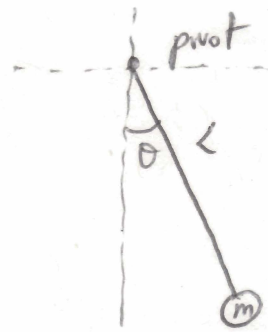
Ch 13 Oscillatory Motion

Examples:

1) Pendulum



Swinging bob

θ follows osc. motion

$\tau = I\alpha \rightarrow$ $\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\sin\theta$

small angle approximation

θ is small

$\rightarrow \sin\theta \approx \theta$

Taylor expansion

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\,\theta} \quad (a)$$

2nd order differential equation.

solution $\boxed{\theta = \theta_m \cos(\omega t)}$ (b)

θ_m: amplitude

ω = angular freq.

(why ω?)

(b) in (a): $\dfrac{d}{dt}\left(\dfrac{d}{dt}\theta_m \cos(\omega t)\right) = -\dfrac{g}{L}\theta_m \cos\omega t$

$\theta_m \dfrac{d}{dt}\left(-\omega \sin\omega t\right)$

$-\cancel{\theta_m}\,\omega^2 \cancel{\cos\omega t} = -\dfrac{g}{L}\cancel{\theta_m}\cancel{\cos\omega t}$

$$\rightarrow \boxed{\omega = \sqrt{\frac{g}{L}}}$$

2) Torsional pendulum:

twisting bob

$\tau = -\kappa\,\theta$

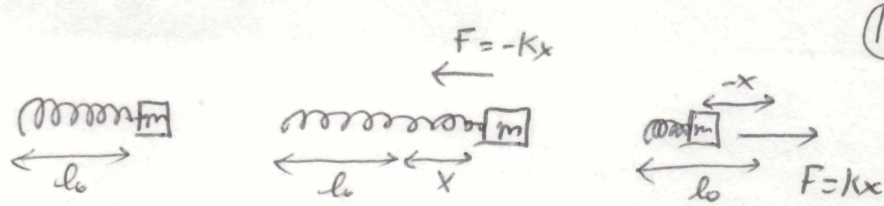
κ Kappa: torsional constant

$\tau = I\alpha$

$-\kappa\theta = I\dfrac{d^2\theta}{dt^2} \rightarrow \boxed{\dfrac{d^2\theta}{dt^2} = -\dfrac{\kappa}{I}\theta}$

$\hookrightarrow \theta = \theta_m \cos\omega t$; $\omega = \sqrt{\dfrac{\kappa}{\;}}$

3) Spring.



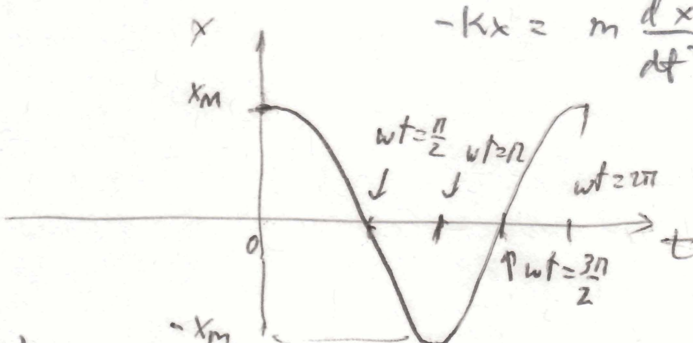
Spring Force always opposes motion of mass m → m undergoes osc. motion.

Newton's Law : $F = ma = m\dfrac{d^2x}{dt^2}$

$\downarrow$

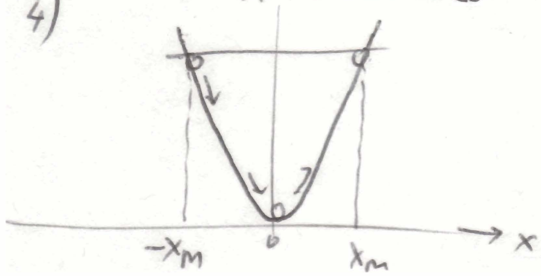
$-kx = m\dfrac{d^2x}{dt^2} \rightarrow \boxed{\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x}$

$\boxed{\omega = \sqrt{\dfrac{k}{m}}}$



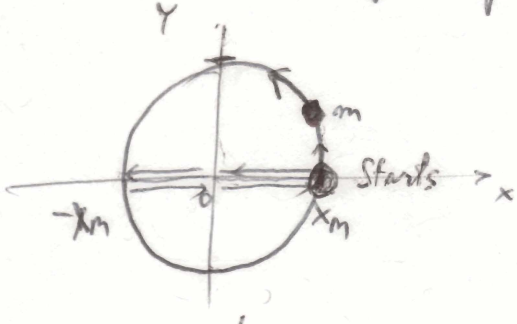
$\leftarrow \boxed{x = x_m \cos \omega t}$

4)



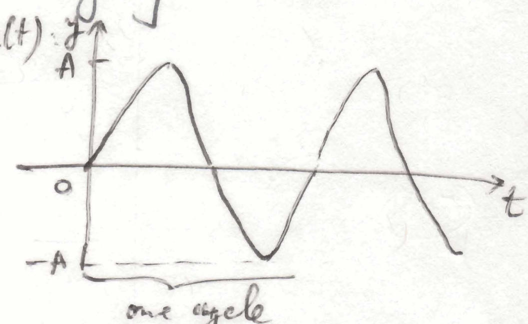
potential well, particle m trapped inside, no friction → osc. motion

5) Coordinates x & y of a particle undergoing UCM

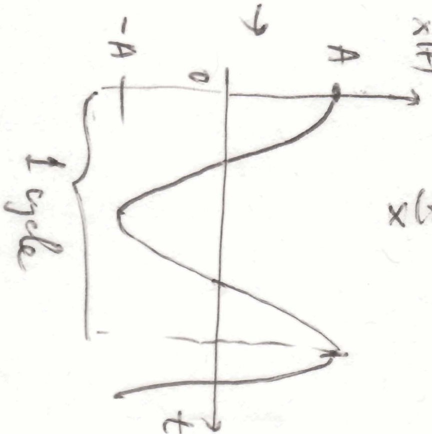


→

Plot y(t):



$y(t) = A \sin \omega t$



$x(t) = A\cos\omega t \rightarrow$ x & y are shifted by 90° or $\frac{\pi}{2}$ rad.

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \rightarrow x \text{ describes SHM osc. motion}$$

$$x(t) = x_m \cos(\omega t) \qquad \omega = \sqrt{\frac{k}{m}}$$

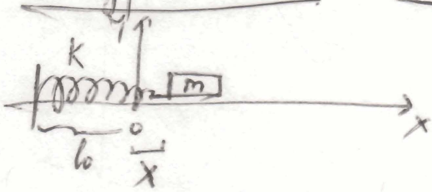
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{b}{m}\frac{dx}{dt} \rightarrow x \text{ describe a damped SHM osc. motion:}$$

$$x(t) = x_m \underbrace{e^{-\frac{bt}{2m}}}_{\text{decaying exponential}} \cos(\omega t + \phi)$$

Phi

decaying exponential: brings amplitude of osc. lower and lower over time.

Energy in SHM: mass + spring system.



$$\omega = \sqrt{\frac{k}{m}}$$

$x = A\cos\omega t$

↳ displacement w.r.t. equilibrium length.

Total energy: $E = KE + \underbrace{U_{Elastic}}_{\text{elastic potential energy}} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

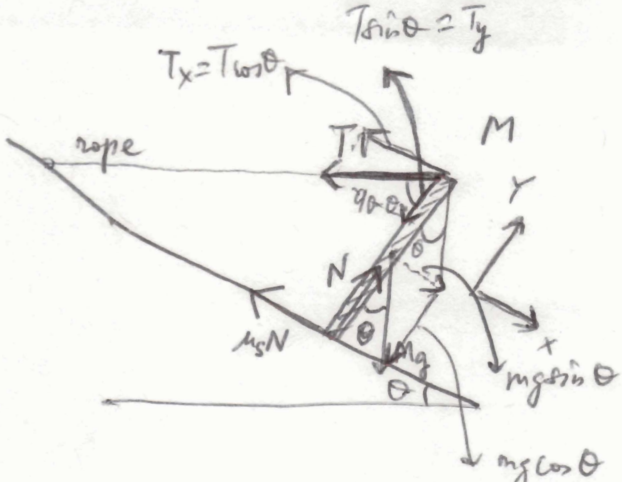
$$v = \frac{dx}{dt} = -A\omega\sin\omega t$$

$$\rightarrow E = \frac{1}{2}m A^2 \underbrace{\omega^2}_{\frac{k}{m}} \sin^2\omega t + \frac{1}{2}k A^2\cos^2\omega t = \frac{1}{2}kA^2 \underbrace{(\sin^2\omega t + \cos^2\omega t)}_{1}$$

→ Total energy of mass + spring system is $\frac{1}{2}kA^2$ (not depending on time)

(when x is large v is small & vice versa)

12.57

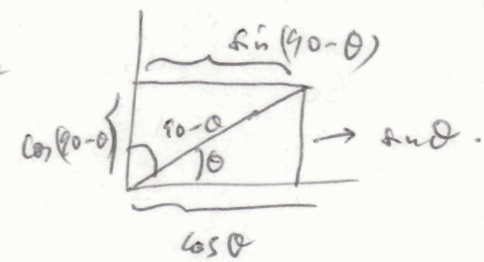


L: length of pole

$\boxed{\mu_s\ ?}$ for pole not slipping

→ Static equilibrium $\begin{cases} \sum_i F_i = 0 \\ \sum_i \tau_i = 0 \end{cases}$

$\sin(90-\theta) = \cos\theta$
$\cos(90-\theta) = \sin\theta$

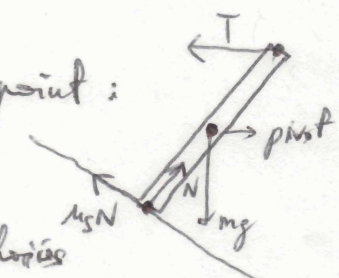


4 forces acting on pole:
$mg \begin{cases} mg\sin\theta\ \hat{i} \\ mg\cos\theta\,(-\hat{j}) \end{cases}$; $N\hat{j}$;
$\mu_s N(-\hat{i})$; $T \begin{cases} T\cos\theta\,(-\hat{i}) \\ T\sin\theta\,(-\hat{j}) \end{cases}$

$$\sum_i F_i = 0 \quad \begin{cases} x: \quad mg\sin\theta - \mu_s N - T\cos\theta = 0 \quad (1) \\ y: \quad -mg\cos\theta - T\sin\theta + N = 0 \quad (2) \end{cases}$$

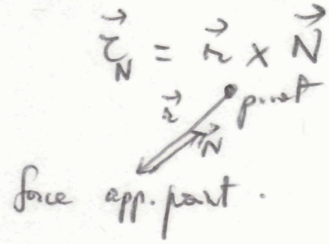
$\sum_i \tau_i = 0$: Determine or $\underline{\underline{\text{choose}}}$ pivot point:

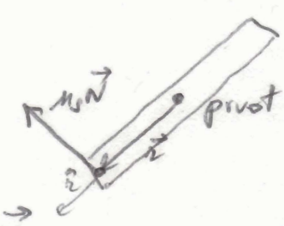
↓ math different for different choices



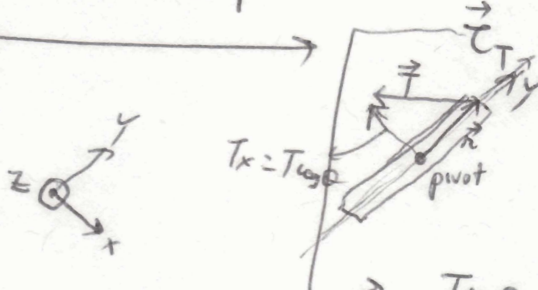
Let's pick CM of pole → pivot : normal force & mg → no torque

$$\vec{\tau}_N = \vec{r} \times \vec{N} = 0$$





$$\vec{\tau}_T + \vec{\tau}_{\mu_s N} = 0$$



Parallel vectors → no cross product:
T_y → no torque

$$T\cos\theta\,\frac{L}{2}\,(\hat{k}) + \mu_s N\,\frac{L}{2}\,(-\hat{k}) = 0 \quad (3)$$

(3) in (1): $\mu_s N = T\cos\theta$

$\longrightarrow mg\tan\theta - 2T\cos\theta = 0 \rightarrow T = \dfrac{mg\sin\theta}{2\cos\theta}$

$$\boxed{T = \frac{mg\tan\theta}{2}}$$

(2) $N = mg\cos\theta + T\sin\theta = mg\cos\theta + \dfrac{mg\tan\theta}{2}\sin\theta$

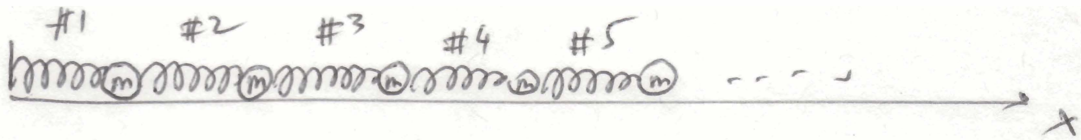
$$\boxed{N = mg\left[\cos\theta + \frac{1}{2}\tan\theta\sin\theta\right]}$$

(3) $\mu_s = \dfrac{T}{N}\cos\theta = \dfrac{\frac{1}{2}\tan\theta\cos\theta}{\cos\theta + \frac{1}{2}\tan\theta\sin\theta} \quad \dfrac{\times\frac{2}{\cos\theta}}{\times\frac{2}{\cos\theta}}$

$$\boxed{\mu_{s\,\text{minimum}} = \frac{\tan\theta}{2 + \tan^2\theta}} \rightarrow \mu_s \geq \mu_{s\,\text{min}}.$$

Ch 14 Wave Motion:

Wave: propagation of a local disturbance (oscillation), not of matter:



osc. in spring #1

1) takes some time to propagate to spring #5

   wave speed $v = \frac{\lambda}{T}$ — $\lambda \to$ lambda → wavelength; $T \to$ period (separation b/w two consecutive peaks)

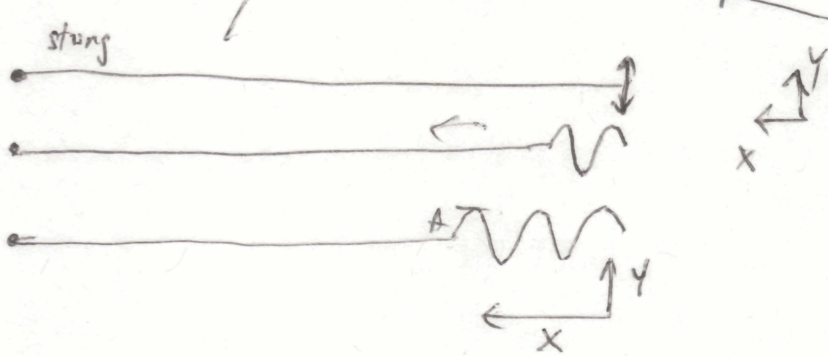
2) spring & bob stay local; only the osc. (disturbance) is propagated!

3) propagation is in space → also an SHM.

   oscillation is in time → Simple Harmonic Motion ($\cos\omega t$) (SHM)

2 types:
- longitudinal: when osc/disturbance is in same direction as propagation (above)
- transverse: osc/disturbance is $\perp$ propagation

↓ example: $y(x,t) = A \sin\left(\underbrace{\frac{2\pi}{\lambda}x}_{\text{space}} - \underbrace{\frac{2\pi}{T}t}_{\text{time}}\right)$
- osc.: $y(t)$ along $\hat{y}$
- propagation: along $\hat{x}$

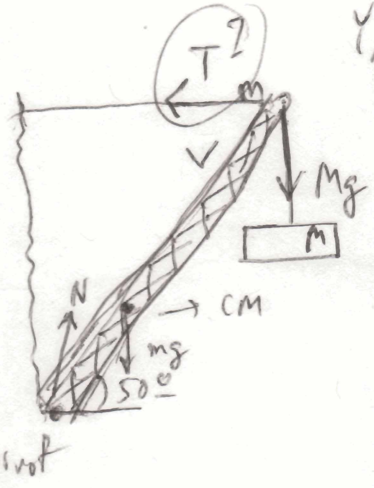


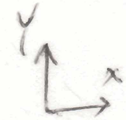
also:

$$y(x,t) = A\sin(kx - \omega t)$$

- k = wave number $= \frac{2\pi}{\lambda}$ (m^{-1})
- ω = ang. freq $= \frac{2\pi}{T}$ (s^{-1})

12.40





$M = 2500\,kg$; $m = 830\,kg$

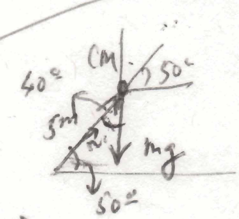
$L = 15\,m$

Forces acting on crane of mass m:

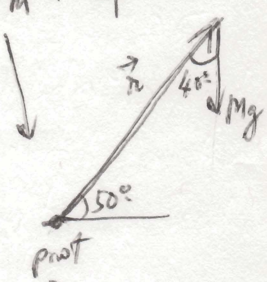
$Mg(-\hat{j})$; $T(-\hat{i})$; $mg(-\hat{j})$; $\vec{N}$

direction depends on contact b/w crane & ground

$\rightarrow \begin{cases} \sum_i \vec{F}_i = 0 & : \text{incomplete info } (\vec{N} : \text{unknown direction}) \\ \sum_i \tau_i = 0, & \rightarrow \text{decide on pivot} \rightarrow \text{at base} \rightarrow \vec{\tau}_m + \vec{\tau}_M + \vec{\tau}_T = 0 \end{cases}$

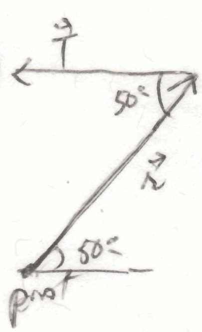


$\vec{\tau}_M = 5\sin 40\, mg\,(-\hat{k})$



$\vec{\tau}_M = 15\sin 40°\, Mg\,(-\hat{k})$

$$\boxed{\cancel{15}_{3}\sin 50°\, T - \cancel{5}\, mg\sin 40 - \cancel{15}_{3}\, Mg\sin 40 = 0}$$



$\vec{\tau}_T = 15\sin 50°\, T\hat{k}$

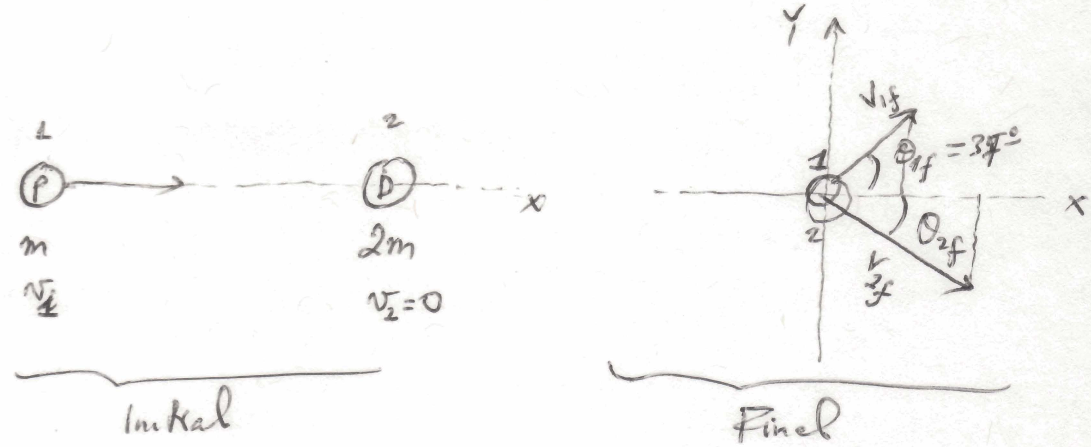
$$T = \frac{mg\sin 40 + 3Mg\sin 40}{3\sin 50°}\ N$$

Exam 3: 9 (Collisions) 10 (Rotations) 11 (C.A.M) 12 (Static Eq.) 13 (SHM)
14 (Waves) 15 (Fluids)

13.50

9.65; 9.70; 9.71

9.65



2D Elastic collision:

Conserv. Mom. x: $m v_1 = m v_{1f} \cos 37° + 2m\, v_{2f} \cos \theta_{2f}$ (1)

y: $0 = m v_{1f} \sin 37° + 2m\, v_{2f} \sin \theta_{2f}$ (2)

Conserv. KE: $\frac{1}{2} m v_1^2 = \frac{1}{2} m v_{1f}^2 + \frac{1}{2} 2m\, v_{2f}^2$ (3)

3 eqs : 3 unknowns: v_{1f}, v_{2f}, θ_{2f}

→ Fraction of KE transfered to deuteron (particle #2): $\dfrac{\frac{1}{2} 2m v_{2f}^2}{\frac{1}{2} m v_1^2} = 2 \dfrac{v_{2f}^2}{v_1^2}$

↳ Need to find v_{2f} as a function of v_1

Math: (1) Isolate term with $\cos\theta_{2f}$, square it to combine with similar manipulation of (2)

(1) → $v_1 - v_{1f} \cos 37° = 2 v_{2f} \cos\theta_{2f}$

(2) → $- v_{1f} \sin 37° = 2 v_{2f} \sin \theta_{2f}$

$(1)^2 + (2)^2$ → $v_1^2 + v_{1f}^2 \cos^2 37 - 2 v_1 v_{1f} \cos 37° + v_{1f}^2 \sin^2 37 = 4 v_{2f}^2 \underbrace{(\cos^2\theta_{2f} + \sin^2 \theta_{2f})}_{1}$

$\boxed{1 = \sin^2\alpha + \cos^2\alpha}$

ⓐ $\boxed{v_1^2 + v_{1f}^2 - 2 v_1 v_{1f} \cos 37 = 4 v_{2f}^2}$

(3) → ⓑ $\boxed{v_1^2 - v_{1f}^2 = 2 v_{2f}^2}$

→ Use ⓐ & ⓑ solve for v_{1f}: ⓑ in ⓐ → $v_1^2 + v_{1f}^2 - 2 v_1 v_{1f} \cos 37 = 2 v_1^2 - 2 v_{1f}^2$

$3 \underbrace{v_{1f}^2}_{x^2} - 2 v_1 \cos 37 \underbrace{v_{1f}}_{x} - v_1^2 = 0 \rightarrow v_{1f} = \dfrac{+ 2 v_1 \cos 37 \pm \sqrt{4 v_1^2 \cos^2 37 + 12 v_1^2}}{6}$

$$\to v_{1f} = \frac{+2v_1\cos 37 \pm 2v_1\sqrt{\cos^2 37+3}}{6} = \begin{cases} \dfrac{v_1\cos 37 + v_1\sqrt{\cos^2 37+3}}{3} \ \checkmark \ + \\ \dfrac{v_1 \underbrace{\cos 37}_{<1} - v_1 \underbrace{\sqrt{\cos^2 37+3}}_{>1}}{3} \ \times \ (-) \end{cases}$$

v_{1f} is the magnitude of $\vec{v}_{1f}$ $\to$ has to be +

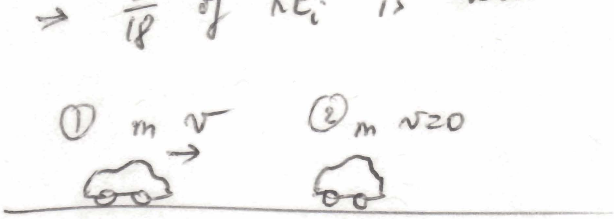
$$\hookrightarrow v_{1f} = \frac{v_1}{3}\Big[\underbrace{\cos 37 + \sqrt{\cos^2 37+3}}_{2.7}\Big] = 0.9\,v_1 \to \boxed{v_{1f} = 0.9\,v_1}$$

<u>Fraction of KE transferred to deuteron:</u> $2\dfrac{v_{2f}^2}{v_1^2} \overset{⑥}{=} \cancel{2}\dfrac{\frac{v_1^2 - v_{1f}^2}{\cancel{2}}}{v_1^2}$

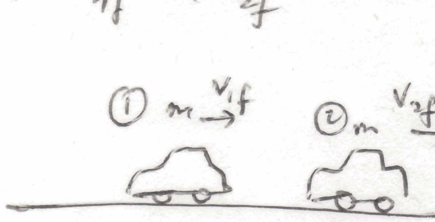
$$= 1 - \frac{v_{1f}^2}{v_1^2} = 1 - 0.9^2$$

$$= 0.18 \to 18\%$$

9.70 1D collision $\boxed{\text{neither elastic nor fully inelastic}}$ $\hookrightarrow v_{1f} \neq v_{2f}$
b/w 2 cars same mass m.
$\to \frac{5}{18}$ of KE_i is lost find v_{1f} & v_{2f}



initial



~~after~~ final

Not fully inelastic $\to$ $v_{1f} \neq v_{2f}$
Not elastic $\to$ some lost of KE ($\frac{5}{18}KE_i$ is L.f)

$$\downarrow\ 1 - \frac{5}{18} = \frac{18-5}{18} = \frac{13}{18}$$

$$\hookrightarrow \begin{cases} \text{Mom: } \cancel{m}v = \cancel{m}v_{1f} + \cancel{m}v_{2f} & ① \\ \text{KE: } \frac{13}{18}v^2 = v_{1f}^2 + v_{2f}^2 & ② \end{cases}$$

Solve 2 eqs for 2 unknowns v_{1f} & v_{2f}

$$①^2 \quad v^2 = v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f}$$

$$② = \frac{13}{18}v^2 + 2v_{1f}v_{2f}$$

$$\frac{5}{18}v^2 = 2v_{1f}v_{2f} \rightarrow \boxed{v_{1f} = \frac{5}{36}v^2\frac{1}{v_{2f}}} \; ④$$

$$④ \text{ in } ①: \left[v = \frac{5}{36}v^2\frac{1}{v_{2f}} + v_{2f}\right] \times v_{2f}$$

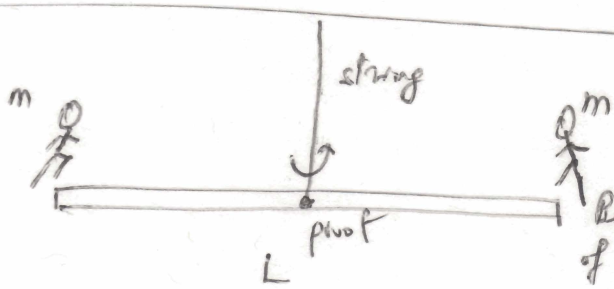
$$v_{2f}v = \frac{5}{36}v^2 + v_{2f}^2 \rightarrow \boxed{v_{2f}^2 - vv_{2f} + \frac{5}{36}v^2 = 0}$$

$$\rightarrow v_{2f} = \frac{v \pm \sqrt{v^2 - \frac{5}{9}v^2}}{2} = \frac{v}{2}\left[1 \pm \sqrt{\frac{4}{9}}\right]$$

$$= \frac{v}{2}\left[1 \pm \frac{2}{3}\right] \begin{cases} \frac{v}{2}\frac{5}{3} = \frac{5}{6}v \\ \frac{v}{2}\frac{1}{3} = \frac{1}{6}v \end{cases}$$

$$v_{2f} = \begin{cases} \frac{5}{6}v \longrightarrow v_{1f} = \frac{1}{6}v \\ \frac{1}{6}v \longrightarrow v_{1f} = \frac{5}{6}v \end{cases}$$

13.50



Beam of Mass M? undergoing torsional oscillations

$m = 75 kg$

$L = 8 m$

Before (no workers on beam) = ω_i

After (with 2 workers on beam) = $\omega_f = 0.8\omega_i$ (20% reduction in angular freq)

Torsional pendulum:

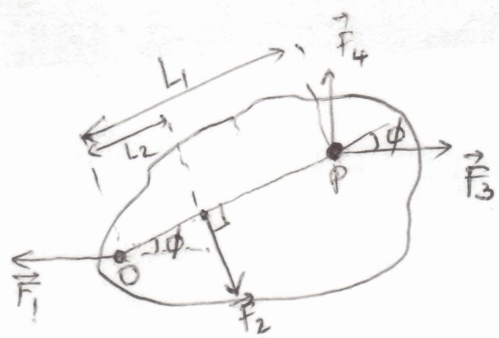
$$\omega = \sqrt{\frac{K}{I}} \quad \begin{matrix} \leftarrow \text{torsional constant} \\ \leftarrow \text{mom. of inertia} \end{matrix}$$

$$\frac{\omega_f}{\omega_i} = 0.8 = \frac{\sqrt{\frac{\cancel{K}}{I_f}}}{\sqrt{\frac{\cancel{K}}{I_i}}} = \sqrt{\frac{I_i}{I_f}} \rightarrow 0.8^2 = \frac{I_i}{I_f} = \frac{\frac{1}{12}ML^2}{\frac{1}{12}ML^2 + 2m\left(\frac{L}{2}\right)}$$

$$0.8^2 = \frac{\frac{1}{12}M}{\frac{1}{12}M + \frac{2\times 75}{4}} \rightarrow \frac{0.8^2}{12}M + 0.8^2\,\frac{75}{2} = \frac{1}{12}M$$

$$\rightarrow \frac{1}{12}M(1 - 0.8^2) = 0.8^2\,\frac{75}{2} \rightarrow M = \frac{12\times 0.8^2\times 75}{(1-0.8^2)\,2} = 800\,\text{kg}.$$

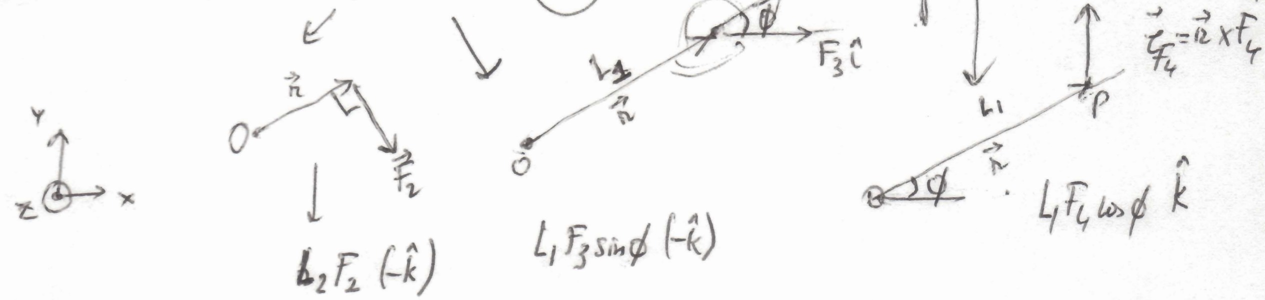
12.17



Static equilibrium $\sum_i \tau_i = 0$

4 forces

a) Pivot at O → $\vec\tau_{F_2} + \vec\tau_{F_3} + \vec\tau_{F_4} = 0$



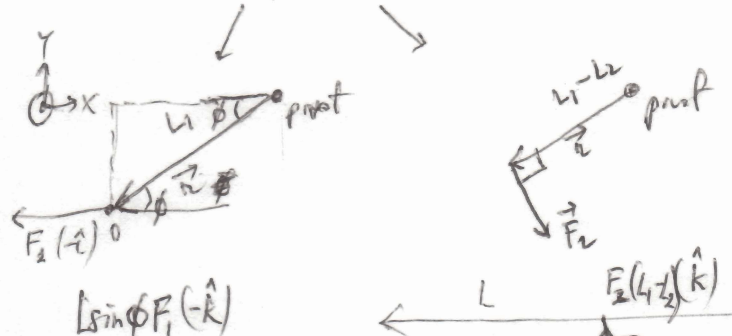
$$\boxed{-L_2F_2 - L_1F_3\sin\phi + L_1F_4\cos\phi = 0}$$

$\sin(180-\phi) = \sin\phi$

Solution in book has switched lengths.

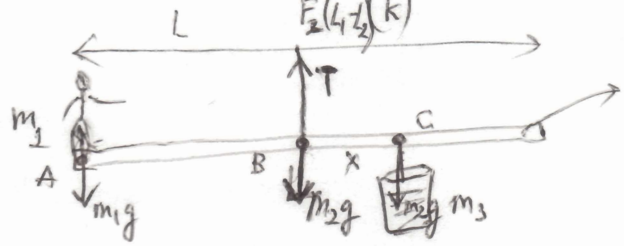
b) Pivot at P: no torques from $\vec F_3$ & $\vec F_4$ (these $\vec r$'s are 0)

$\vec\tau_{F_1} + \vec\tau_{F_2} = 0$



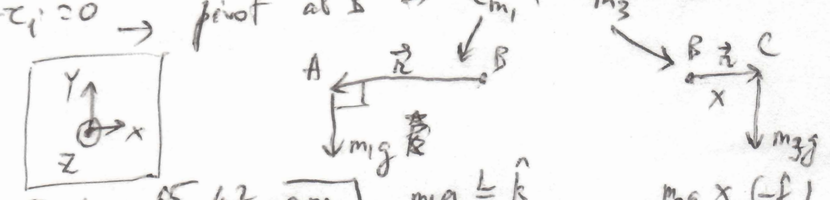
$$\boxed{-L\sin\phi F_1 + F_2(L_1-L_2) = 0}$$

12.23



$L = 4.2$ m
$m_1 = 65$ kg.
$m_3 = 190$ kg.

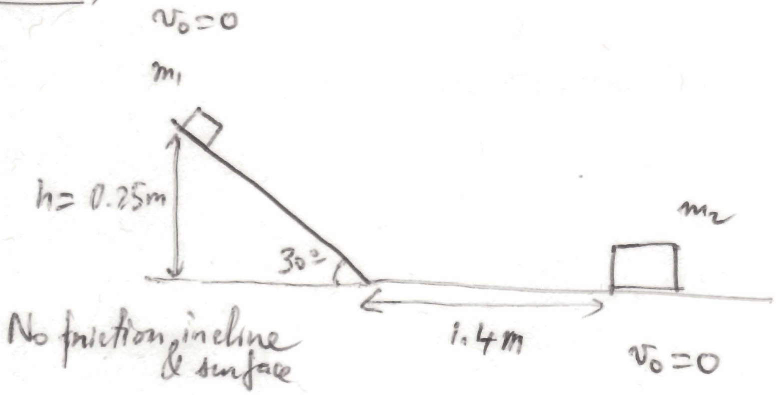
Static equilibrium of Beam: on beam
$\sum_i F_i = 0 \to T - m_1g - m_2g - m_3g = 0$
$\sum_i \tau_i = 0 \to$ pivot at B → $\vec\tau_{m_1} + \vec\tau_{m_3} = 0$



$m_1 g\frac{L}{2} - m_3 g x = 0 \to x = \frac{m_1}{m_3}\frac{L}{2} = \frac{65}{190}\frac{4.2}{2} = \boxed{0.72 m}$

13.45; 14.61
9.71; 13.64; 13.70
12.28; 12; 29;

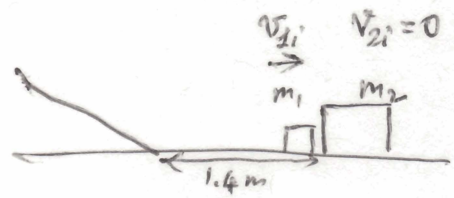
9.71:



$m_1 = 0.2kg$
$m_2 = 0.8kg$
Elastic collision
Conserv: $\begin{cases} \text{momentum} \\ KE \end{cases}$

$\underbrace{\text{After 1}^{st}\text{ elastic collision}}$ when will they collide again?

m_1 return to incline then back down while m_2 starts moving slowly to the right.

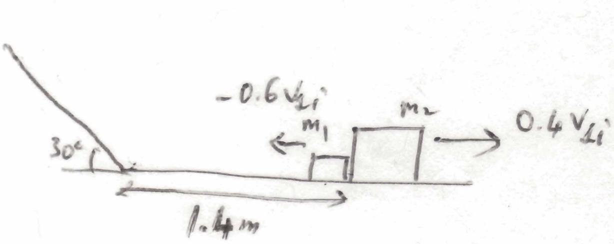


<u>1st collision</u>: 1D elastic collision

$$\begin{cases} v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i} + \dfrac{2m_2}{m_1 + m_2} \cancel{v_{2i}}^{\,0} \\ v_{2f} = \dfrac{2m_1}{m_1 + m_2} v_{1i} + \dfrac{m_2 - m_1}{m_1 + m_2} \cancel{v_{2i}}^{\,0} \end{cases}$$

Need v_{1i}: $\rightarrow$ $\cancel{m_1} gh = \frac{1}{2} \cancel{m_1} v_{1i}^2 \rightarrow \boxed{\begin{aligned} v_{1i} &= \sqrt{2gh} \\ &= \sqrt{2 \times 9.8 \times 0.25} \\ &= 2.2 \text{ m/s} \end{aligned}}$ (no friction → same speed throughout 1.4 m distance)

$$\rightarrow \begin{cases} v_{1f} = \dfrac{0.2 - 0.8}{1} v_{1i} = -0.6 v_{1i} \\ v_{2f} = \dfrac{0.4}{1} v_{1i} = 0.4 v_{1i} \end{cases}$$

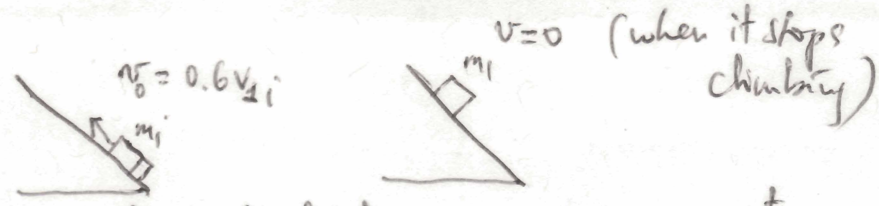


<u>After 1st collision</u>:

m_2: $\begin{cases} \text{back to bottom of incline: } t_a = \dfrac{1.4}{0.6 v_{1i}} \\ \text{up incline:} \end{cases}$

$\rightarrow m_1 g \sin 30 \rightarrow F_{net} = m_1 a$

$- \cancel{m_1} g \sin 30 = \cancel{m_1} a \rightarrow \boxed{a = -g \sin 30}$



constant deceleration: $v = v_0 + a t_b$

$$0 = 0.6v_{1i} - g\sin 30\, t_b$$

up incline: $t_b = \dfrac{0.6v_{1i}}{g\sin 30} = \dfrac{1.2v_{1i}}{g}$, $\sin 30 = \frac{1}{2}$

m_1
- back to bottom incline: $t_a = \dfrac{1.4}{0.6v_{1i}}$
- up & down incline: $2t_b = \dfrac{2.4v_{1i}}{g}$
- starting from bottom incline: trying to catch up with m_2 for the 2nd collision:

separation b/w m_1 & m_2: $1.4m + v_2(t_a + 2t_b) = 1.4 + 0.4v_{1i}\left(\dfrac{1.4}{0.6v_{1i}} + \dfrac{2.4v_{1i}}{g}\right)$

$$= 1.4 + 0.4\left(\frac{1.4}{0.6} + \frac{2.4 \times v_{1i}^2}{9.81}\right)$$

$$= 1.4 + 0.4\left(\frac{1.4}{0.6} + \frac{2.4 \times 2.2^2}{9.81}\right) = 2.8\ m$$

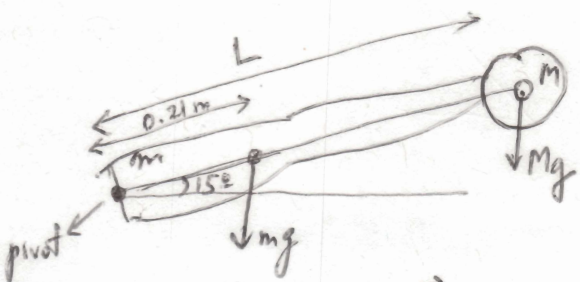
From now on: $\begin{cases} m_1: & x_1 = v_1 t \\ m_2: & x_2 = 2.8 + v_2 t \end{cases}$

2nd collision happens when $x_2 = x_1 \rightarrow 0.6v_{1i}t = 2.8 + 0.4v_{1i}t$

$$\rightarrow t = \frac{2.8}{0.2 \times 2.2} = 6.36\ s$$

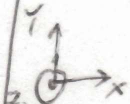
12.28

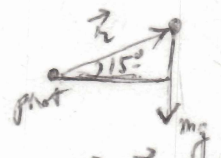
$m = 4.2 kg$
$L = 0.56 m$
$M = 6 kg$

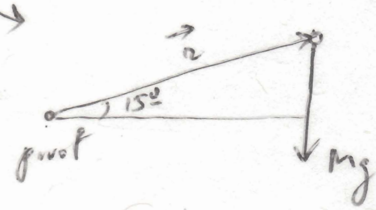


arm 15° above horizontal

a) $\vec{\tau}_{\text{about shoulder}} = \vec{\tau}_m + \vec{\tau}_M$







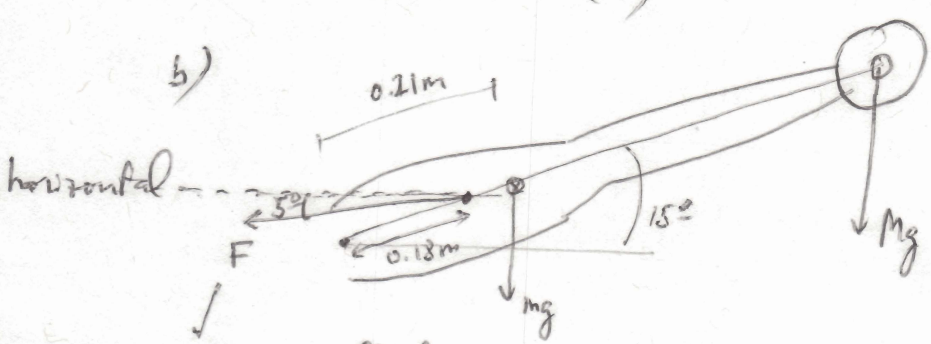
$\vec{r} \times \vec{F}$

$0.21 \cos 15 \, mg \, (-\hat{k})$ $\quad\quad$ $0.56 \cos 15 \, Mg \, (-\hat{k})$

$9.81 \cos 15 \left[0.21 \times 4.2 + 0.56 \times 6 \right] (-\hat{k})$

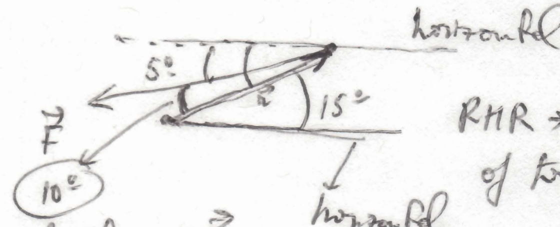
$= 40.2 \text{ Nm } (-\hat{k})$

b)

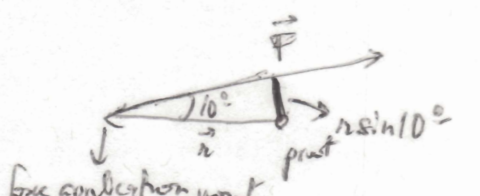


Force by deltoid muscle

Static equilibrium: $\sum_i \tau_i = 0 \rightarrow \vec{\tau}_F = 40.2 \text{ Nm } (\hat{k})$



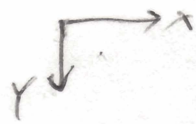
RHR → direction of torque is $+\hat{k}$!



b/w direction of $\vec{F}$ & direction of $\vec{r}$ → $\tau_F = r \sin 10° \, F$

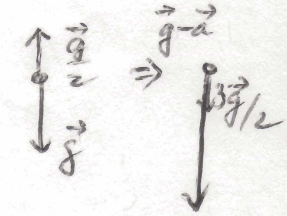
$$F = \frac{\tau_F}{r \sin 10°} = \frac{40.2}{0.18 \sin 10°} = 1286 N$$

13.45 **Pendulum in a rocket**: SHM $\rightarrow \omega = \sqrt{\frac{g}{L}}$



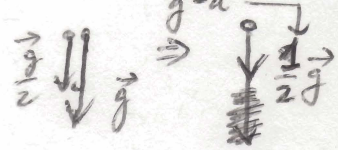
a) Rocket at rest : $T_a = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$ $\quad \downarrow \vec{g} \quad \vec{g} = g\hat{j}$

b) Rocket accelerates upward at $\vec{a} = \frac{g}{2}(-\hat{j})$



$T_b = 2\pi\sqrt{\frac{L}{|\vec{g}-\vec{a}|}}$ $\qquad T_b = 2\pi\sqrt{\frac{L}{3\frac{g}{2}}} = 2\pi\sqrt{\frac{2L}{3g}}$

c) Rocket accelerates downward at $\vec{a} = \frac{g}{2}\hat{j}$



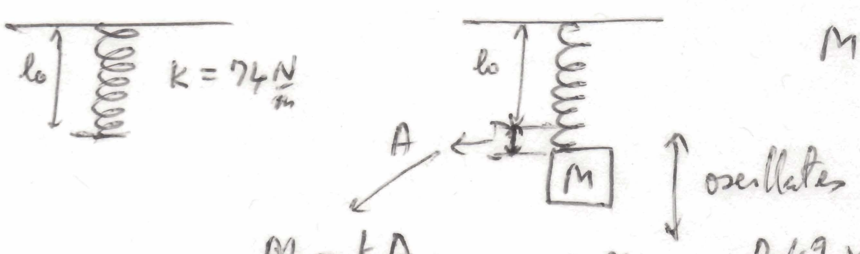
$T_c = 2\pi\sqrt{\frac{L}{\frac{g}{2}}} = 2\pi\sqrt{\frac{2L}{g}}$

passenger inside elevator: $\begin{cases} \text{elevator up: you feel heavier} \\ \text{elevator down: " " lighter.} \end{cases}$

d) Rocket in free fall: $\vec{a} = g\hat{j} \rightarrow \vec{g} - \vec{a} = 0$

$T = 2\pi\sqrt{\frac{L}{0}} \rightarrow \infty \rightarrow$ pendulum ceases to oscillate.

13.64 a)



$M = 0.49\,kg$

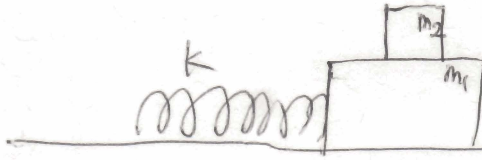
$Mg = kA \rightarrow A = \frac{Mg}{k} = \frac{0.49 \times 9.81}{74} = 0.065\,m$

b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{M}}} = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{0.49}{74}} = 0.5\,s.$

13.90

$m_1 = 0.5\,kg$

$k = 8.7\,N/m$



← friction (μ_s)?

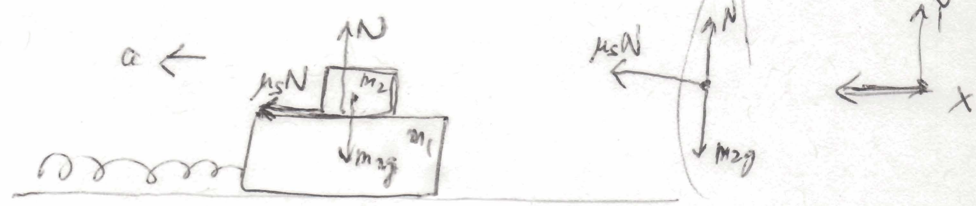
← No friction here

System: m_1 & m_2 & spring oscillate w/ $T = 1.8\,s$

$x = A = 0.35\,m \rightarrow m_2$ starts to slip

SHM → mass & spring → $\omega = \sqrt{\dfrac{k}{m_1+m_2}} \rightarrow T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m_1+m_2}{K}}$

μ_s: Focus on m_2 → Forces on m_2:



→ 2nd Newton's Law: $F_{net} = m_2 a$

$\mu_s N = m_2 a$

$\mu_s \cancel{m_2} g = \cancel{m_2} a \rightarrow \boxed{a = \mu_s g}$ is acceleration of m_1 & m_2 when m_1 is not slipping

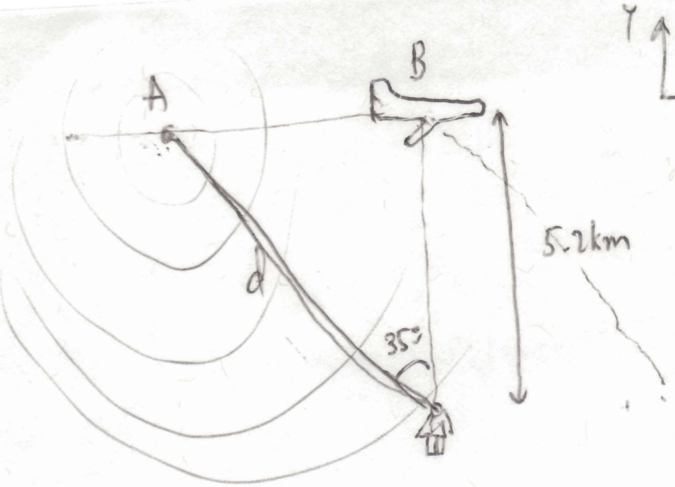
Info to find μ_s: m_2 starts slipping at $x = A = 0.35\,m$

SHM: $x = A\cos(\omega t) \rightarrow v = \dfrac{dx}{dt} = -A\omega\sin\omega t \rightarrow \boxed{a = \dfrac{dv}{dt} = -A\omega^2\cos\omega t}$

$x = A \rightarrow \cos\omega t = 1 \rightarrow a = -A\omega^2 \rightarrow A\omega^2 = \mu_s g$

$\rightarrow \mu_s = \dfrac{A\omega^2}{g} = \dfrac{0.35\left(\dfrac{2\pi}{1.8}\right)^2}{9.81} = 0.44$

14.61



Y, x

$v_{sound} = 330\, m/s$

v_{plane} ?

sound: wave: propagation of disturbance in air density, not instantaneous
→ but at average speed of 330 m/s

source is traveling along x. if source was static: • starts blinking & beeping at same time.

$v_{light} = c = 3 \times 10^8\, m/s$

$v_{sound} = 330 \frac{m}{s}$

see it before you hear it.

plane: uniform motion: $v_{plane} = \frac{AB}{t}$ → t: time for sound to travel distance d → $t = \frac{d}{v_s}$

$$v_{plane} = \frac{d \sin 35°}{\frac{d}{v_s}} = v_s \sin 35° = 330 \sin 35° = 189\, m/s$$