Ch 14 Wave Motion (Cont.)

Transverse wave: $y(x,t) = A \sin(kx - \omega t)$

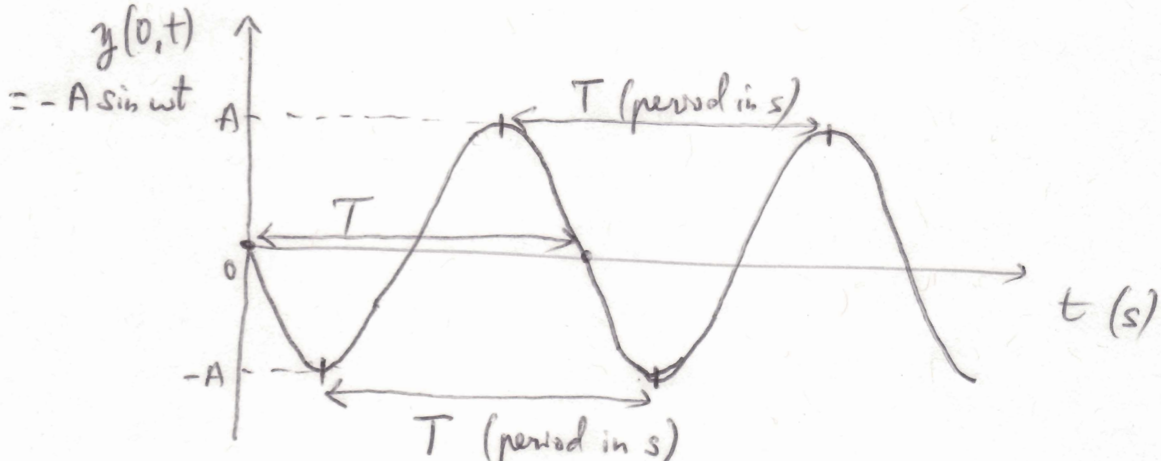
$\hookrightarrow \begin{cases} \text{disturbance or oscillation along } \hat{y} \\ \text{propagation is along } + \hat{x} \end{cases}$

A: amplitude ; k: wave number $= \frac{2\pi}{\lambda}$ (m^{-1});

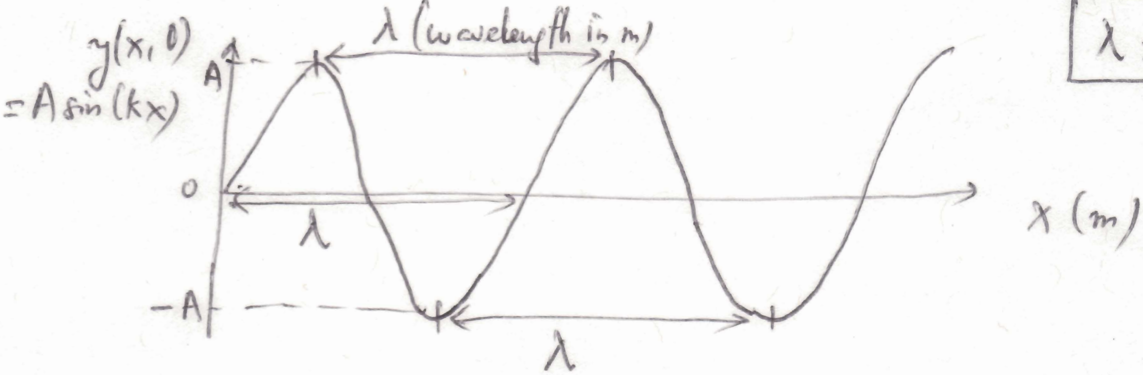
λ: wavelength

ω: angular frequency $= \frac{2\pi}{T}$ (s^{-1})

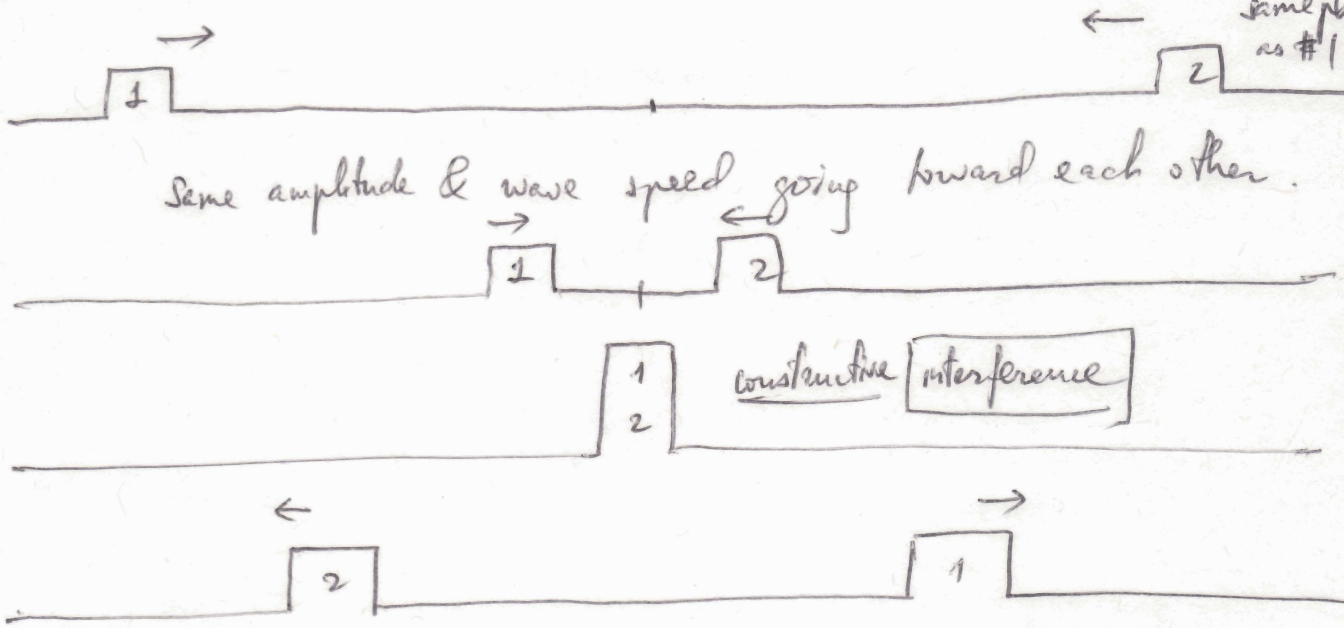
T: period



$\boxed{\lambda \neq T}$



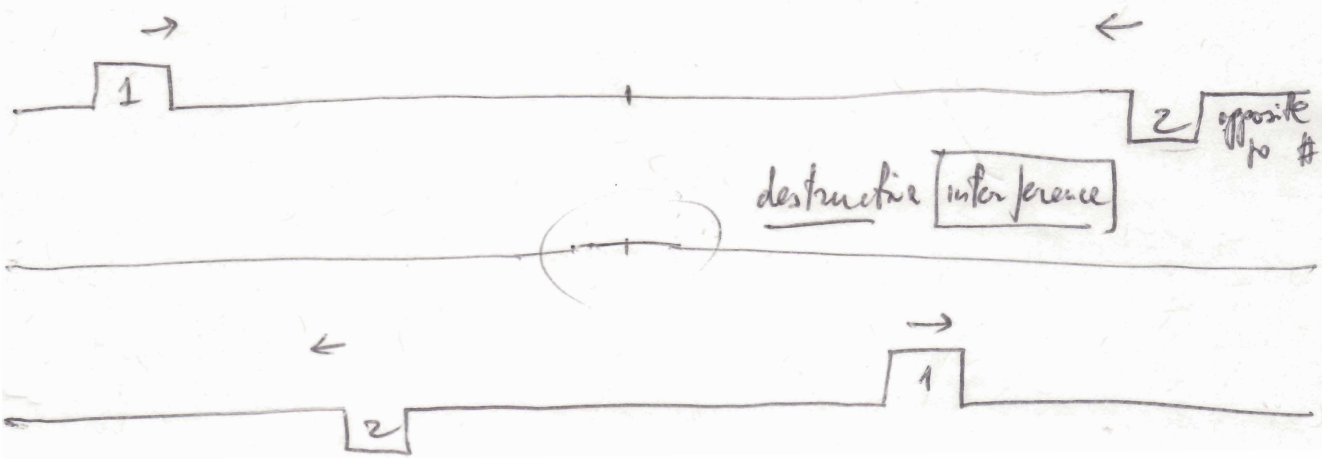
Wave superposition (unique properties of waves)



same phase as #1

Same amplitude & wave speed going toward each other.

constructive $\boxed{\text{interference}}$



opposite phase to #1

destructive $\boxed{\text{interference}}$

Quantitative expression for wave superposition or interference:

Beat phenomenon: superpositions of 2 waves at a same location $x=0$

$$\begin{cases} \#1 : \text{ with angular freq } \omega_1 \\ \#2 : \quad " \quad " \quad " \quad \omega_2 \end{cases} \Big\} \text{ same amplitude } A$$

$$y(x,t) = A \underbrace{\cos}_{\text{even function}} (kx - \omega t) \qquad \begin{cases} y_1(0,t) = A \cos \omega_1 t \\ y_2(0,t) = A \cos \omega_2 t \end{cases}$$

$\cos(-\omega t) = \cos(\omega t)$

$$y_1(0,t) + y_2(0,t) = A\left[\underbrace{\cos w_1 t + \cos w_2 t}_{2\cos\left(\frac{w_1 - w_2}{2}t\right)\cos\left(\frac{w_1+w_2}{2}t\right)}\right]$$

[Trig: $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\hookrightarrow$ Resulting wave: $\underbrace{\left[2A\cos\left(\frac{w_1 - w_2}{2}\right)t\right]}_{\text{amplitude: modulated with a frequency that is } \frac{w_1 - w_2}{2}} \underbrace{\cos\left(\frac{w_1+w_2}{2}t\right)}_{\text{resulting freq.}}$

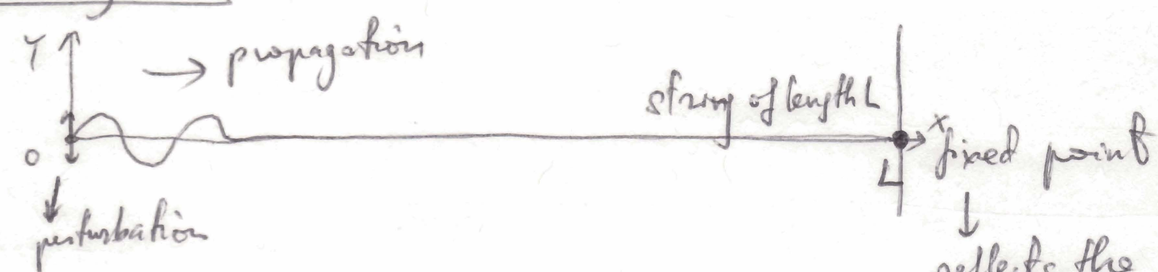
<u>Beat phenomenon:</u> $w_2 = w_1 + \varepsilon$

$\hookrightarrow$ small difference ($<10\%$)

$\rightarrow$ Resulting wave also carries a low frequency modulation on its amplitude.

$\longrightarrow$ <u>beats</u> (check with sound waves) observed.

<u>Standing waves:</u>



$\downarrow$ reflects the incoming wave throws in a 180° phase shift

After reflection $\rightarrow$ 2 waves on string:

$$y_T(x,t) = y_1(x,t) + y_2(x,t)$$
$$= \underbrace{A\cos(kx - wt)}_{\text{incoming}} - \underbrace{A\cos(kx + wt)}_{\text{reflected}}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$y_T(x,t) = -2A\sin kx\sin\left(-\frac{\cancel{2}\omega t}{\cancel{2}}\right) = 2A\sin kx\sin\omega t$$

Fixed point at $x = L \rightarrow y_T(L,t) = 0 \rightarrow \sin kL = 0$

$\rightarrow kL = n\pi$; $n = 1, 2, 3, \ldots$

$$\frac{2\pi}{\lambda}L = n\pi$$

$$\rightarrow \lambda = \frac{2\cancel{\pi}}{n\cancel{\pi}}L = \frac{2L}{n};\ n = 1, 2, 3, \ldots$$

↳ Standing waves : only when $\lambda = \frac{2L}{n}$; $n = 1, 2, 3, 4$, etc.

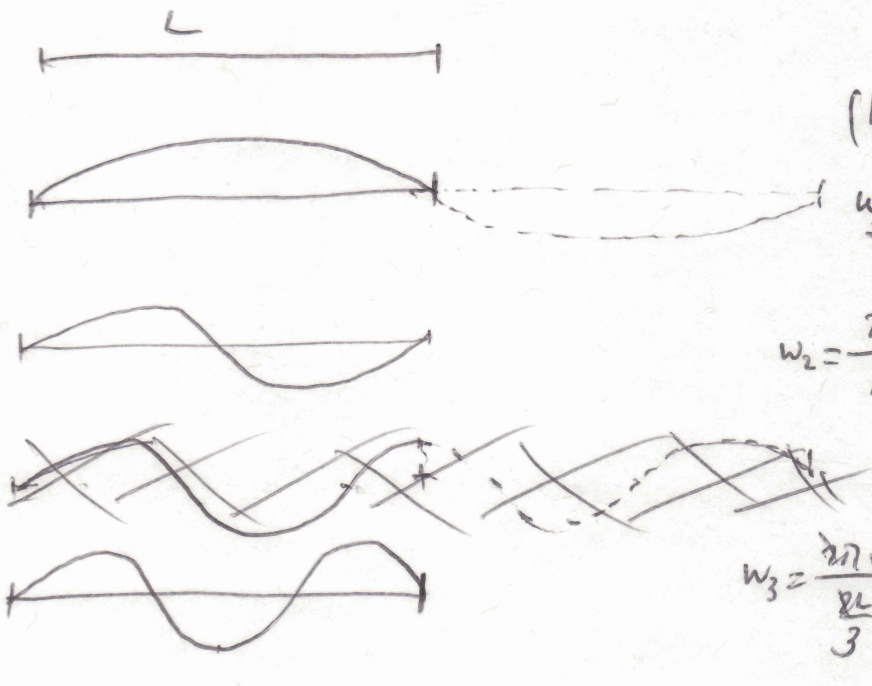
Only certain waves can stand within a string of length L: those with wavelengths $\frac{2L}{1}$; $\frac{2L}{2}$; $\frac{2L}{3}$; etc.

↳ Longest wavelength can stand in a string of length L: $\rightarrow 2L$:

wave speed

$$\uparrow$$

$$v = \frac{\lambda}{T} = \frac{\lambda}{\frac{2\pi}{\omega}} = \frac{\omega}{k}$$



(Lowest, fundamental)

$n = 1 \rightarrow \lambda = 2L$

$$\omega_1 = \frac{2\pi v}{\lambda} = \frac{\pi v}{L}$$

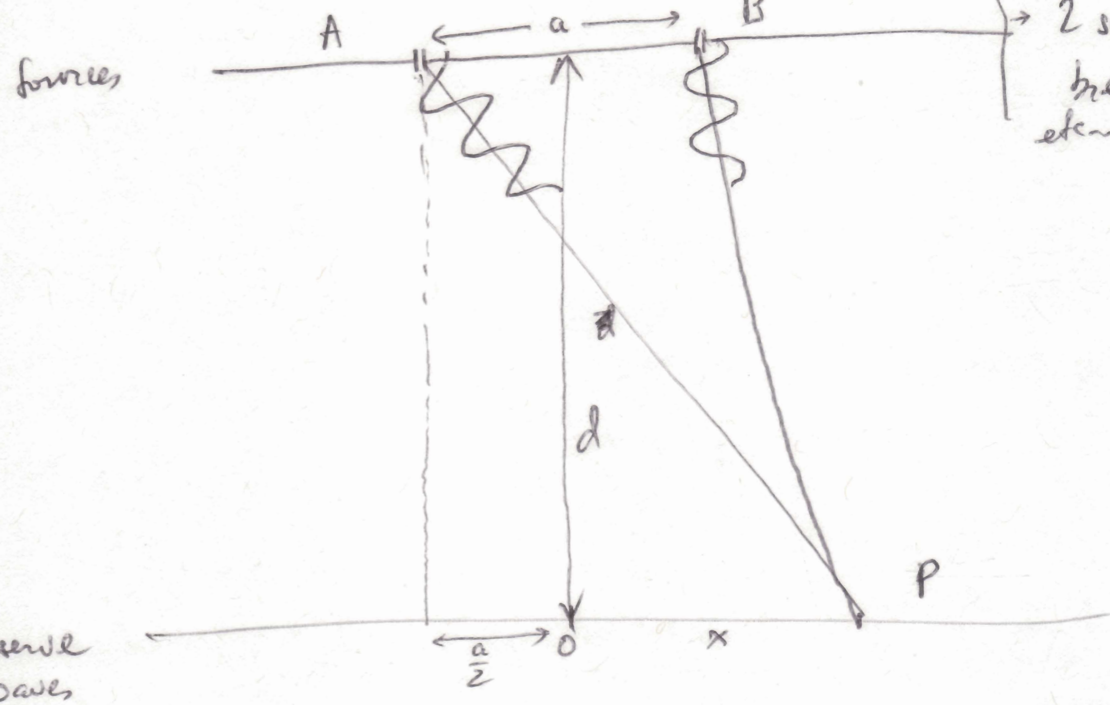
$n = 2 \rightarrow \lambda = L$

$$\omega_2 = \frac{2\pi v}{L} = 2\omega_1$$

$n = 3 \rightarrow \lambda = \frac{2L}{3}$

$$\omega_3 = \frac{2\pi v}{\frac{2L}{3}} = \frac{3\pi v}{L} = 3\omega_1$$

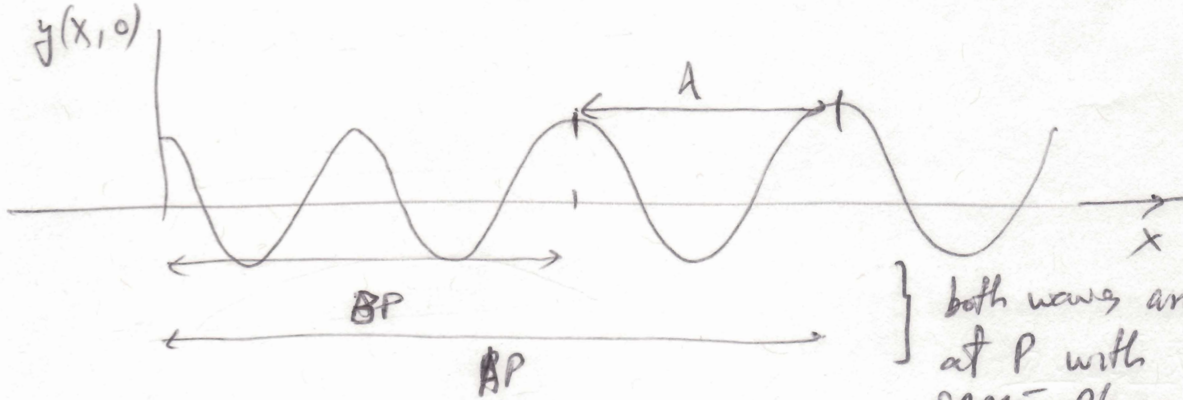
Constructive & Destructive Interference: 2 sources



→ 2 loudspeakers for sound waves
→ 2 small openings in break water
etc.

observe waves @ P

wave A travelled AP
wave B travelled BP
} both have same amplitude A, same ω & K (2 identical waves)



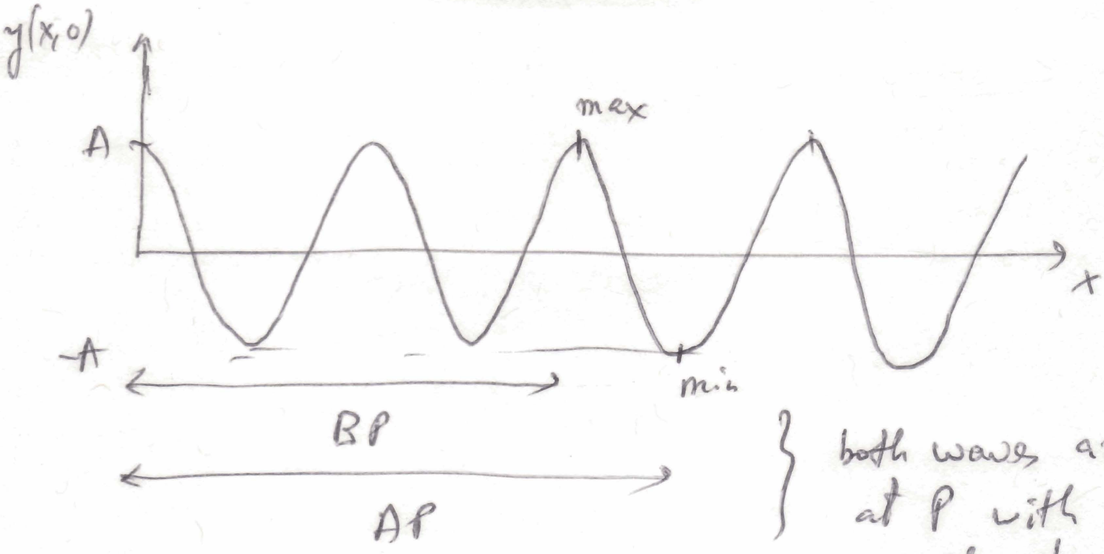
} both waves arrive at P with SAME phase

→ $\boxed{\text{constructive interference}}$

→ AP − BP = λ → can get location of P or of 1st maximum or 1st constructive interference spot!

$$AP = \sqrt{d^2 + \left(\frac{a}{2}+x\right)^2}$$
$$BP = \sqrt{d^2 + \left(x - \frac{a}{2}\right)^2}$$

} AP − BP = λ → solve for x!

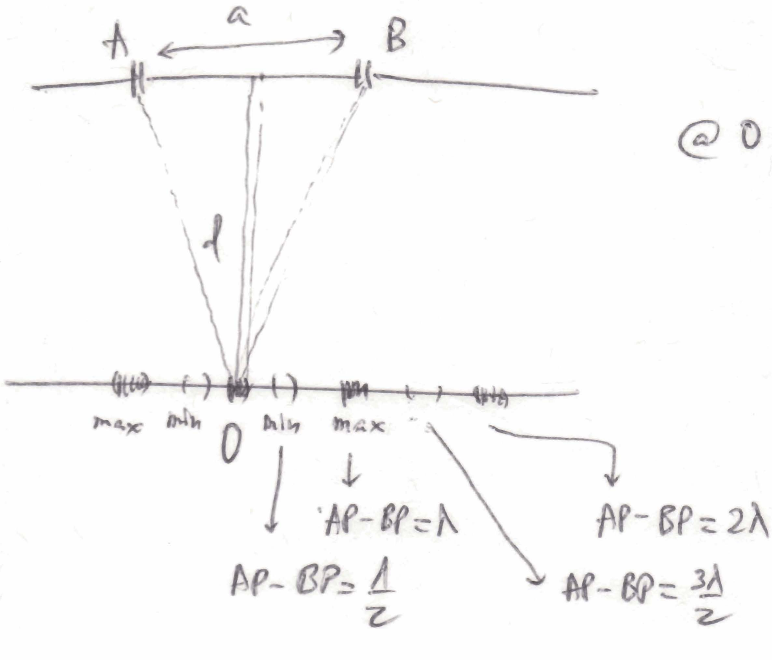


both waves arrive at P with opposite phase

$\rightarrow$ $\boxed{\text{destructive interference}}$

$$AP - BP = \frac{\lambda}{2} \rightarrow \sqrt{d^2 + \left(x + \frac{a}{2}\right)^2} - \sqrt{d^2 + \left(x - \frac{a}{2}\right)^2} = \frac{\lambda}{2}$$

solve for x: loc. of 1st minimum or destructive interference spot.

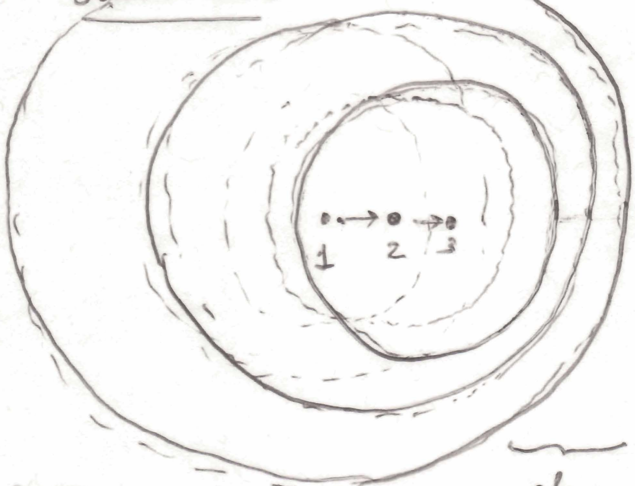


@ 0: both waves arrive at same phase

$\rightarrow$ always a max.

Doppler Effect: moving source of wave

Sound wave:



further apart

longer λ

$\downarrow$

lower f

$\downarrow$

$\lambda' = \lambda + uT$

$\downarrow$

receding source

closer: smaller λ

$\downarrow$

higher ω or f

$\downarrow$

$\lambda' = \lambda - uT$

u $\rightarrow$ speed of source

T $\rightarrow$ period of wave

approaching source

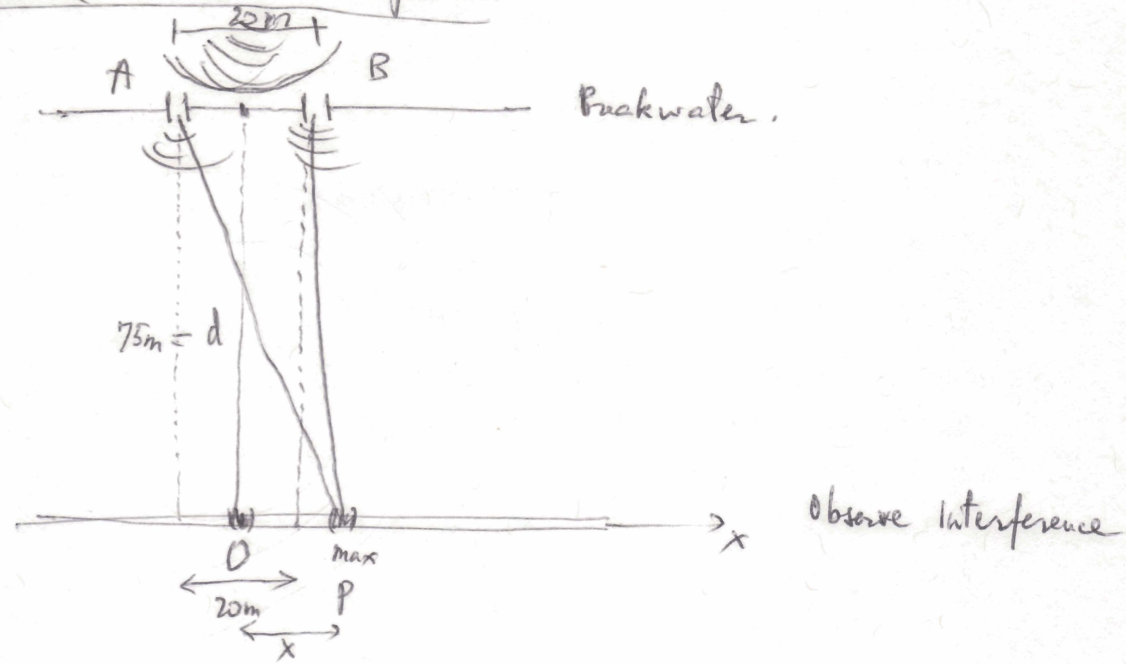
$$\boxed{f' = \frac{f}{1 \pm \frac{u}{v}}}$$

u $\rightarrow$ source speed

v $\rightarrow$ wave speed

Doppler Effect.

Constructive & destructive interference:



P: 1st max:

$AP - BP = \lambda \longrightarrow$ of water wave

$$\sqrt{d^2+(x+10)^2} - \sqrt{d^2+(x-10)^2} = \lambda \rightarrow d^2+(x+10)^2 + d^2+(x-10)^2 - 2\sqrt{d^2+(x+10)^2}\sqrt{d^2+(x-10)^2} = \lambda^2$$

$$2d^2 + 2x^2 + 200 - \lambda^2 = 2\sqrt{d^2+(x+10)^2}\sqrt{d^2+(x-10)^2}$$

$\boxed{\lambda = 16m}$

data

$$\cancel{11250}\ \left[11194 + 2x^2\right]^2 = 4\left(5725 + x^2 + 20x\right)\left(5725 + x^2 - 20x\right)$$

$$11194^2 + 4\times 11194 x^2 + 4x^4 = 4\times 5725^2 + 4\times 2\times 5725 x^2 - 4\times 20^2 x^2 + 4x^4$$

simplify: $y = x^2 \rightarrow$ quadratic eq in $y \rightarrow$ solve for y

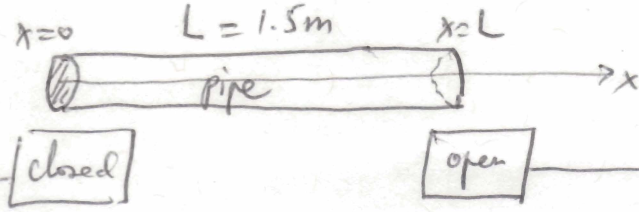
$\hookrightarrow x = \pm\sqrt{y} \rightarrow \boxed{x = \pm 33\,m}$

Find 1st min $\rightarrow$ replace λ by $\frac{\lambda}{2}$ in previous eq

Find 2nd max $\longrightarrow$ replace λ by 2λ

Find 2nd min: " λ " $\frac{3\lambda}{2}$ . . .

14.72 15.57



$f_n = 225\,Hz$

$f_{n+1} = 375\,Hz$

a) find fundamental freq f_0 b) Sound speed.

Standing waves : $y = y_{incoming} + y_{reflected}$

$y_{incoming} \to A\cos(kx - \omega t)$ $\quad y_{reflected} \to -A\cos(kx+\omega t)$

$= 2A \sin kx \sin\omega t$

$y(0,t) = 0$ $\qquad y(L,t) = max$

$\to \sin kL = \pm 1 \to kL = \frac{\pi}{2}; \frac{3\pi}{2}; \frac{5\pi}{2};$

odd multiple of $\frac{\pi}{2}$

$= (2n+1)\frac{\pi}{2}$

$(n = 0,1,2,3, etc..)$

$$\Rightarrow \boxed{\frac{2\pi}{\lambda}L = (2n+1)\frac{\pi}{2} \quad (n = 0,1,2,3,4,\dots)}$$

$v = \frac{\lambda}{T} = \lambda f \to v = \lambda_n f_n$ (n $\leftrightarrow$ mode)

same wave speed for all modes n $\to$ (propagation in a same medium $\to$ air)

$$\boxed{f_n = \frac{v}{\lambda_n} = v\frac{(2n+1)}{4L} \quad n = 0,1,2,3,4,\dots}$$

(133)

$$f_n = \frac{v}{4L}(2n+1) \rightarrow f_{n+1} = \frac{v}{4L}\left(2(n+1)+1\right)$$

$$= \frac{v}{4L}(2n+3)$$

$$\rightarrow \boxed{\frac{f_{n+1}}{f_n} = \frac{\cancel{\frac{v}{4L}}}{\cancel{\frac{v}{4L}}} \frac{(2n+3)}{(2n+1)}} = \frac{375\ \cancel{Hz}}{225\ \cancel{Hz}} = \frac{125\times \cancel{3}}{75\times\cancel{3}} = \frac{5}{3}$$

$$\cancel{(2n+3)225 = (2n+1)37}$$

$$(2n+3)3 = (2n+1)5$$

$$6n+9 = 10n+5 \rightarrow 4n = 4 \rightarrow \boxed{n=1}$$

$$\boxed{f_1 = 225\,Hz}$$

$\rightarrow$ Fundamental fq : $f_0 = \frac{v}{4L}$.

$$\cancel{\frac{f_{n+1}}{f_n} = \frac{5}{3} \rightarrow \frac{f_1}{f_0} = \frac{5}{3} \rightarrow f_0 = \frac{3}{5} f_1 = \frac{3}{5}225}$$

$n=0$

$$\frac{f_{n+1}}{f_n} = \frac{2n+3}{2n+1} \underset{n=0}{\Rightarrow} \frac{f_1}{f_0} = \frac{3}{1} \rightarrow \boxed{f_0 = \frac{f_1}{3} = \frac{225}{3} = 75\,Hz}$$

b) $f_0 = \frac{v}{4L} \rightarrow v = f_0 4L = 75\,s^{-1}\,4\times 1.5\,m = 450\,m/s$

Ch 15 Fluid Motion

Gas : ρ can be variable (gas is compressible)
Liquid : ρ is constant (liquid is incompressible)

ρ → density

→ Density $\rho \equiv \dfrac{\text{mass}}{\text{volume}} = \dfrac{dm}{dV}$ (S.I : $\dfrac{kg}{m^3}$)

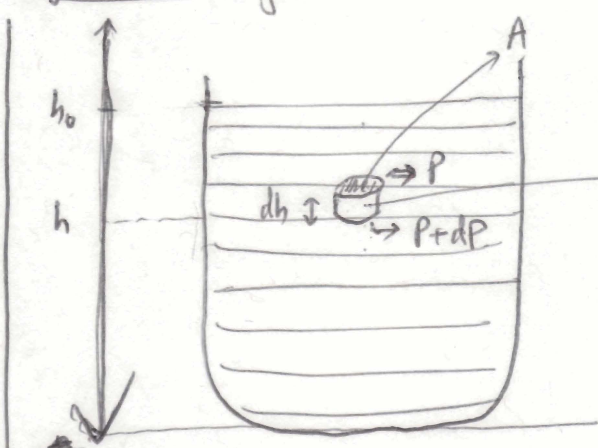
→ Pressure P = $\boxed{\text{normal}}$ force per unit area (SI. $\dfrac{N}{m^2}$ = Pa or Pascal)

$P = \dfrac{F}{A}$ or $\dfrac{dF}{dA}$ → direction is not relevant
P is not a vector

→ Alternative unit: Atm (Atmosphere)

$$\boxed{1\,\text{Atm} = 1.013 \times 10^5\ \text{Pa}}$$

Hydrostatic equilibrium:



→ Vase filled with water

Let's focus on this infinitesimal piece of water of mass dm and cross-sectional area A

dP : infinitesimal increase in pressure from top to bottom of the tiny cylinder of water

→ piece of water in equilibrium:

$\underbrace{(P+dP)A}_{\text{upward force}} - \underbrace{PA}_{\text{downward force}} = g\,dm \rightarrow \cancel{A}dP = g\,dm = g\rho \underbrace{dV}_{\cancel{A}dh}$ (volume of piece of water)

$$\boxed{\frac{dP}{dh} = g\rho}$$

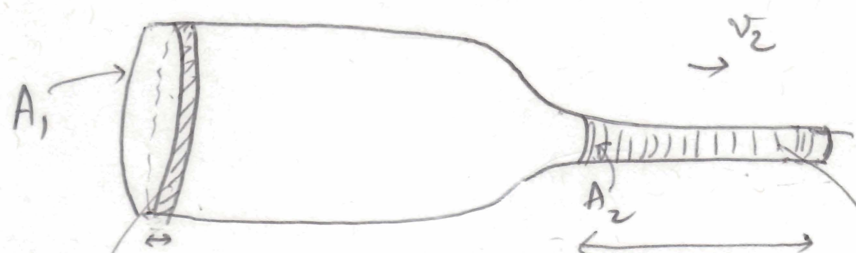
$$dP = g\rho \, dh \rightarrow P = \int_{h_0}^{h} g\rho \, dh = g\rho (h - h_0) \quad (h > h_0)$$

If $h_0 = 0 \rightarrow \boxed{P = g\rho h}$

$$\int_{h_0}^{h} dh = h \Big]_{h_0}^{h} = h - h_0$$

~~Bouyancy~~ Bouyancy: $\rightarrow F_{bouyant} = P \cdot A = g\rho \underbrace{hA}_{vol.}$

Conservation of mass: (no leaking or loss of fluid molecules)



→ a disk of fluid of mass m_1

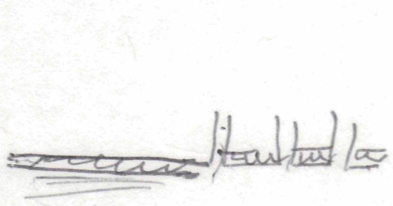
→ same disk of fluid of mass $m_2 = m_1$

$m_1 = \rho_1 V_1$ $\qquad$ $m_2 = \rho_2 V_2$ → same fluid $\rho_1 = \rho_2$ (incompressible fluid)

conservation of mass →

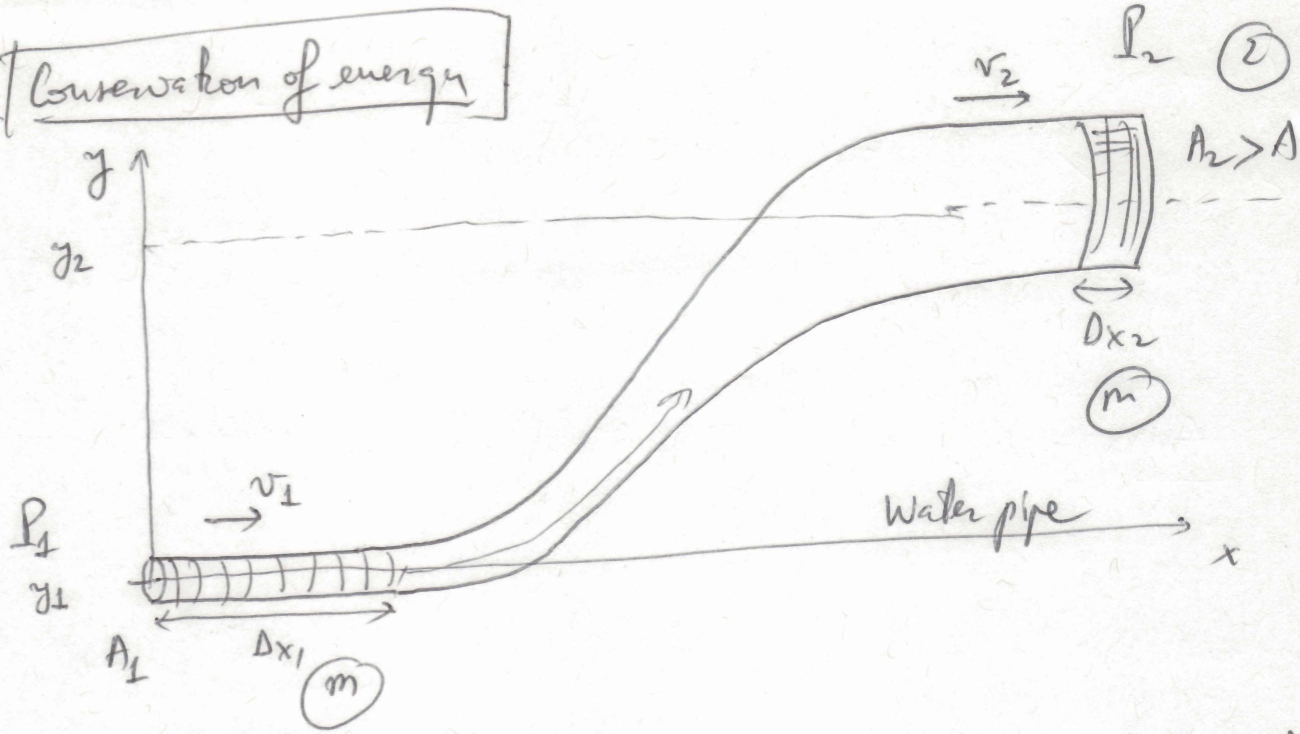
$$m_1 = m_2 \rightarrow \rho V_1 = \rho V_2 \rightarrow V_1 = V_2$$

$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$



→ $\boxed{vA \text{ is constant}}$ → larger cross-sectional area → lower speed.

Conservation of energy



1

Difference in pressure from 1 to 2 : water went from 1 up to 2 → Work done by pressure is:

$$\Delta W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta X_2$$

$$\boxed{\Delta W = \Delta KE + \Delta P.E.}$$

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

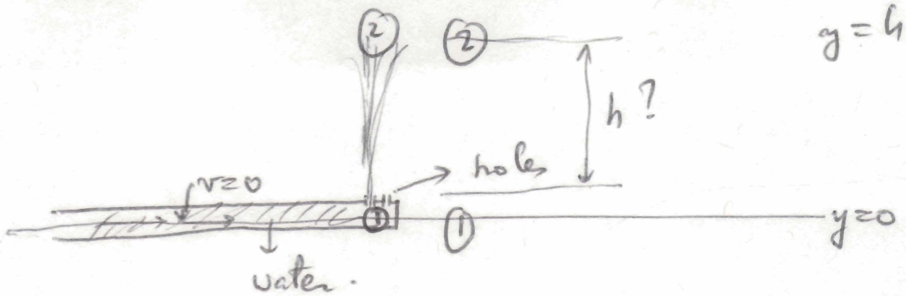
$$\rightarrow \boxed{\frac{1}{2} m v^2 + mgy + PA\Delta x = \text{constant}}$$

Rearrange: dividing by $V = A\Delta x$

$$\frac{1}{2}\frac{m}{V} v^2 + \frac{m}{V} gy + P = \text{const.}$$

$$\boxed{\frac{1}{2}\rho v^2 + \rho g y + P = \text{const}} \rightarrow \text{Bernoulli's equation.}$$

(15.57)



→ Conservation of energy or Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$$P_1 + \frac{1}{2}\cancel{\rho 0^2} + \rho g 0 = P_2 + \frac{1}{2}\rho 0^2 + mgh$$

$$\rightarrow h = \frac{P_1 - P_2}{\rho g}$$

$$= \frac{140\,\text{kPa} + \cancel{P_{atm}} - \cancel{P_{atm}}}{10^3 \frac{kg}{m^3}\, 9.81\, m/s^2}$$

$\boxed{\rho_{water} = 1000\, kg/m^3}$

$$\boxed{h = \frac{140}{9.81}\, m = 14.3\, m}$$

Ch 9 Elastic / inelastic collisions

Ch 10 Rotations

Ch 11 Angular momentum

Ch 12 Static Equilibrium : $\sum_i F_i = 0$; $\sum_i \tau_i = 0$

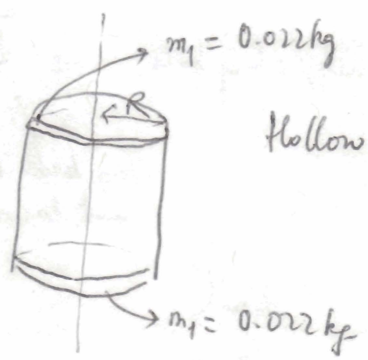
Ch 13 Osc.

Ch 14 Wave motion

Ch 15 Fluid Motion.

10.28 ; 34 ; 58

a)



Hollow cylinder. $m = 0.065$ kg ; $R = 0.071$ m

$$I = I_{cylinder} + 2 I_{disks}$$

Table 10.2 $\quad mR^2 + \cancel{2}\frac{1}{2} m_1 R^2$

$$= 0.065 \times 0.071^2 + 0.022 \times 0.071^2$$

$$\boxed{I = 4.39 \times 10^{-4}\ kg\,m^2}$$

b) $\alpha = 3.4\ rad/s^2 \rightarrow \tau$?

$$\tau = I\alpha = 4.39 \times 10^{-4} \times 3.4 = 1.5 \times 10^{-3}\ Nm.$$

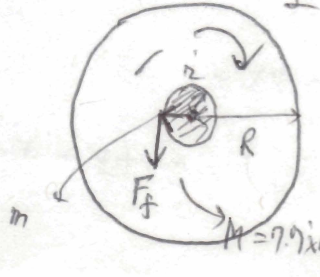
10.34 Friction (tangential) 34 kN slows (decelerates) wheel from $\omega_0 = 360$ rpm to $\omega_f = 0$:

$$\omega_f = \omega_0 - \alpha t \Rightarrow t = \frac{\omega_0}{\alpha}$$

$\omega_f = 0$

$$\tau = I\alpha \rightarrow \alpha = \frac{\tau}{I} = \frac{rF}{\frac{1}{2}MR^2} \longrightarrow t = \frac{\omega_0 \frac{1}{2}MR^2}{rF} = \frac{360 \cdot \frac{2\pi}{60} \frac{1}{2} 7.7\times10^4 \times 2.4^2}{0.205^2 \times 34\times10^3}$$

$$= 1200\ s \quad \text{or} \quad 20\ min$$



$R = 2.4$ m (wheel)

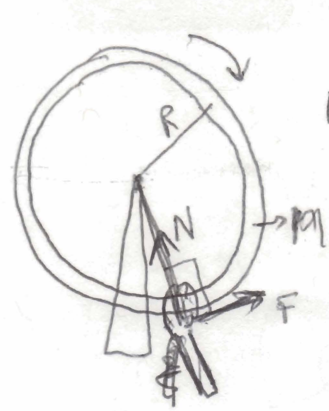
$r = \frac{0.41\,m}{2}$ (shaft)

$I_{Disk} = \frac{1}{2}MR^2$

$$I = \frac{1}{2}MR^2 + \underbrace{\frac{1}{2}mr^2 - \frac{1}{2}(\pi r^2)\frac{M}{\pi(R^2 - r^2)} r^2}_{\simeq 0}$$

139

10.58



$R = 0.33m$

$t = 3.1s.$

$w_0 = 230 rpm$

$M = 1.9 kg$

$N = 2.7N$

$\hookrightarrow F = \mu_k N = 0.46 \times 2.7 N$

$\downarrow$

$\tau = RF.$

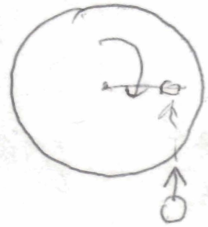
$\tau = I.\alpha \Rightarrow \underset{\substack{\downarrow \\ \text{angular deceleration}}}{\alpha} = \frac{\tau}{I} = \frac{RF}{I} = \frac{R\mu N}{MR^2} = \frac{\mu N}{MR}$

$w_f = w_0 - \underbrace{\alpha t}_{\text{in rpm}} \Rightarrow w_f = 230 \, rpm - \underbrace{\underbrace{\frac{0.46 \times 2.7}{1.9 \times 0.33}}_{\frac{rad}{s}} \cdot 3.1 \frac{1 rev}{2\pi rad} \cdot \frac{60 s}{min}}_{58.6 \, rpm}$

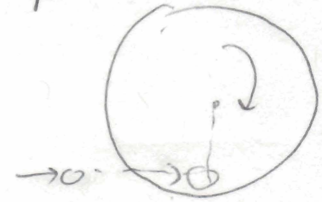
$\boxed{w_f = 171 \, rpm}$

11.47

2D sketch from above

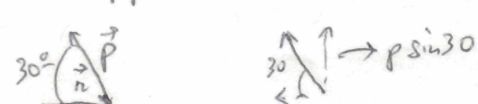


or



$L_{\text{before clay hits turntable}} = L_{\text{after clay hits turntable}}$

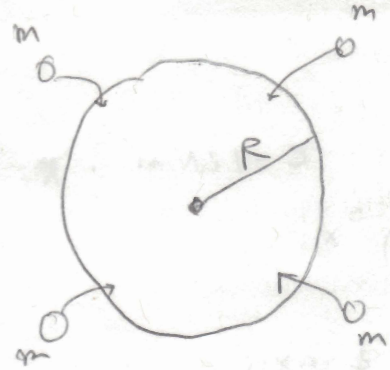
$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \boxed{L = rp}$ $\quad \vec{r} \quad \vec{p} \longrightarrow \tau = \frac{dL}{dt} \Rightarrow \tau = 0 \rightarrow L$ conserved.



$L = rp\sin 30$

$\hookrightarrow L = rmv = mr^2 \frac{v}{r} = I\omega \longrightarrow (I\omega)_{before} = (I\omega)_{after}.$

11.26



$R = 1.5\,m$ $\quad m = 25\,kg.$

$I = 120\ kg\,m^2$

$\omega_0 = 0.5\,\frac{rev}{s}$

System of 4 children + disk

$\tau_{ext} = 0 \rightarrow L_{before} = L_{after}$

a) $I\omega_0 = (I + 4mR^2)\,\omega_f \rightarrow \omega_f = \underbrace{\frac{120}{120 + 100\times 1.5^2}}_{\text{dimensionless}} 0.5\frac{rev}{s}$

$= 0.174\,\frac{rev}{s}$

b) Energy lost to friction $= \underbrace{\frac{1}{2}I\omega_0^2}_{\text{initial}} - \underbrace{\frac{1}{2}(I + 4mR^2)\,\omega_f^2}_{\text{final}}$

↳ Rot: $\frac{1}{2}I\omega^2$

$\frac{1}{2}\,120\,\underbrace{0.5^2}_{\times(2\pi)^2} - \frac{1}{2}(120 + 225)\underbrace{0.174^2}_{\times(2\pi)^2} = \left(60\times 0.25 - \frac{345}{2}\times 0.174^2\right)4\pi^2\,J$

$= 386\,J$

14.53

$y(x,t) = A\cos(kx - \omega t) + A\cos(kx - \omega t + \phi)$

$= A[\cos(kx - \omega t) + \cos(kx - \omega t + \phi)]$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ $\quad \alpha = kx - \omega t$, $\beta = \alpha + \phi$

$= 2A\cos\left(kx - \omega t + \frac{\phi}{2}\right)\cos\left(-\frac{\phi}{2}\right)$

$$\boxed{y(x,t) = \underbrace{2A\cos\frac{\phi}{2}}_{\text{amplitude of the superposition wave}}\ \underbrace{\cos\left(kx - \omega t + \frac{\phi}{2}\right)}_{\text{S Harmonic wave}}}$$

in phase $\phi = 0$ → amplitude $2A$

→ opposite phase $\phi = 180$ → amplitude 0

13.37

Eq. 13.19 $\rightarrow x(t) = \underbrace{A e^{-\frac{bt}{2m}}}\cos(\omega t + \phi)$ $\Rightarrow \frac{b}{2m} = 2.8\,s^{-1}$

vibration amplitude $\hat{A}$

$$\begin{cases} \hat{A}(t=0) = A \\ \hat{A}(t) = \frac{\cancel{A}}{2} = \cancel{A} e^{-\frac{bt}{2m}} \end{cases}$$

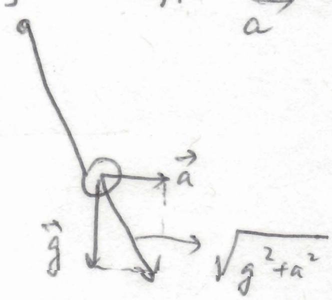
$$\ln \boxed{\frac{1}{2} = e^{-2.8t}}$$

$$-\ln 2 = -2.8t \rightarrow t = \frac{\ln 2}{2.8} = 0.248\,s$$

13.79

Before

During take-off $\vec{a}$



pendulum $\rightarrow \boxed{\omega_i = \sqrt{\frac{g}{L}}}$

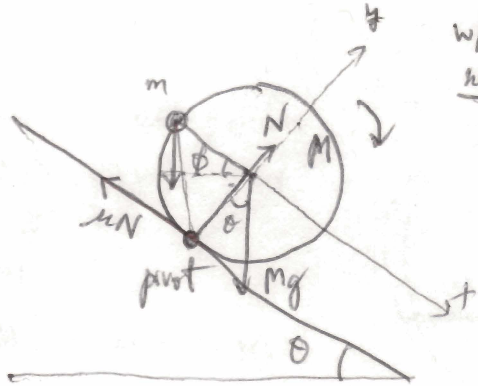
$$\omega_f = \sqrt{\frac{\sqrt{g^2+a^2}}{L}}$$

$$\frac{\omega_f^4}{\omega_i^4} = \frac{\frac{g^2+a^2}{\cancel{L^2}}}{\frac{g^2}{\cancel{L^2}}} = 1 + \left(\frac{a}{g}\right)^2 \rightarrow a = g\sqrt{\frac{\omega_f^4}{\omega_i^4} - 1}$$

$$= 9.81\sqrt{\frac{91^4}{90^4} - 1}$$

$$= 2.1\,m/s^2$$

12.61



w/friction non-slipping $\rightarrow$ can roll

$M = 1.5\,kg$; $m = 0.95\,kg$

$\theta = 20^\circ$

ϕ? wheel in static equilibrium

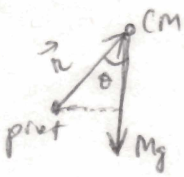
$\hookrightarrow \sum_i F_i = 0$

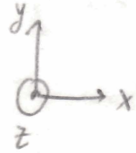
$\hookrightarrow \sum_i \tau_i = 0$

On wheel: 4 forces: Mg ; N ; μN ; mg

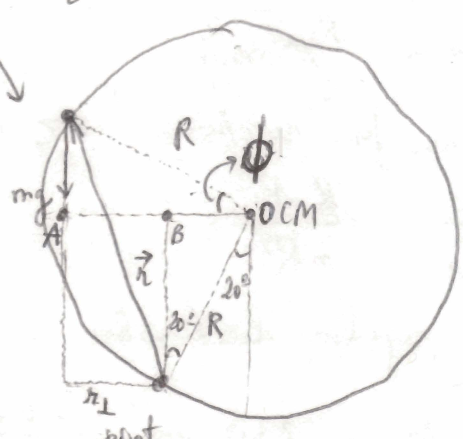
$\sum_i \tau_i = 0 \rightarrow$ pivot: candidates $\begin{cases} \text{CM of wheel } (Mg) \\ \text{loc of } m \ (mg) \\ \text{Contact point } (N \,\&\, \mu N) \checkmark \end{cases}$

$$\boxed{\vec{\tau}_M + \vec{\tau}_m = 0}$$





$(R \sin\theta\, Mg)(-\hat{k})$



$r_\perp = AO - BO = R\cos\phi - R\sin 20^\circ$

$r_\perp mg \ (\hat{k})$

$$-\cancel{R}\sin 20^\circ M\cancel{g} + (\cancel{R}\cos\phi - \cancel{R}\sin 20^\circ)\, m\cancel{g} = 0$$

$$\rightarrow \cos\phi = \frac{M\sin 20^\circ + m \sin 20^\circ}{m}$$

$$= \left(\frac{M}{m} + 1\right)\sin 20^\circ \rightarrow \phi = \cos^{-1}\left[\left(\frac{1.5}{0.95} + 1\right)\sin 20^\circ\right]$$

$$\boxed{\phi = 28.1^\circ}$$