Physic 113 Summer 2009

Ch. 1 Doing Physics.

Dimensional Analysis:

Speed: $\underbrace{[v]}_{\text{dimension of speed}} = \dfrac{[\Delta s]}{[\Delta t]} = \dfrac{L}{T}$ (length) / (time)

Δ : "delta"

Δs : "change of or increment of space" or position

Acceleration: $[a] = \dfrac{[\Delta v]}{[\Delta t]} = \dfrac{\frac{L}{T}}{T} = \dfrac{L}{T^2}$

Kinetic energy: $[K.E.] = \left[\frac{1}{2}mv^2\right] = [\frac{1}{2}]\cdot[m]\cdot[v]^2$

$1 \cdot M \cdot \dfrac{L^2}{T^2}$

$\downarrow$

All numeric constants (π, G, k, ...)

Application of dimensional analysis: check a formula.

$v = \frac{1}{2}gh^2 \Rightarrow [\frac{1}{2}]\cdot[g]\cdot[h]^2 = 1 \cdot \frac{L}{T^2} \cdot L^2 = \frac{L^3}{T^2}$

$v = \sqrt{gh} \Rightarrow \sqrt{[g]\cdot[h]} = \sqrt{\frac{L}{T^2}\cdot L} = \left(\frac{L^2}{T^2}\right)^{1/2} = \frac{L}{T}$ ✓

g = acceleration of gravity

h = height or vertical position (lenght)

Dimensional analysis limitation: it won't correct your numeric constant. You need to do curve fitting or data modeling

Units : S.I. = system of international units

L	:	m	(meter)
T	:	s	(second)
M	:	kg	(kilo = 10^3 → kg = 1000 g)
Area	:	m^2	
Volume	:	m^3	
Energy	:	$kg \frac{m^2}{s^2}$ = J	(Joule)

Unit conversion (in Appendix C)

1m	1nm	1µm	1cm	1mm	1km	1 light year	1mi	1ft	1in
1m	10^{-9} m	10^{-6} m	10^{-2} m	10^{-3} m	10^3 m	9.46×10^{15} m	1609 m	0.3048 m	2.54 cm

..

1 lb = 0.454 kg

1 h = 3600 s ; 1 day : 86400 s ; etc..

$1\,km^2 = (10^3)^2 (m)^2 = 10^6\,m^2$; $1\,cm^2 = 10^{-4}\,m^2$;

$1\,mm^2 = 10^{-6}\,m^2$

$1\,km^3 = 10^9\,m^3$; $1\,cm^3 = 10^{-6}\,m^3$; $1\,mm^3 = 10^{-9}\,m^3$

etc..

Accuracy & Significant Figures

- Scientific notation:
  ↳ useful in multiplication & division

$$\Delta s = 3\,105\,000\ m$$
$$= \underbrace{3.105}_{\text{coefficient less than 10}} \times \underbrace{10^6}_{\text{power of 10}}\ m$$
$$= 3.105\text{E}6$$
(in calculator)

$$\Delta t = 3000\,s = 3\times10^3\,s$$

speed: $v = \frac{\Delta s}{\Delta t} = \frac{3.105\times10^6}{3\times10^3} = \frac{3.105}{3}10^3$

$$= 1.035\times10^3\ m/s$$

- $\underline{\text{Accuracy}}$: # of decimal digits

$\pi = 3.1416$ is more accurate than 3.14

Addition & substraction:

$3.1416 - 1.14 \stackrel{\text{calculator}}{=} 2.0016$

$= 2.00$

keeping same accuracy as the least accurate term in the equation.

Significant Figures ; 6 370 000 — 3 s.f.'s (zeroes at end do not count)

6 370 001 — 7 s.f.'s (middle zeroes count)

Multiplication & Division , keep smallest number of significant figures except for numeric constants.

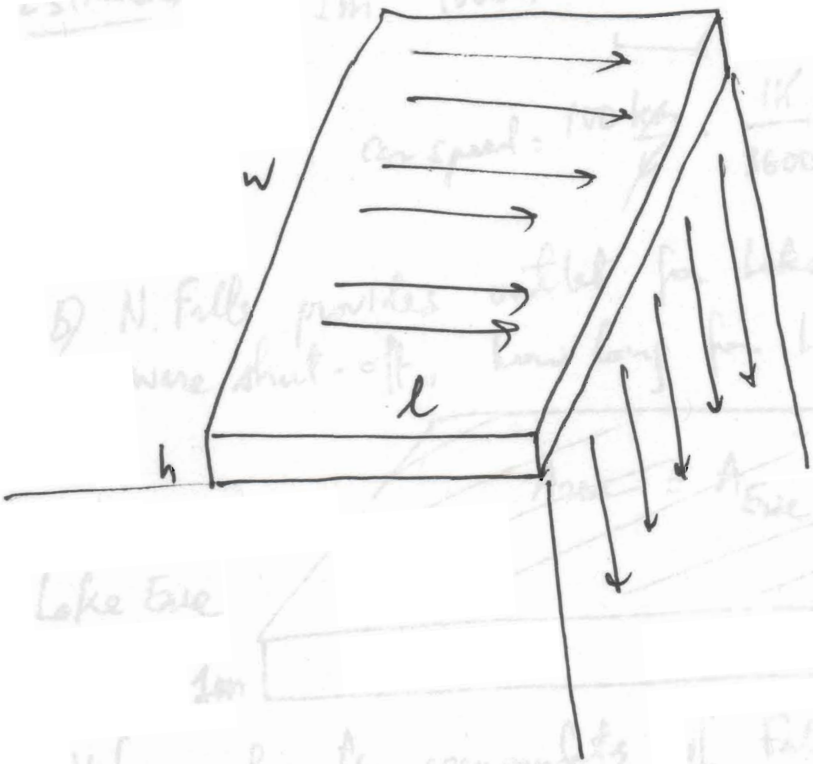
Earth's circumference : $2\pi R_E = 2 \times 3.1416 \times \underbrace{6.37 \times 10^6}_{3\text{ s.f.'s}}$

$= 4.002398 \times 10^7$

$= 4.00 \times 10^7$ m

1.40 a) Estimate volume of water flowing over Niagara Falls in 1 s.

(your guess : $9\times10^4\ m^3/s$; $1.5\times10^5\ m^3/s$)



$\boxed{\text{slab}}$ of water on top of the Falls

$\rightarrow$ vol = hwl

Flow rate = volume of water per unit time.

rate $\downarrow$ per unit time

$$= \frac{hwl}{t} \begin{cases} 1)\ \frac{h}{t} wl \\ 2)\ h\frac{w}{t} l \\ 3)\ hw\left(\frac{l}{t}\right) \checkmark \end{cases}$$

speed of water that will go over the Falls.

Flow rate: $h\, w \left(\dfrac{\ell}{t}\right) = 1\,m \cdot 1000\,m \cdot 5\,m/s = 5000\,\dfrac{m^3}{s}$

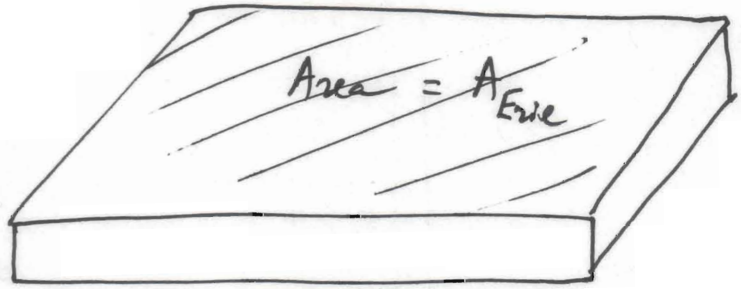
speed of water (m/s)

Estimate: $1\,m \quad 1000\,m \quad 5\,m/s$

Car speed: $100\,\dfrac{\cancel{km}}{\cancel{h}} \cdot \dfrac{1\cancel{h}}{3600\,s} \cdot \dfrac{1000\,m}{1\,\cancel{km}} = \dfrac{100}{3.6} = 26\,m/s$

b) N. Falls provides outlet for Lake Erie. If N. Falls were shut-off, how long for Lake Erie to rise 1m

Lake Erie



1m

Volume of water accumulates if Falls were shut off:

$A_{Erie} \cdot 1\,m$

How long for this volume to accumulate?

time?

How fast it accumulates is: flow rate over N. Falls:

$5000\,\dfrac{m^3}{s}$

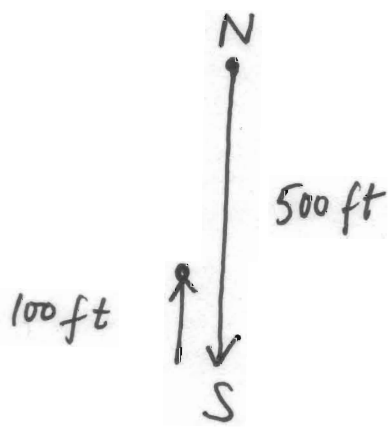
$$\frac{A_{Erie} \cdot 1\,m}{5000\,\frac{m^3}{s}} = \frac{75\,km \times 375\,km \times 1\,m}{5000\,m^3/s} = \frac{75 \times 375 \times 10^6 \times 1\,\cancel{m^3}}{5 \times 10^3\,\frac{\cancel{m^3}}{s}}$$

$$= 75^2 \times 10^3\,s = 5625 \times 10^3\,s \cdot \frac{1\,day}{84600\,s} = 66\text{ days.}$$

Ch2: Motion in a Straight Line:

Average Motion:

$$\text{Speed} = \frac{\text{distance}}{\text{time}} \quad ; \quad \text{Velocity} = \frac{\text{displacement}}{\text{time}}$$



Total time : 6 min

$$\text{Speed}: \frac{600\text{ ft}}{6\text{ min}} = \underline{100\text{ ft/min}}$$

$$\text{velocity} = \frac{400\text{ ft}}{6\text{ min}} = \underline{66.67\text{ ft/min}}$$

< speed

Since direction of motion is considered.

Average velocity: (unit in SI : m/s)

$$\bar{v} = \frac{\Delta x}{\Delta t}$$; Δ : delta = "displacement" or "change of"

Instantaneous velocity: (unit in SI: m/s)

$$v = \underbrace{\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}} = \frac{dx}{dt}$$ ("derivative of x w.r.t. t")

example: $x = at^3 \rightarrow v = 3at^2$

$x = at^n \rightarrow v = nat^{n-1}$

Other units: mi/h; km/h; m/s; etc...

$$\boxed{\text{Calculus}: \frac{dt^n}{dt} = nt^{n-1}}$$

Acceleration: change of velocity over time

Average Acceleration: $\bar{a} = \dfrac{\Delta v}{\Delta t}$; Δ: delta = "change of"
(unit in SI: m/s^2)

Instantaneous acceleration: $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$
(unit SI: m/s^2)

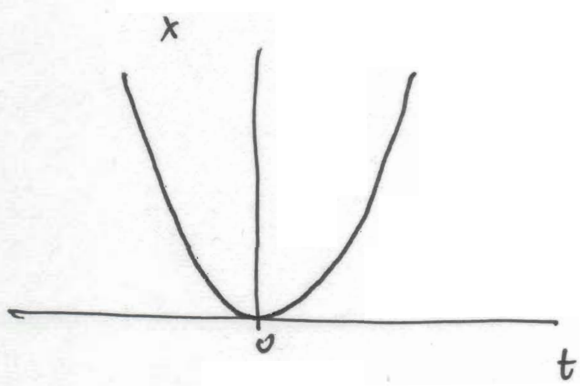
→ Example: $x = bt^3 \rightarrow v = \dfrac{dx}{dt} = 3bt^2$

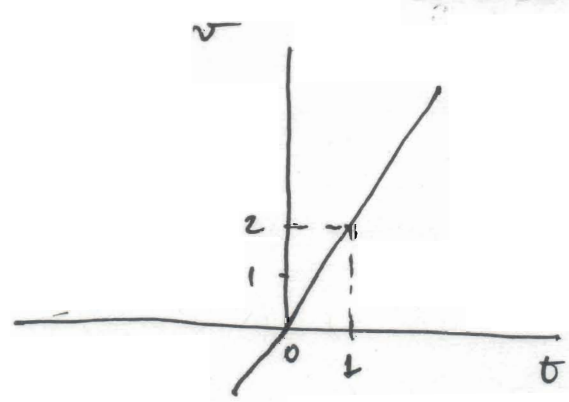
$\rightarrow a = \dfrac{dv}{dt} = 6bt$

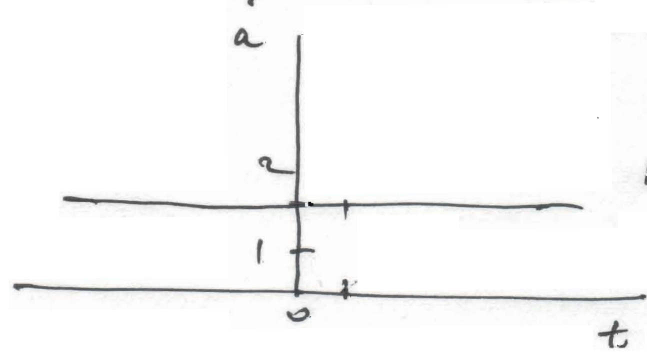
$\dfrac{d}{dt}(3bt^2) = 3b\underbrace{\dfrac{d}{dt}(t^2)}_{2t}$

→ Example: $x = bt^2 \longrightarrow v = 2bt$
$(b=1)$

$a = 2b$







Note: $\bar{a} = a$!

From these definitions we now derive our 2 equations of motion (kinematic equations) for constant acceleration in 1D:

<u>Constant acceleration</u>: $\bar{a} = a$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} \quad \Rightarrow \quad v - v_0 = at$$

v_0: v @ t=0 or initial velocity

$$\boxed{v = v_0 + at} \quad (1)$$

1st kinematic eq. (constant acceleration 1D)

x_0: x @ t=0 or initial position

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \rightarrow x = x_0 + \bar{v}t \quad (A)$$

On the other hand, the average velocity can also be calculate as:

$$\bar{v} = \frac{1}{t-0}\int_0^t v\,dt$$

$$\overset{(1)}{=} \frac{1}{t}\int_0^t (v_0 + at)\,dt$$

$$= \frac{1}{t}\left[v_0 t + \frac{1}{2}at^2\right]_0^t = \frac{1}{t}\left[v_0 t + \frac{1}{2}at^2\right]$$

$$= v_0 + \frac{1}{2}at$$

$$= \frac{1}{2}v_0 + \underbrace{\frac{1}{2}a.0}_{0} + \frac{1}{2}v_0 + \frac{1}{2}at$$

$$= \frac{1}{2}\left[\underbrace{(v_0 + a.0)} + \underbrace{(v_0 + a.t)}_{\shortparallel} \right]$$

$$= \frac{1}{2}\left[\quad v_0 \quad + \quad v \quad \right]$$

$$\Rightarrow \bar{v} = \frac{1}{2}(v_0 + v) \qquad (B)$$

(B) into (A):

$$x = x_0 + \bar{v}t = x_0 + \frac{1}{2}(v_0 + v)t$$

$$\overset{(1)}{=} x_0 + \frac{1}{2}(v_0 + \underbrace{v_0 + at}_{v})t$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \quad (2)$$

2nd kinematic equation
(constant acceleration in 1D)

Summary: to describe a constant acceleration motion in 1D:

1) $v = v_0 + at$

2) $x = x_0 + v_0 t + \frac{1}{2}at^2$

3) $\dfrac{v^2 - v_0^2}{x - x_0} = 2a$ (no time variable involved)

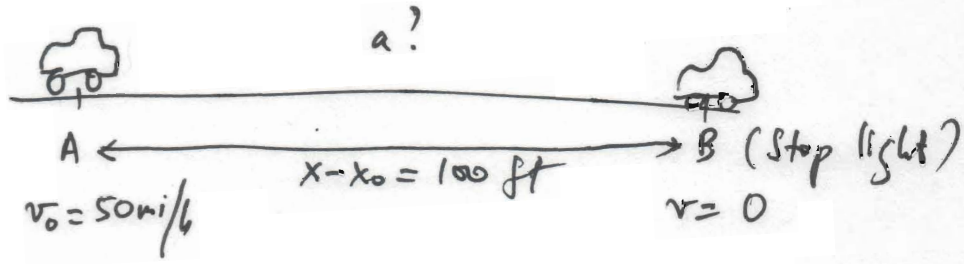
can derive from 1) & 2)

x_0: initial position ; x = final position	a: constant accel.
v_0 = initial velocity ; v = final velocity	t = time

Application of kinematic equations (constant acceleration, 1D)

(2.35) Facts:
- car with $v_0 = 50\,mi/hr$
- decelerates to $v = 0$ over $x - x_0 = 100\,ft$
  $\downarrow$
  (constant deceleration)

Question: a?



→ Find appropriate equation among our 3 kinematic equations: obviously eq. 3 fits well with our data:

$$2a = \frac{0 - 22.35^2}{30.48} \Rightarrow a = - \, 8.192\,\frac{m}{s^2}$$

↳ acceleration opposite to direction of motion, or deceleration

Make sure units are in S.I.

$$v_0 = 50\,\frac{\cancel{mi}}{\cancel{h}} \cdot \frac{1609\,m}{\cancel{mi}} \cdot \frac{\cancel{h}}{3600\,s} = 22.35\,m/s$$

$$x - x_0 = 100\,\cancel{ft} \cdot \frac{0.3048\,m}{1\,\cancel{ft}} = 30.48\,m$$

2.17 1500m swim + 40 km bike ride + 10 km run

Total time: 1h 49 min 31 s

$\bar{v} = ?$ $\quad \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{1500\,m + 40000\,m + 10000\,m}{3600\,s + 49 \times 60\,s + 31\,s}$

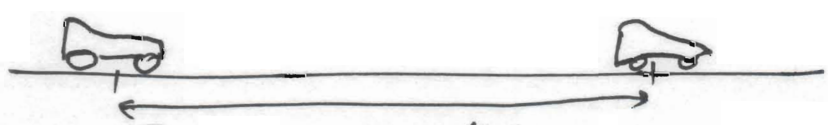
$= 7.84\,m/s.$

2.61 Facts: a constant

$x - x_0 = 140\,m$

$t = 3.6\,s$

a) @ $x - x_0 = 140\,m$; $v = 53\,m/s$



$x - x_0 = 140\,m$

$v_0 ?$ $\quad t = 3.6\,s$ $\quad v = 53\,m/s$

Question v_0? There is time info → $\begin{cases} 1)\ v = v_0 + at \\ 2)\ x - x_0 = v_0 t + \frac{1}{2}at^2 \end{cases}$

Alternative #1 : * Find a by eliminating what I don't know or v_0 in Eq 2) using Eq 1)

$v_0 = v - at \rightarrow x - x_0 = (v - at)t + \frac{1}{2}at^2$

$= vt - \frac{1}{2}at^2$

$\rightarrow a = \dfrac{-2(x - x_0 - vt)}{t^2} = -2\,\dfrac{140 - 53 \times 3.6}{3.6^2} = 7.83\,\frac{m}{s^2}$

* $v_0 = v - at = 53 - 7.83 \times 3.6 = \boxed{24.8\,m/s}$

Alternative #2: * Eliminate a from 2) using 1)

1) $v = v_0 + at \rightarrow a = \dfrac{v - v_0}{t}$

2) $x - x_0 = v_0 t + \frac{1}{2} a t^2$

$$v_0 t = (x - x_0) - \frac{1}{2} a t^2$$

$$= (x - x_0) - \frac{1}{2} \frac{v - v_0}{\cancel{t}} t^{\cancel{2}}$$

$$v_0 t = x - x_0 - \frac{1}{2} v t + \frac{1}{2} v_0 t$$

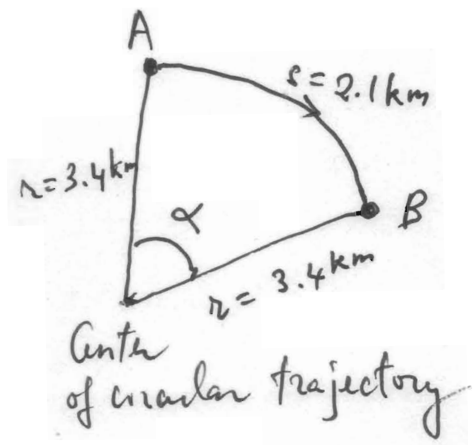
$$\frac{1}{2} v_0 t = x - x_0 - \frac{1}{2} v t$$

$$v_0 = \frac{2(x - x_0)}{t} - v$$

$$= \frac{2(140)}{3.6} - 53$$

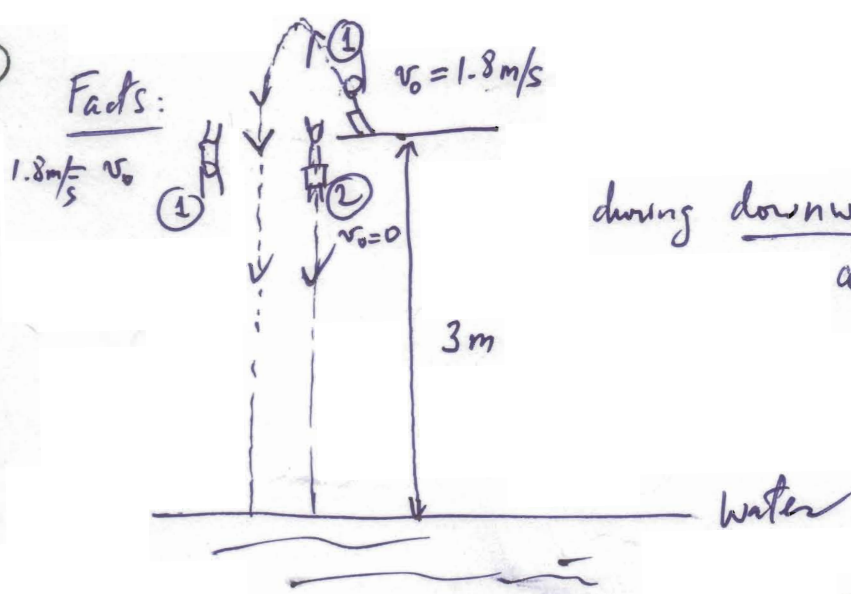
$$\boxed{v_0 = 24.8 \text{ m/s}}$$

1.17 Fact: jetliner follows circular path $s = 2.1\,km$; $r = 3.4\,km$



$$\alpha = \frac{arc}{radius} = \frac{2.1\,km}{3.4\,km} = 0.62\,rad \times \frac{180^\circ}{\pi\,rad} = 35.4^\circ$$

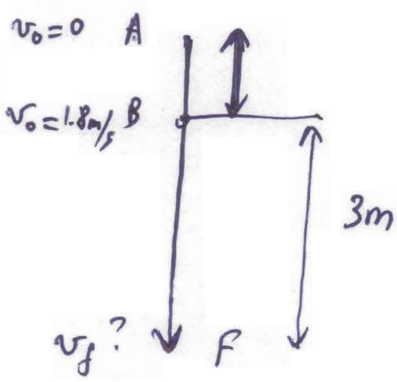
2.69 Facts:



during downward motion:
$a = +g$

a) Speeds at water level for the divers?

Diver #1:



on the way down we can choose A or B as the initial point for diver #1

For example: I) consider motion of diver #1 between B & F: → no time information: → eq. 3:

$$\frac{v_f^2 - v_0^2}{x - x_0} = 2a \rightarrow v_f = \sqrt{2a(x-x_0) + v_0^2}$$

$$= \sqrt{2 \times 9.81 \times 3 + 1.8^2}$$

$$= 7.88\ m/s$$

II) If we consider instead motion of diver #1 b/w A & F:

→ I need AB: 2) $(x-x_0)_{BA} = v_0 t - \frac{1}{2} g t^2$ → upward motion B to A. $a = -g$

1) $v_A = v_B - gt$

$0 = 1.8 - 9.81\, t \Rightarrow t = \frac{1.8}{9.81}\ s = 0.18s.$

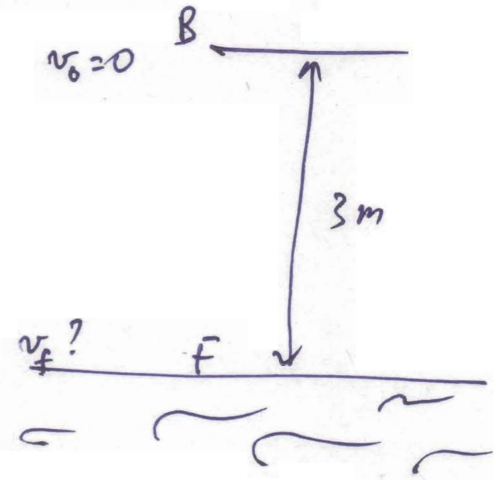
→ $(x-x_0)_{BA} = 1.8 \times 0.18 - \frac{1}{2} 9.81 \times 0.18^2 = 0.17m$

→ AF = 3.17m.

AF: $\frac{v_f^2 - \cancel{v_A^2}^{\,0}}{(x-x_0)_{AF}} = 2g \Rightarrow v_f = \sqrt{2g(x-x_0)_{AF}} = \sqrt{2 \times 9.81 \times 3.17}$

$= 7.88\ m/s$

Diver #2: $v_0 = 0$



$$\frac{v_f^2 - v_0^2}{3} = 2 \times 9.81$$

$$v_f = \sqrt{2 \times 9.81 \times 3}$$

$$= 7.67\ m/s$$

b) Which diver hits water first? Look at their motion b/w B & F : $v_f = v_0 + at = v_0 + gt$

$$t_1 = \frac{v_{f1} - v_{01}}{g} = \frac{7.88 - 1.8}{9.81} = 0.62 s$$

$$t_2 = \frac{v_{f2} - v_{02}}{g} = \frac{7.67 - 0}{9.81} = 0.782 s.$$

diver #1 enters water first by $(0.782 - 0.62) s = 0.162 s$.

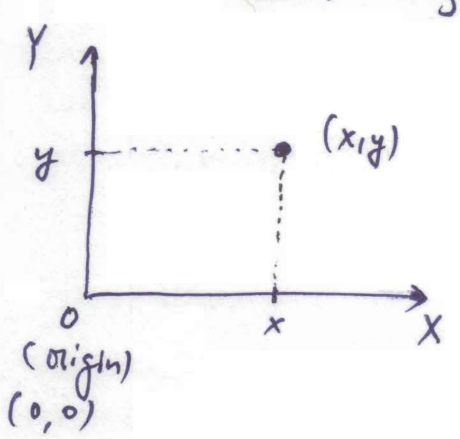
Ch3 Motion in 2 and 3 Dimensions:

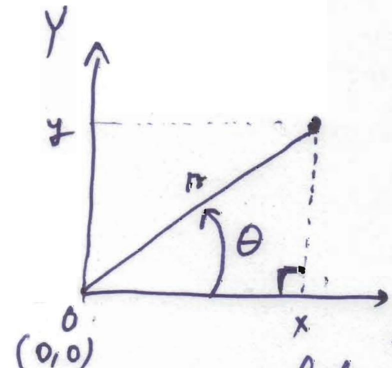
In 1D: on a straight line, position is determined by one number x

In 2D: on a plane, position is determined by two numbers:

$$\begin{cases} (x, y) \longrightarrow \text{Cartesian coordinates} \\ \text{or } (r, \theta) \longrightarrow \text{Polar coordinates} \end{cases}$$

r ↓ radius, θ ↓ angle "theta"





r: measured from the origin
θ: measured from the x axis

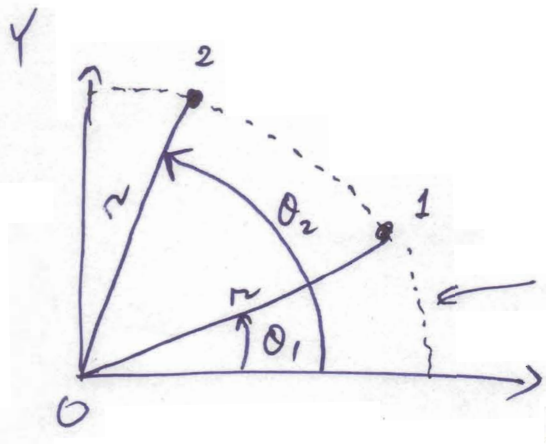
Cartesian ⟶ Polar

$$\vec{r} = (x, y) \longrightarrow \vec{r} = (r, \theta) = \left(r = \sqrt{x^2 + y^2},\ \theta = \tan^{-1}\frac{y}{x}\right)$$

↓ position vector

Polar ⟶ Cartesian

$$\vec{r} = (r, \theta) \longrightarrow \vec{r} = (x, y) = (x = r\cos\theta,\ y = r\sin\theta)$$



a quarter of a circle

Clearly we need 2 coordinads to determine a point in 2 dimensions !

For motion description: in 2D

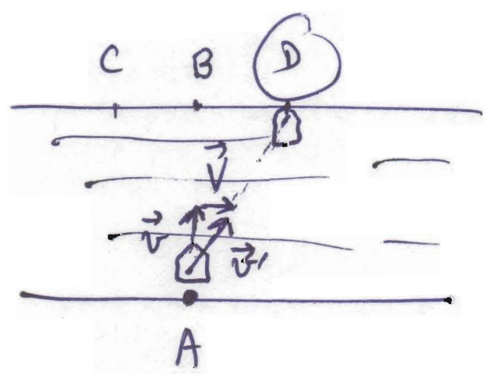
Position vector: $\vec{r} = (x, y) = (r, \theta)$

Velocity vector: $\vec{v} = (v_x, v_y) = (v, \theta_v)$

Acceleration vector: $\vec{a} = (a_x, a_y) = (a, \theta_a)$

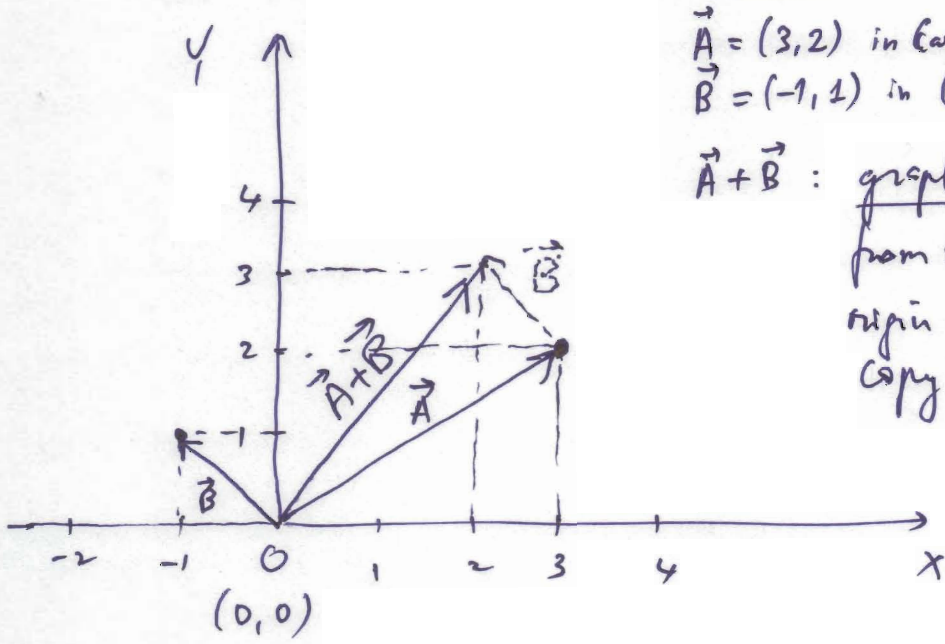
Question:

Where will you end up if you head straight across the river?



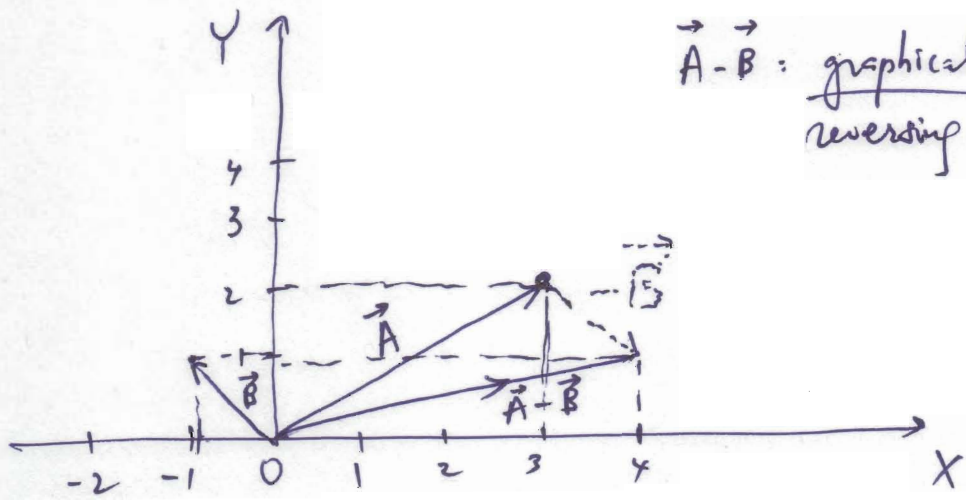
river w/ small velocity $\vec{V}$

Adding & Substracting vectors : { - Graphically / - Mathematically



$\vec{A} = (3,2)$ in Cartesian
$\vec{B} = (-1,1)$ in Cartesian.

$\vec{A}+\vec{B}$: graphically: draw a copy of $\vec{B}$ from the tip of $\vec{A}$, then connect origin of $\vec{A}$ to the tip of this copy of $\vec{B}$



$\vec{A}-\vec{B}$: graphically: repeat same but reversing direction of $\vec{B}$

Mathematically: use Cartesian coordinates (best for addition & substraction) (polar coordinates will work better for multiplication & division)

$$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} = 3\hat{\imath} + 2\hat{\jmath}$$
$$\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} = -1\hat{\imath} + 1\hat{\jmath}$$

($\hat{\imath}$: unit vector in the x-direction; $\hat{\jmath}$: unit vector in the y-direction)

$$\vec{A}+\vec{B} = (A_x+B_x)\hat{\imath} + (A_y+B_y)\hat{\jmath}$$
$$= (3-1)\,\hat{\imath} + (2+1)\,\hat{\jmath} = 2\hat{\imath}+3\hat{\jmath}$$

$$\vec{A}-\vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$
$$= (3-(-1))\hat{i} + (2-1)\hat{j}$$
$$= 4\hat{i} + 1\hat{j}$$