightarrow \vec{v}_f = -500\ \frac{m}{s}\ \hat{j}$$

$$\vec{a} = \underbrace{-3.3\ \hat{j} - 3.9\ \hat{i}}\ \ \frac{m}{s^2}$$

$$\left\{ \begin{array}{l} a_x = -3.9\ m/s^2 \\ a_y = -3.3\ m/s^2 \end{array} \right\}$$ Cartesian coords of the average acceleration



(acc. is in the 3rd quadrant)

What are the polar coords for this average acceleration:

$$a = \sqrt{(-3.9)^2 + (-3.3)^2} = 5.1\ m/s^2$$ Magnitude of the average accel. vector.

$$\theta_a = 220.5°$$

Note: calculator gives:

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

$$= \tan^{-1}\left(\frac{-3.3}{-3.9}\right)$$

$$= \tan^{-1}\left(\frac{3.3}{3.9}\right)$$

$$= 40.5°$$

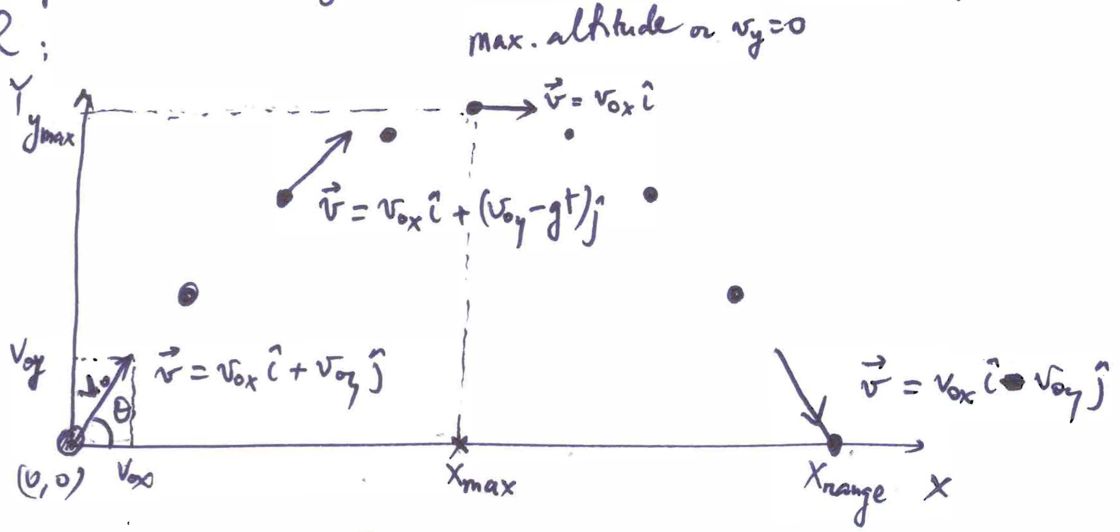
(since it simplified the minus signs!)

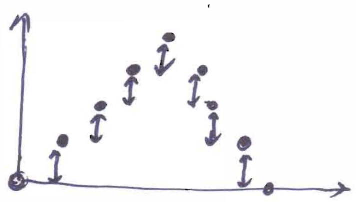
Need to add 180° to get the correct angle.

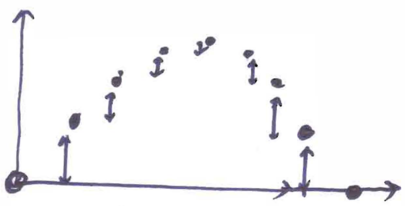
Projectile Motion :

In the movie for the rolling cart with a ball ejected vertically by a spring : the ball followed a projectile motion.

Snapshots at regular time interval of the position of the ball :







constant Δy for each snapshot

$\Delta y \downarrow$ (upward motion) → b/c deceleration of gravity.
$\Delta y \uparrow$ (downward motion) → b/c acceleration of gravity.
✓
Δx : are the same ! b/c constant velocity along x direction.

→ Characteristics of a projectile motion :
1) Vertical motion under effect of gravity
2) Horizontal motion is at constant velocity

→ Examples projectile motion :
ball launched by a catapult or trebuchet, or a canon , short-range missile

To describe the $\boxed{\text{projectile motion}}$ mathematically: use 2D Kinematic equation for constant acceleration, keeping in mind:

1) Vertical motion under constant acceleration of gravity (deceleration on the way up, acceleration on the way down)
2) Horizontal motion is uniform or at constant velocity

1) $\vec{v} = \vec{v}_0 + \vec{a}t \longrightarrow$ 1) $\begin{cases} v_x = v_{0x} \\ v_y = v_{0y} - gt \end{cases}$ ($a = -g$: on the way up to (x_{max}, y_{max}))

$$= v_0 \sin\theta - gt$$

2) $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \rightarrow$ 2) $\begin{cases} x = x_0 + v_{0x}t = x_0 + v_0 \cos\theta \, t \\ y = y_0 + v_0 \sin\theta \, t - \frac{1}{2}gt^2 \end{cases}$

Also: $(x_0, y_0) = (0, 0)$

2) $\rightarrow \quad x = v_0 \cos\theta \, t \rightarrow t = \dfrac{x}{v_0 \cos\theta}$

$$y = \cancel{v_0} \sin\theta \frac{x}{\cancel{v_0} \cos\theta} - \frac{1}{2}g \frac{x^2}{v_0^2 \cos^2\theta}$$

$$\boxed{y = x \tan\theta - \frac{g}{2} \frac{x^2}{v_0^2 \cos^2\theta}}$$

Trajectory equation for a projectile motion: a parabola in the XY plane as we described earlier.

Max. altitude point: $(x_{max}, y_{max}) = \left(\frac{v_0^2 \sin 2\theta}{2g}, \frac{v_0^2 \sin^2\theta}{2g} \right)$

↳ Proof: From the kinematic eqs in 2D (const. acceleration)

1) $v_y = \underbrace{v_0 \sin\theta}_{v_{0y}} - gt$

@ (x_{max}, y_{max}): $v_y = 0 \rightarrow t_{max} = \frac{v_0 \sin\theta}{g}$

2) $x_{max} = v_{0x} t_{max} = v_0 \cos\theta \cdot \frac{v_0 \sin\theta}{g}$

$= \frac{v_0^2 \cos\theta \sin\theta}{g} = \frac{v_0^2 \sin 2\theta}{2g}$

$\boxed{\cos\theta \sin\theta = \frac{\sin 2\theta}{2}}$

Trig. identity.

$y_{max} = v_{0y} t_{max} - \frac{1}{2} g t_{max}^2$

$= v_0 \sin\theta \frac{v_0 \sin\theta}{g} - \frac{1}{2}\cancel{g} \frac{v_0^2 \sin^2\theta}{g^{\cancel{2}}}$

$= \frac{v_0^2 \sin^2\theta}{2g}$

Range: $(x_{range}, y_{range}) = (2x_{max}, 0)$

$= \left(\frac{v_0^2 \sin 2\theta}{g}, 0 \right)$

$\boxed{\text{Uniform}}$ Circular Motion (UCM)

↳ Constant speed along the circular trajectory (not constant velocity)

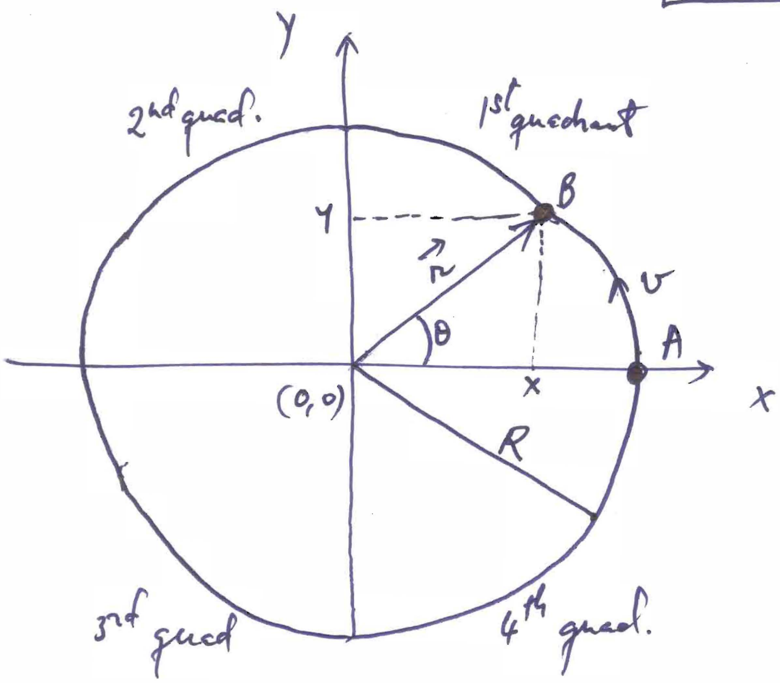
↳ includes { magnitude (speed), direction }

↳ In a UCM the speed is constant, but direction is changing

↳ Since $\vec{v}$ is changing → there is an acceleration

in UCM: $\vec{a} = \dfrac{d\vec{v}}{dt} \longrightarrow \boxed{|\vec{a}| = \dfrac{|\vec{v}|^2}{R}}$ or $\boxed{a = \dfrac{v^2}{R}}$

R: Radius of circular trajectory



If ~~affect~~ object takes time t to go from A to B:

$$\theta = \frac{\text{arc}}{R} = \frac{vt}{R}$$

$$\vec{r} = x\hat{i} + y\hat{j} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$\vec{r} = R\left[\cos\frac{vt}{R}\,\hat{i} + \sin\frac{vt}{R}\,\hat{j}\right] \Rightarrow |\vec{r}| = R$$

(position vector in a UCM has a fixed magnitude $= R$, but a changing direction)

$$\vec{v} = \frac{d\vec{r}}{dt} = R\left[-\frac{v}{R}\sin\frac{vt}{R}\,\hat{i} + \frac{v}{R}\cos\frac{vt}{R}\,\hat{j}\right]$$

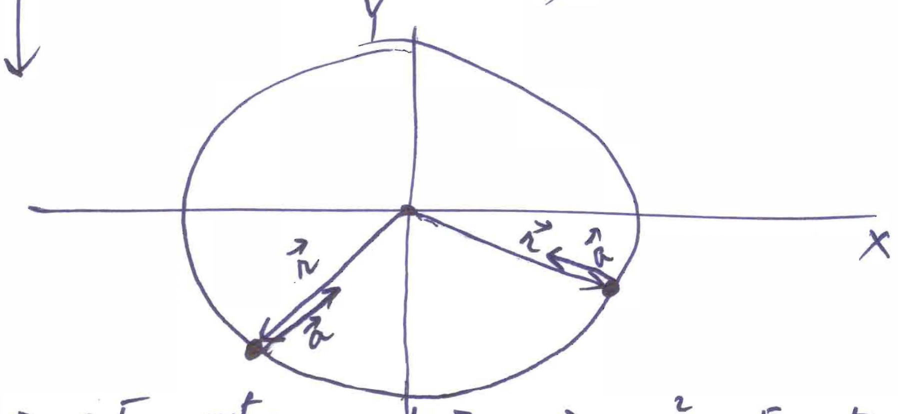
$$= v\left[-\sin\frac{vt}{R}\,\hat{i} + \cos\frac{vt}{R}\,\hat{j}\right] \Rightarrow |\vec{v}| = v$$

(Velocity vector in UCM has a fixed magnitude $= v$, but a changing direction)

$$\vec{a} = \frac{d\vec{v}}{dt} = v\left[-\frac{v}{R}\cos\frac{vt}{R}\,\hat{i} - \frac{v}{R}\sin\frac{vt}{R}\,\hat{j}\right]$$

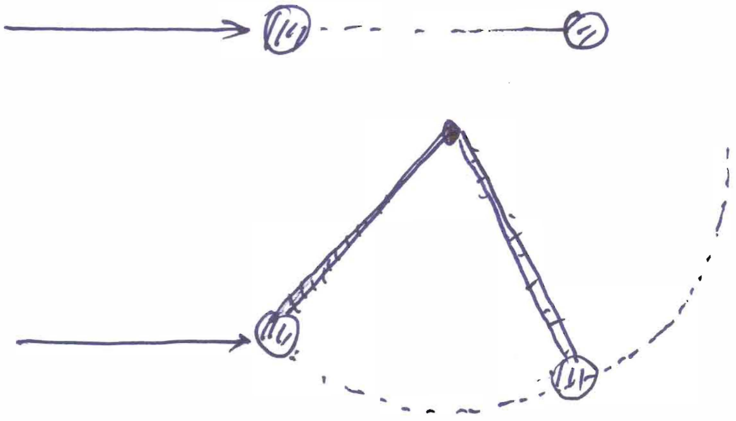
$$= -\frac{v^2}{R}\left[\cos\frac{vt}{R}\,\hat{i} + \sin\frac{vt}{R}\,\hat{j}\right] \Rightarrow \boxed{|\vec{a}| = \frac{v^2}{R}}$$

(Acceleration vector in UCM has a fixed magnitude $= \frac{v^2}{R}$, but a changing direction)



Acceleration is always towards the center of the circular trajectory, since it is keeping the object on the circle

$$\vec{r} = R\left[\cos\frac{vt}{R}\,\hat{i} + \sin\frac{vt}{R}\,\hat{j}\right]; \quad \vec{a} = \frac{v^2}{R}(-1)\left[\cos\frac{vt}{R}\,\hat{i} + \sin\frac{vt}{R}\,\hat{j}\right]$$

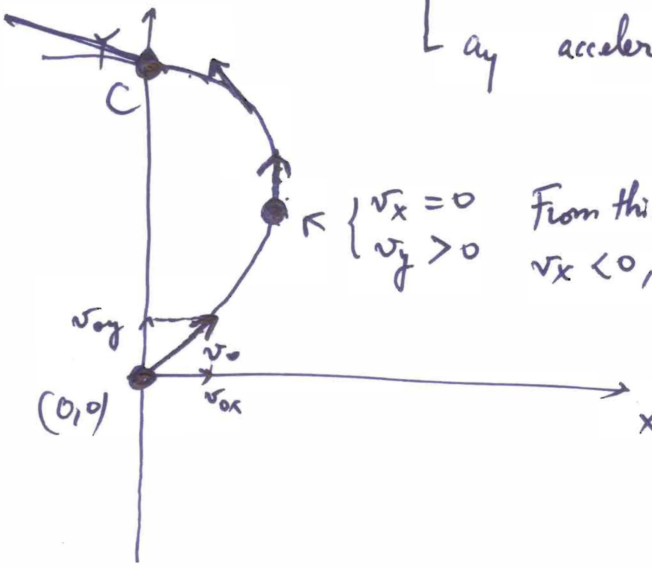


3.60 Facts:

$$\left\{\begin{array}{l} \vec{v}_0 = 11\hat{i} + 14\hat{j} \text{ m/s} \quad @ \ (x=0, y=0) \\ \vec{a} = \underbrace{-1.2\hat{i}}_{a_x} + \underbrace{0.26\hat{j}}_{a_y} \text{ m/s}^2 \quad \text{(const. accel.)} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{2D} \end{array}\right.$$

a_x decelerates particle in the x-direction (~~will~~ braking to a zero v_x then pushing the particle in the negative x-direction)

a_y accelerates particle in +y direction.



From this turning point on $v_x < 0$, $v_y > 0$

a) This is why we are asked "when does the particle cross the Y-axis"

$$2D \begin{cases} 1) & \vec{v} = \vec{v}_0 + \vec{a}t \quad \begin{cases} v_x = v_{ox} + a_x t \\ v_y = v_{oy} + a_y t \end{cases} \\ 2) & \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \quad \begin{cases} x = 0 + v_{ox}t + \frac{1}{2}a_x t^2 \\ y = 0 + v_{oy}t + \frac{1}{2}a_y t^2 \end{cases} \end{cases}$$

(we've written all possible equations for a constant acceleration motion in 2D)

@ crossing point C: $x = 0 \rightarrow 0 = 11t + \frac{1}{2}(-1.2)t^2$

$\rightarrow 0 = 11 - 0.6t$

$\rightarrow \boxed{t = \frac{11}{0.6} = 18.3s}$

time to cross the Y-axis or time for particle to go back to a $x=0$ position.

b) What is y @ crossing point C

$$y = 14(18.3) + \frac{1}{2}0.26(18.3)^2 = 300\,m$$

c) How fast & in what direction does it go @ crossing point C or what is $\vec{v}$ @ C

$$\underbrace{\begin{aligned} v_x &= 11 - 1.2(18.3) = -10.96\,m/s \\ v_y &= 14 + 0.26(18.3) = 18.8\,m/s \end{aligned}}_{\text{Cartesian components of the velocity @ C}}$$

$\downarrow$ 2nd quad.

$v = \sqrt{(-10.96)^2 + 18.8^2} = 21.7\,m/s$

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{18.8}{-10.96} = \underbrace{-60°}_{4^{th}\text{ quad.}}$$

$\downarrow$

Fixed: $\theta_v = -60° + 180° = \underline{120°}$

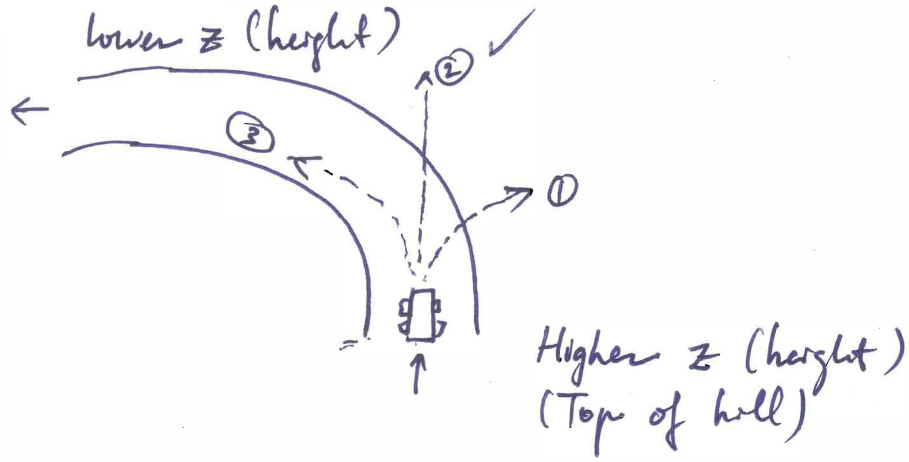
Ch 4 Force & Motion :

$\downarrow$

$\vec{r}, \vec{v}, \vec{a}$

We will complete the picture by relating force to those quantities $(\vec{r}, \vec{v}, \vec{a})$ we used to describe motions in 1, 2, 3D, through the Newton's Laws

Visual experiment: curved road, down-hill, icy conditions



Vehicle would follow path ②: the agent or force that would normally pull the vehicle toward the center of curvature is absent. That is the friction force b/w tires & road is absent.

Conclusion: a force (or agent) is needed to change a motion (described by $\vec{v}$).
(vehicle entering our curve heading in $+y$ direction, would continue in $+y$ direction if there is no friction that allows it to turn)

* If there are several forces (or agents), the net force is the one that will change motion.

When a vehicle is turning @ constant speed (UCM):
$\vec{v}$ is changing direction, $\vec{a}$ is needed that has fixed magnitude $\left(\frac{v^2}{R}\right)$ and always points toward the center of curvature;

Change of motion → change of $\vec{v}$ → $\vec{a}$
→ net force acting on vehicle } $\boxed{\text{Force is a vector}}$

1st Newton's law: a body in uniform motion will stay in uniform motion; a body at rest will stay at rest, unless there is a net force acting on the body (Law of inertia)

2nd Newton's Law: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\left\{ \begin{array}{l} \vec{p}: \text{ linear momentum} \\ \vec{p} \equiv m\vec{v} \end{array} \right.$

$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt} = \frac{dm}{dt}\vec{v} + m\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}$$

$$= m\vec{a}$$

↓

$m =$ constant → $\frac{dm}{dt} = 0$

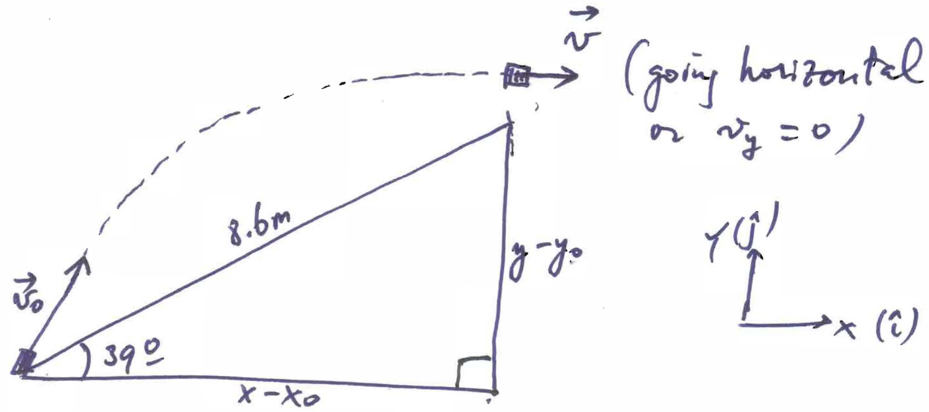
Dimension: $[F] = \frac{[p]}{[t]} = \frac{[m][v]}{[t]} = \frac{M\frac{L}{T}}{T} = M\frac{L}{T^2}$

↓

Units (SI): $kg\frac{m}{s^2} \equiv N$ (Newton)

<u>3rd Newton's Law:</u> If A exerts a force on B, B exerts an equal & opposite force on A (Law of action & reaction)

3.66 <u>Facts:</u>



Chocolate bar $\left\{ \begin{array}{l} \vec{v}_o \; ? \\ \vec{v} = v_x \hat{i} \end{array} \right\}$ It follows the 1st half of a <u>projectile motion</u> under the effect of gravity $\rightarrow \vec{a} = 0\hat{i} - g\hat{j}$

(1) $\vec{v} = \vec{v}_o + \vec{a}t \quad \left\{ \begin{array}{l} v_x = v_{ox} \\ v_y = v_{oy} - gt = 0 \end{array} \right.$ (a)

(2) $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2 \quad \left\{ \begin{array}{l} x - x_o = v_{ox} t \\ y - y_o = v_{oy} t - \frac{1}{2} g t^2 \end{array} \right.$

$\rightarrow \left\{ \begin{array}{ll} 8.6 \cos 39 = v_{ox} t & \text{(b)} \\ 8.6 \sin 39 = v_{oy} t - \frac{9.81}{2} t^2 & \text{(c)} \end{array} \right.$

(a) $\Rightarrow t = \frac{v_{oy}}{9.81} \rightarrow$ (c) $8.6 \sin 39 = \frac{v_{oy}^2}{9.81} - \frac{\cancel{9.81}}{2} \frac{v_{oy}^2}{9.81^{\cancel{2}}} = \frac{v_{oy}^2}{2 \times 9.81}$

$\Rightarrow v_{oy} = \sqrt{8.6 \sin 39° \times 2 \times 9.81} = 10.3 \text{ m/s}$

(b) $v_{ox} = \frac{8.6 \cos 39°}{10.3/9.81} = 6.36 \text{ m/s}$

$\vec{v}_0 = \underbrace{6.36\,\hat{i} + 10.3\,\hat{j}\ \frac{m}{s}}_{1^{st}\text{ quadrant}} \rightarrow \begin{cases} v_0 = \sqrt{6.36^2 + 10.3^2} \\ \quad = 12.1\,m/s \end{cases}$

$\theta_{v_0} = \tan^{-1}\frac{10.3}{6.36}$

$= 58.3°$ ✓

Alternative solution,

Observation:

$$\left.\begin{cases} x - x_0 = 8.6\cos 39° \\ y - y_0 = 8.6\sin 39° \\ a_x = 0 \\ a_y = -9.81\,m/s^2 \end{cases}\right\}$$

No time information!

↓

can use the 3rd Kin. eq in 2D:

$$\frac{v_x^2 - v_{0x}^2}{x - x_0} = 2a_x \quad\rightarrow\quad v_x = v_{0x} \text{ (uniform motion in x)}$$

$$\frac{v_y^2 - v_{0y}^2}{y - y_0} = 2a_y$$

$$v_{0y}^2 = \underbrace{v_y^2}_{0} - 2a_y(y - y_0) = -2(-9.81)\,8.6\sin 39°$$

$$v_{0y} = \sqrt{2 \times 9.81 \times 8.6 \times \sin 39°} = 10.3\,m/s$$

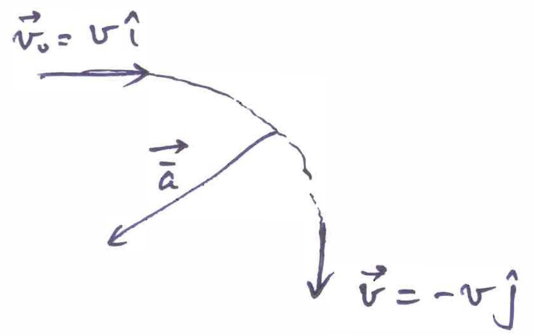
$$v_x = v_{0x} = \frac{x - x_0}{t} = \frac{x - x_0}{\frac{v_{0y}}{g}} = \frac{8.6\cos 39°}{\frac{10.3}{9.81}} = 6.36\,m/s$$

$\downarrow$

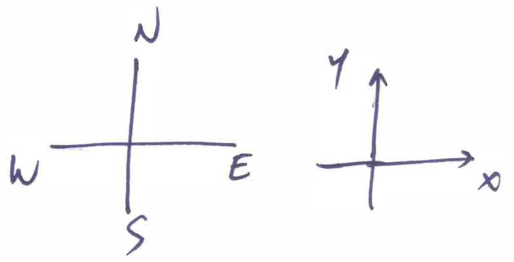
$\underset{\overset{\parallel}{0}}{v_y} = v_{0y} - gt \rightarrow t = \frac{v_{0y}}{g}$

→ Same result: $\vec{v}_0 = 6.36\,\hat{i} + 10.3\,\hat{j}\ m/s \begin{cases} 12.1\,m/s \\ \theta_{v_0} = 58.3° \end{cases}$

3-29



constant speed v



Average acceleration vector : $\vec{\bar{a}} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v} - \vec{v}_0}{\Delta t}$

$$= \frac{-v\hat{j} - v\hat{i}}{\Delta t}$$

$$= \frac{v}{\Delta t}\underbrace{(-\hat{i} - \hat{j})}_{3^{rd}\ quad.}$$

Direction of $\vec{\bar{a}}$ => $\theta_{\bar{a}} = \tan^{-1}\dfrac{-1}{-1} = 45°$ ← calculator.

$\theta_{\bar{a}} = 45° + 180° = 225°$,

(makes sense since acceleration has to point towards center of curvature)

Ch 4 Force & Motion (Cont.)

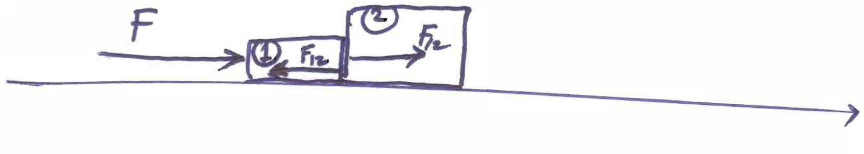
1st Newton's Law: Law of inertia

2nd Newton's Law: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\left\{ \begin{array}{l} \vec{p} = \text{linear momentum} \\ \vec{p} = m\vec{v} \end{array} \right.$

$$\vec{F}_{net} = \underbrace{\frac{dm}{dt}}_{\frac{dm}{dt}=0}\vec{v} + \underbrace{m\frac{d\vec{v}}{dt}}_{m\vec{a}}$$

3rd Newton's Law: Law of action & reaction
(If A exerts a force on B, B exerts an equal and opposite force on A)

Two boxes on horizontal surface w/ no friction



F is applied on box 1, moving both to the right $(+x)$

What force is being applied on box 2 ? $\left\{ \begin{array}{l} \cancel{F} \\ \tilde{F} \end{array} \right.$

a) 1 exerts a force F_{12} on 2 → the application point of F_{12} lays within 2

b) By the Law of action & reaction 2 exerts an equal and opposite force on 1

c) Along x $\left\{ \begin{array}{l} \text{Net force on box \#1} \rightarrow \{ \; F\hat{i} - F_{12}\hat{i} = (F - F_{12})\hat{i} \\ \text{Net force on box \#2} \rightarrow \{ \; F_{12}\hat{i} \end{array} \right.$

d) Along x : Net force on both boxes : $F\hat{i}$

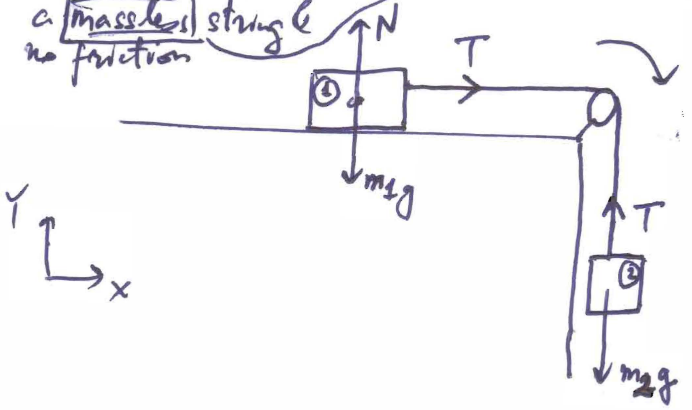
(Conclusion: on a system, net force is the external force, all internal by action & reaction $F_{12}\hat{i}$ & $F_{12}(-\hat{i})$ cancel by pairs!)

Applying Newton's Laws: Find F_{net} on each object.

Two boxes $\boxed{\text{connected}}$ via a $\boxed{\text{massless}}$ string & no friction

$a_1 = a_2 = a$

$T_1 = T_2 = T$



① : Tension force T
weight $m_1 g$ & normal N by table

$$\vec{F}_{net\,1} = T\hat{i} + \underbrace{(N - m_1 g)}_{0}\hat{j}$$

② Tension force T
weight $m_2 g$

$$\vec{F}_{net\,2} = 0\hat{i} + (T - m_2 g)\hat{j}$$

$$\vec{F}_{net\,1} = m_1 \vec{a}$$

$$T\hat{i} = m_1 a \hat{i}$$

$$T = m_1 a$$

$$\vec{F}_{net\,2} = m_2 \vec{a}$$

$$(T - m_2 g)\hat{j} = m_2 a(-\hat{j})$$

$$T - m_2 g = -m_2 a$$

$$m_1 a - m_2 g = -m_2 a$$

$$(m_1 + m_2)a = m_2 g \rightarrow \boxed{a = \frac{m_2}{m_1 + m_2} g}$$

→ If m_2 is doubled → $a' = \frac{2m_2}{m_1+2m_2} g > \underbrace{\frac{2m_2}{2m_1+2m_2} g}_{a}$

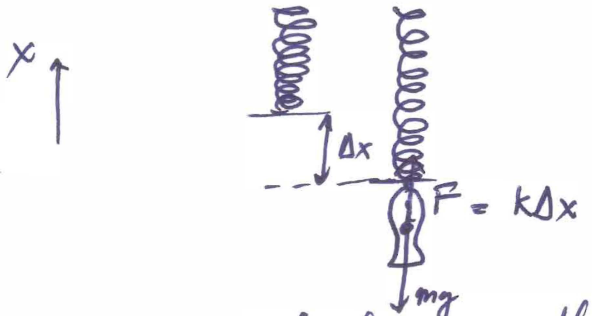
$a' > a$ but $2a > a'$ $\quad (2a > a' > a)$

→ If both m_1 & m_2 are doubled: $a'' = a$

4.38 Spring scale to weigh a fish

↳ $k = 340 \frac{N}{m}$

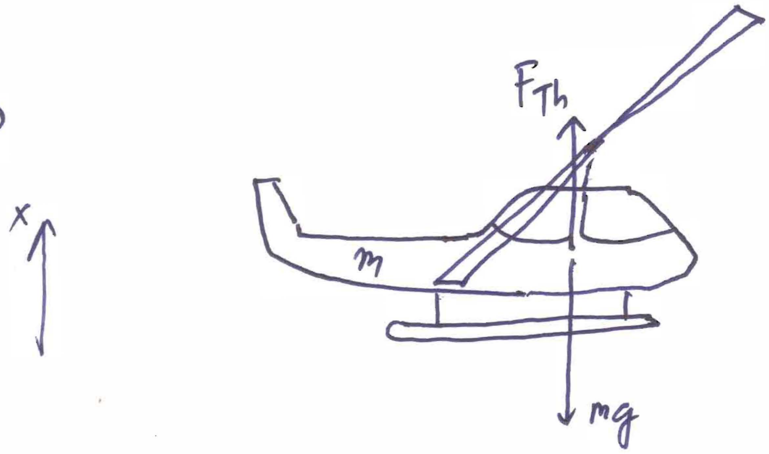
↳ $m = 6.7 kg$ } Spring stretch from its natural length Δx?



Let's focus on the fish: $\vec{F}_{net} = (k\Delta x - mg)\hat{i} = 0$

$\Delta x = \frac{mg}{k} = \frac{6.7 \times 9.81}{340} = 0.193 m$

4.53



$m = 4300\,kg$

F_{th}: is that exerted by the air on the blades (by law of action & reaction, blades exert an equal & opposite force downward on the air)

a) Hovering @ constant altitude: $\Rightarrow a = 0 \Rightarrow F_{net} = 0$

$F_{net} = F_{th} - mg = 0 \rightarrow F_{th} = mg = 4300 \times 9.81 = 42 \times 10^3 N$

$F_{\text{Downward on air}} = -42 \times 10^3 N = -42\,kN$

b) Dropping @ 21 m/s with $a = (+) 3.2\,m/s^2$ $(\vec{a} = a\hat{i})$
(speed decreasing @ $3.2\,m/s^2$)

$\boxed{\text{downward deceleration = upward acceleration.}}$

2nd Newton's law: $\vec{F}_{net} = m\vec{a}$

$$(F_{th} - mg)\hat{i} = ma\,\hat{i}$$

$$F_{th} = m(a+g)$$

$$= 4300\,(+3.2 + 9.81)$$

$$= 55.9\,kN$$

$$\downarrow$$

$F_{\text{Downward on air}} = -55.9\,kN$

Note: Free fall object gets downward acceleration g increasing speed.
b) is the opposite: decreasing speed while going down

c) Rising @ 17 m/s with $\underbrace{\text{speed increasing}}_{\text{upward acceleration}}$ @ 3.2 m/s²

$a = 3.2\,m/s^2$

$$(F_{th} - mg)\,\hat{i} = ma\,\hat{i}$$

$$F_{th} = m(a+g)$$

$$= 55.9\ kN$$

$$F_{\text{Downward on air}} = -55.9\ kN$$

d) Rising @ steady 15 m/s

constant speed $\rightarrow a = 0$

$$F_{th} = mg = 4300 \times 9.81 = 42\ kN$$

$$F_{\text{Downward on air}} = -42\ kN.$$

e) Rising @ 15 m/s with speed decreasing @ 3.2 m/s²

upward deceleration : $a = -3.2\,m/s^2$

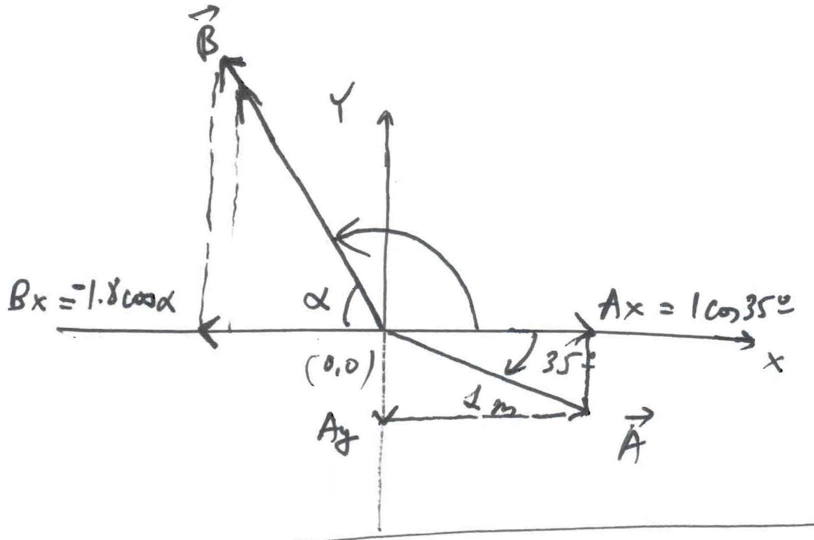
$$F_{th} = m(a+g) = 4300(-3.2+9.81)$$

$$= 28.4\ kN$$

$$F_{\text{Downward on air}} = -28.4\ kN.$$

43

3.49



$\vec{B} \rightarrow \begin{cases} B = 1.8\text{ m} \\ \alpha\ ? \end{cases}$

So $\vec{A}+\vec{B}$ is purely vertical (no x component)

$$\vec{A}+\vec{B} = \underbrace{(A_x + B_x)}_{0 \text{ (purely vertical)}}\hat{i} + (A_y + B_y)\hat{j}$$

$$A_x + B_x = 0$$

$$\cos 35° - 1.8\cos\alpha = 0 \rightarrow \alpha = \cos^{-1}\left(\frac{\cos 35°}{1.8}\right) = 62.9°$$

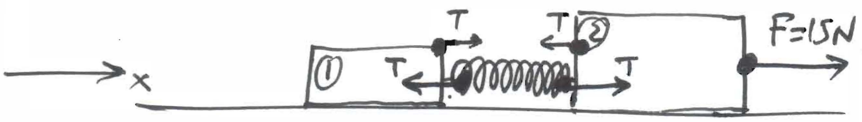
$\rightarrow$ Angle of $\vec{B}$ w.r.t. +x axis is $180 - 62.9° = \boxed{117.1°}$

3.52

For an object: $\vec{r} = 12t\,\hat{i} + (15t - 5t^2)\,\hat{j}$ m

a) $\vec{r}(t=2s) =$

4.51



Spring { massless
$k = 140\,N/m$

$m_1 = 2kg$ $m_2 = 3kg$
$a_1 = a_2 = a$
No friction

Spring stretch from its natural length?

$$\Delta x = \frac{T}{k}$$ (T is tension force on spring)

focus on ①: $F_{net_①} = T = m_1 a$
focus on ②: $F_{net_②} = F - T = m_2 a$

} system of 2 equations w/ 2 unknowns T & a

$$T = m_1 a \rightarrow a = \frac{T}{m_1}$$

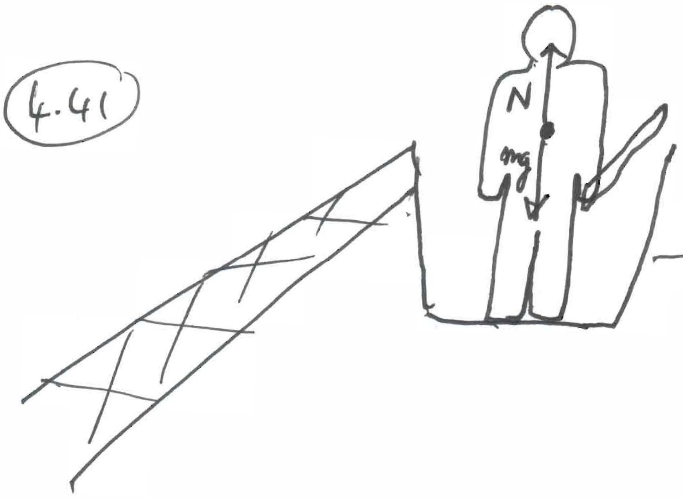
$$\rightarrow F - T = m_2 \frac{T}{m_1} \rightarrow F = T\left(1 + \frac{m_2}{m_1}\right)$$

$$T = \frac{F}{1 + \frac{m_2}{m_1}} = \frac{15}{1 + \frac{3}{2}}$$

$$T = 6N$$

$$\Delta x = \frac{T}{k} = \frac{6\,N}{140\,\frac{N}{m}} = 0.0429\,m.$$

4.41



$m = 74\,kg$

→ "Cherry picker" lift bucket

<u>Free-body diagram of forces:</u>
on the tree surgeon:

↑ N (Normal force exerted by bucket on surgeon)

↓ mg (weight)

a) Bucket at rest → $a = 0$

2nd Newton's Law: $F_{net} = ma = 0$

$N - mg = 0 \rightarrow \boxed{N = mg}$

$= 74 \times 9.81$
$= 725\,N$

b) Bucket moving upward @ <u>steady</u> speed $v = 2.4\,m/s$

↳ $a = 0$

$F_{net} = ma = 0$

$N - mg = 0 \rightarrow N = mg = 725\,N$

c) Bucket moving downward @ <u>steady</u> speed $v = 2.4\,m/s$

↳ $a = 0$

$\rightarrow N = 725\,N$

d) Bucket <u>accelerating</u> upward @ $a = 1.7\,m/s^2$

$F_{net} = ma$

$N - mg = ma \rightarrow N = m(g+a) = 74(9.81+1.7)$

$\boxed{N = 851\,N}$

c) Bucket accelerating downward @ $a = \ominus 1.7 \, m/s^2$

$$F_{net} = ma$$
$$N = m(g+a) = 74(9.81 - 1.7)$$
$$\boxed{N = 599 \, N}$$

4.47

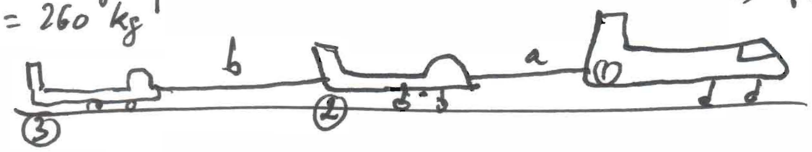
3 "objects" connected → $a_1 = a_2 = a_3 = a$

$\vec{a} \longrightarrow$

→ Massless ropes
→ Same tension along a rope
→ Frictionless.

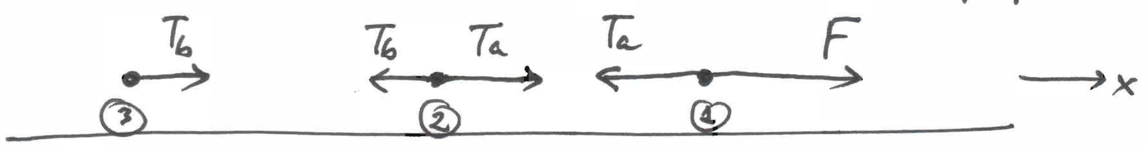
$m_1 = 2200 kg$
$m_2 = 310 kg$
$m_3 = 260 kg$

$a = 1.9 \, m/s^2$



Draw all forces involved for each object: to derive the correct F_{net} for each object!

↗ Thrust force from propeller



Newton's 2nd Law for each object:

(1) $$\underbrace{F - T_a}_{F_{net(1)}} = m_1 a$$

(2) $$\underbrace{T_a - T_b}_{F_{net(2)}} = m_2 a$$

(3) $$\underbrace{T_b}_{F_{net(3)}} = m_3 a$$

a) What is F?

work back from ③ → ② → ①:

$$T_a - m_3 a = m_2 a \rightarrow T_a = (m_2 + m_3)a$$

$$F = T_a + m_1 a = (m_1 + m_2 + m_3)a \checkmark$$

$$= (2200 + 310 + 260)\,1.9 = 5.26\ kN$$

b) Tension in 1st rope: T_a.

$$T_a = (m_2 + m_3)a = (310 + 260)\,1.9 = 1.08\ kN$$

c) Tension in 2nd rope: T_b

$$T_b = m_3 a = 260 \times 1.9 = 494 N$$

d) Net force on 1st glider (②)

$$\left\{\begin{array}{lll} T_a - T_b = & 1080 - 494 & = 589 N \\ m_2 a = & 310 \times 1.9 & = 589 N \end{array}\right.$$

down.

Measuring forces:

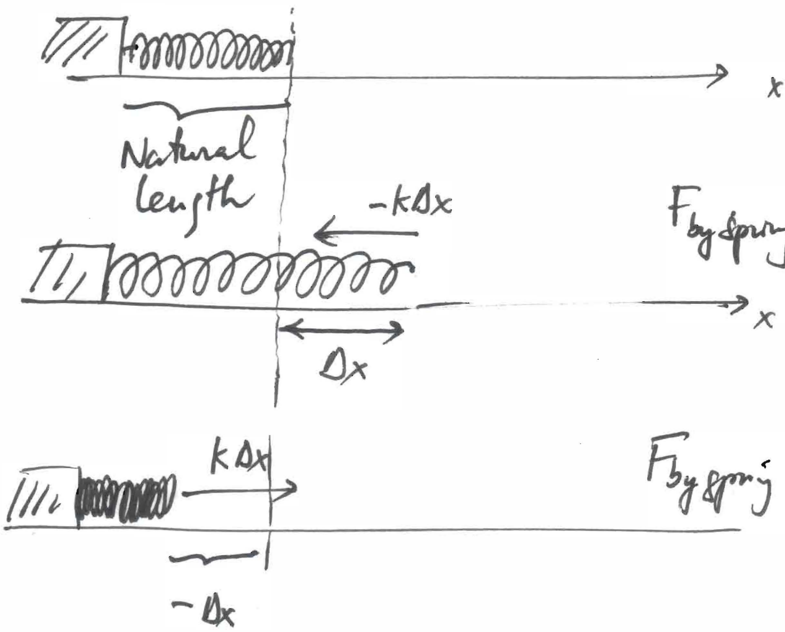
Spring scale : using Hooke's Law:

$$F_{\text{by spring}} = -k\,\Delta x$$

$-$: spring pulls back

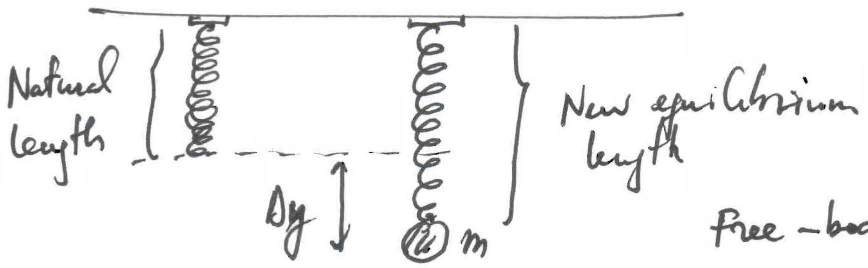
k : spring constant, SI: $\frac{N}{m}$

Δx : stretch or displacement from natural length

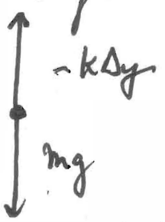


$$F_{\text{by spring}} = -k\,\Delta x$$

$$F_{\text{by spring}} = -k(-\Delta x) = k\,\Delta x$$



Free-body diagram for m:



$$F_{net} = mg - k\Delta y = ma$$

If system is static $\rightarrow a = 0 \rightarrow \boxed{mg = k\Delta y} \rightarrow \Delta y = \frac{mg}{k}$

Frictional force: when an object is in contact with a surface

Static friction: (static contact)

$F_s = \mu_s N$ (N = Normal force exerted by surface on object)

" mu sub s"

or coefficient of static friction

↳ texture, material, roughness...

F_s = threshold to push or pull the object.

Kinetic friction: (moving contact)

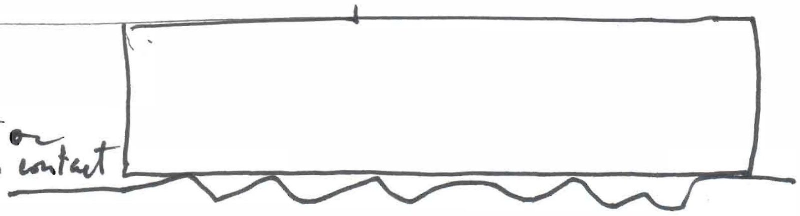
$F_k = \mu_k N$

↳ coefficient of kinetic friction

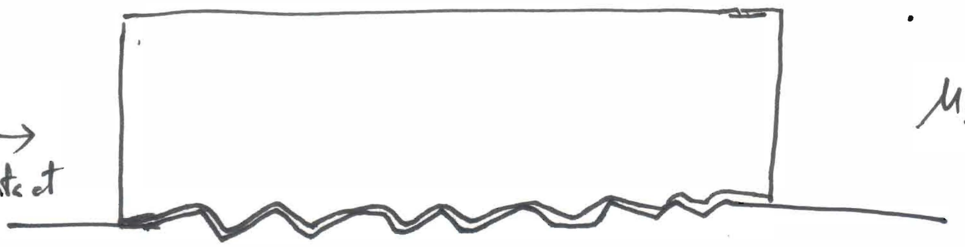
↳ same materials :

Box on floor:

recent or moving contact



after a while → static contact

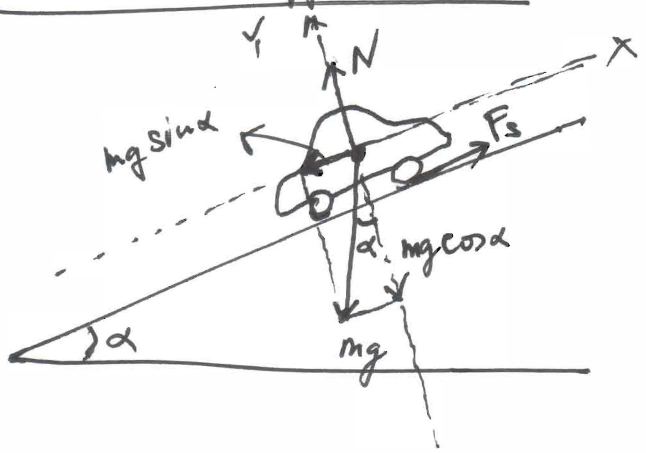


$\mu_s > \mu_k$

Observation: when you try to push a heavy box, you would apply increasing force until you reach the threshold, (to overcome the static friction F_s) then it starts moving fast → this is because you still applying the threshold force while F_s is reduced to F_k as the object starts moving, this gives a net force forward → accelerate the object forward

Static friction application :

Car is parked on a slope of angle α.



Forces on car :
- weight mg
- Normal force N ($\perp$ to surface)
- $\boxed{\text{Static friction}}$ F_s. pointing up hill keeping car not sliding down hill b/c $mg\sin\alpha$

Friction : direction not predefined, always opposing motion!
(unlike weight direction is always pointing vertically downward)

F_{net} on car & 2nd Newton's Law :

x-component : $F_s - mg\sin\alpha = ma_x$

$\mu_s N - mg\sin\alpha = 0$ car is parked

y-component : $N - mg\cos\alpha = ma_y$, $a_y = 0$ (car is not jumping up or down).

$$\mu_s mg\cos\alpha - mg\sin\alpha = 0$$

or $\mu_s\cos\alpha - \sin\alpha = 0$ or $\boxed{\mu_s \geq \tan\alpha}$

For car not to slide down.

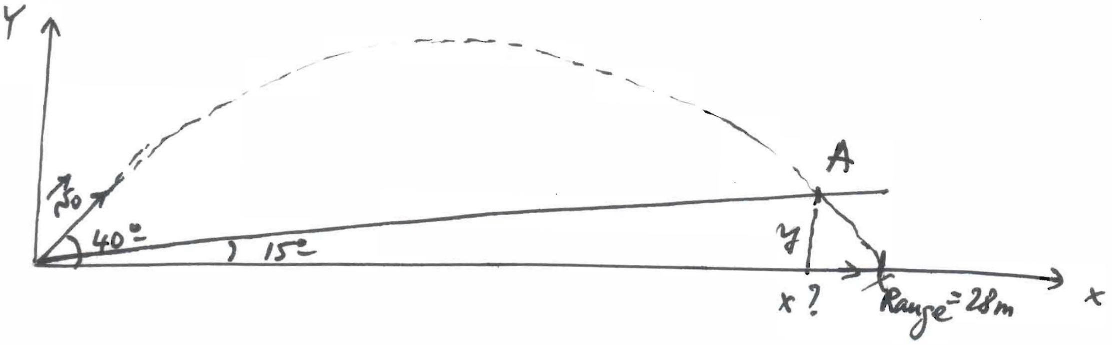
Ch5: Using Newton's Laws

- Static equilibrium
- Multiple objects
- Frictional forces
- Circular motion

Common strategies:

1) Understand the problem (making sense of question) & draw a sketch
2) Select a convenient coordinate system
   ↳ most forces pointing along either x or y-axis.
3) Make a free-body diagram of forces acting on each object (this will allow us to derive the net force on each object correctly). Draw components x & y for forces not already lined along these axes
4) Write 2nd Newton's law for each object. for each component as needed
   (airplane & two gliders: we just need the x-component)
   (car on a slope → both components)
5) Solve for what being asked for: obtain numeric solutions with correct units in SI.

3.76 Ball under projectile motion : { 28m on level ground. when $\vec{v}_0$ @ 40° wrt x.



On a slope the range (for same $\vec{v}_0$) will be less since the slope intersects with the parabola (projectile trajectory) at a prior point.

$$\tan 15° = \frac{y}{x}$$

A on the parabola, it's a point of the trajectory equation:

$$y = x \tan\theta - \frac{g}{2 v_0^2 \cos^2\theta} x^2$$

angle of initial vel. wrt x.

$$\rightarrow \left\{ \begin{array}{l} a)\ y = x \tan 40° - \dfrac{g}{2 v_0^2 \cos^2 40°} x^2 \\ b)\ y = x \tan 15° \\ c)\ x_{Range} = \dfrac{v_0^2 \sin 2\theta}{g} = 28\ m \quad (\theta \rightarrow 40°) \end{array} \right\} \text{3 equations \& 3 unknowns}$$

$$c) \rightarrow v_0^2 = \frac{9.81 \times 28}{\sin 80°} = 278.9.$$

$$a)\ \cancel{x} \tan 15° = \cancel{x} \tan 40° - \frac{9.81}{2 \times 278.9 \times \cos^2 40} x^{\cancel{2}}$$

$$x = \frac{(\tan 40° - \tan 15°)\ 2 \times 278.9 \times \cos^2 40}{9.81}$$

$$\boxed{x = 19.06 m}$$