b) How fast are you moving?

$$v_{mx} = -\frac{4.5}{65} 12 \cos(37.78°) = -0.658\ m/s$$

→ Collisions $\begin{cases} \text{Inelastic: 2 colliding objects stick together after collision: KE is not conserved.} \\ \text{Elastic: KE is conserved} \end{cases}$

↓

$\vec{F}_{net,\,external} = 0 \rightarrow \vec{P}_i = \vec{P}_f$

$\boxed{\text{1D Elastic Collision}}$

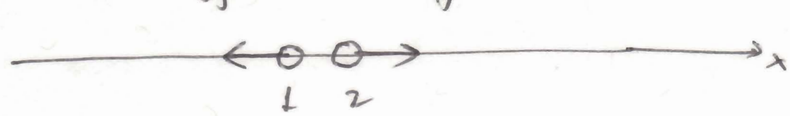
$v_{1i} = 6.9 \times 10^6\ m/s$ $\qquad$ $v_{2i} = -11 \times 10^6\ m/s$

initial

$m_1 = m_2 = m$

v_{1f} ? $\qquad$ v_{2f} ?

final



2 unknowns: v_{1f} & v_{2f} → 2 equations $\begin{cases} 1) & m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ 2) & \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \end{cases}$

↓

$m_1 = m_2 = m \rightarrow \begin{cases} 1) & v_{1i} + v_{2i} = v_{1f} + v_{2f} \\ 2) & v_{1i}^2 + v_{2i}^2 = v_{1f}^2 + v_{2f}^2 \end{cases}$

1) $\underbrace{(6.9 - 11) \times 10^6}_{-4.1 \times 10^6} = v_{1f} + v_{2f} \rightarrow 4.1^2 \times 10^{12} = \underbrace{v_{1f}^2 + v_{2f}^2}_{(6.9^2 + 11^2)10^{12}} + 2 v_{1f} v_{2f}$

2) $(6.9^2 + 11^2) \times 10^{12} = v_{1f}^2 + v_{2f}^2$

$$\rightarrow \quad v_{1f} = \frac{[4.1^2 - (6.9^2 + 11^2)]10^{12}}{2\,v_{2f}} = \frac{-151.8 \times 10^{12}}{2\,v_{2f}} = \frac{-75.9 \times 10^{12}}{v_{2f}}$$

1) $$-4.1 \times 10^6 = -\frac{75.9 \times 10^{12}}{v_{2f}} + v_{2f} = \frac{-75.9 \times 10^{12} + v_{2f}^2}{v_{2f}}$$

$$-4.1 \times 10^6\, v_{2f} = -75.9 \times 10^{12} + v_{2f}^2$$

$$v_{2f}^2 + 4.1 \times 10^6\, v_{2f} - 75.9 \times 10^{12} = 0$$

$$v_{2f} = \frac{-4.1 \times 10^6 \pm \sqrt{4.1^2 \times 10^{12} + 4 \times 75.9 \times 10^{12}}}{2}$$

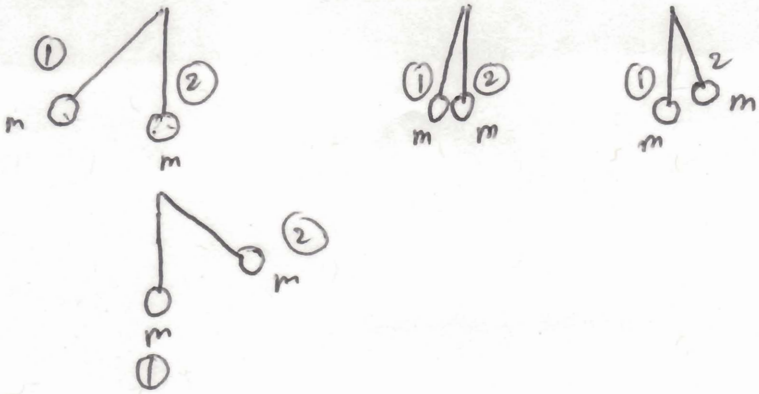
$$= \frac{-4.1 \times 10^6 \pm 17.9 \times 10^6}{2} \begin{cases} -11 \times 10^6 \text{ m/s} \\ \boxed{+6.9 \times 10^6 \text{ m/s}} \end{cases}$$

m_2 going in $+X$
after collision
since $m_2 = m_1$!

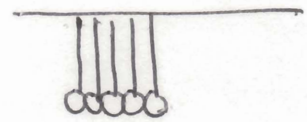
$$\rightarrow \quad v_{1f} = \frac{-75.9 \times 10^{12}}{v_{2f}} = \frac{-75.9 \times 10^{12}}{6.9 \times 10^6} = \boxed{-11 \times 10^6 \text{ m/s}}$$

F

→ exchange of speeds in 1D elastic collision !



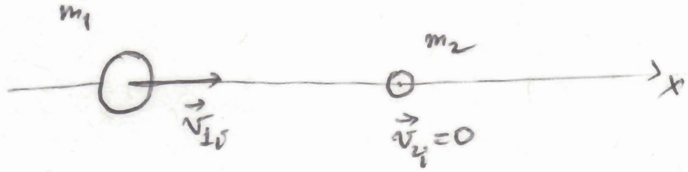
exchange of speeds in 1D elastic collision (all same mass)



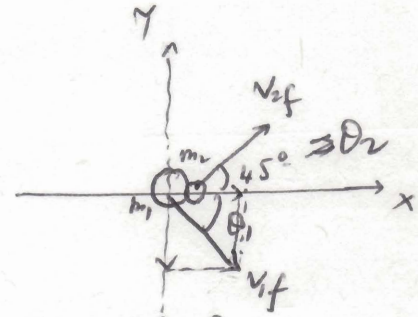
2D Elastic Collision :

$m_1 = 3.2\,kg$; $m_2 = 0.35\,kg$

$v_{1i} = 1\,m/s$ $v_{2i} = 0$



initial



final

3 unknowns $\left\{ \begin{array}{l} v_{1f} \;\&\; \theta_1 \\ v_{2f} \end{array} \right\} \to$ need 3 equations $\begin{cases} 1)\ \vec{P}_{ix} = \vec{P}_{fx} \\ 2)\ \vec{P}_{iy} = \vec{P}_{fy} \\ 3)\ KE_i = KE_f \end{cases}$

$\cos 45 = \sin 45 = \frac{1}{\sqrt{2}}$

1) $m_1 1 + m_2 \cdot 0 = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos 45°$

$\checkmark \quad \checkmark \quad \checkmark$

2) $0 = m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin 45°$

$\checkmark \quad \checkmark \quad \checkmark$

3) $\frac{1}{2} m_1 1^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

$\checkmark \quad \checkmark$

All physics is already written down $\to$ math manipulation :

2) $\to$ $m_2 v_{2f} \frac{1}{\sqrt{2}} = - m_1 v_{1f} \sin\theta_1$

$\to$ 1) $\cancel{m_1} = \cancel{m_1} v_{1f} \cos\theta_1 - \cancel{m_1} v_{1f} \sin\theta_1$

$1 = v_{1f} (\cos\theta_1 - \sin\theta_1)$

1) $$m_1 - m_2 v_{2f} \frac{1}{\sqrt{2}} = m_1 v_{1f} \cos\theta_1$$

2) $$- m_2 v_{2f} \frac{1}{\sqrt{2}} = m_1 v_{1f} \sin\theta_1$$

$$①^2 + ②^2 \Rightarrow m_1^2 + \underbrace{\frac{1}{2} m_2^2 . v_{2f}^2} - \sqrt{2}\, m_1 m_2 v_{2f} + \underbrace{\frac{1}{2} m_2^2 v_{2f}^2} = m_1^2 v_{1f}^2 \underbrace{(\cos^2\theta_1 + \sin^2\theta_1)}_{1}$$

ⓐ $$\boxed{m_1^2 - \sqrt{2}\, m_1 m_2 v_{2f} = m_1^2 v_{1f}^2 - m_2^2 v_{2f}^2}$$

3) $$\left[m_1 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \right] \times m_1$$

ⓑ $$\boxed{m_1^2 = m_1^2 v_{1f}^2 + m_1 m_2 v_{2f}^2}$$

ⓐ-ⓑ $$\left[- \sqrt{2}\, m_1 \cancel{m_2} v_{2f} = - m_2^{\cancel{2}} v_{2f}^2 - m_1 \cancel{m_2} v_{2f}^2 \right] \times (-1)$$

$$m_1 \cancel{v_{2f}} \sqrt{2} = \underbrace{m_2 v_{2f}^2 + m_1 v_{2f}^2}_{(m_1 + m_2) v_{2f}^{\cancel{2}}}$$

$$\boxed{m_1 \sqrt{2} = (m_1 + m_2) v_{2f}} \rightarrow v_{2f} = \frac{m_1}{m_1 + m_2} \sqrt{2}$$

$$= \frac{3.2}{3.2 + 0.35} \sqrt{2}$$

$$\boxed{v_{2f} = 1.27 \, m/s}$$

③ $$m_1 - m_2 v_{2f}^2 = m_1 v_{1f}^2 \rightarrow v_{1f} = \sqrt{\frac{m_1 - m_2 v_{2f}^2}{m_1}}$$

$$= \sqrt{\frac{3.2 - 0.35 \times 1.27^2}{3.2}} = 0.908 \, m/s$$

$$\boxed{v_{1f} = 0.908 \, m/s}$$

2) $$\sin\theta_1 = - \frac{m_2 v_{2f}}{m_1 v_{1f}} \frac{1}{\sqrt{2}}$$

$$\theta_1 = \sin^{-1} \left[- \frac{0.35 \times 1.27}{3.2 \times 0.908} \frac{1}{\sqrt{2}} \right] \rightarrow \boxed{\theta_1 = -6.21^\circ}.$$

Ch. 10 Rotational Motion

Constant acceleration

Linear Motion

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at \quad (1)$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad (2)$$

$$\frac{v^2 - v_0^2}{x - x_0} = 2a \quad (3)$$

$$F = ma$$

Rotational Motion

$$\bar{\omega} = \frac{\omega_0 + \omega}{2}$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\frac{\omega^2 - \omega_0^2}{\theta - \theta_0} = 2\alpha$$

$$\tau = I\alpha$$

ω : "omega" : angular velocity

α : "alpha" : angular acceleration

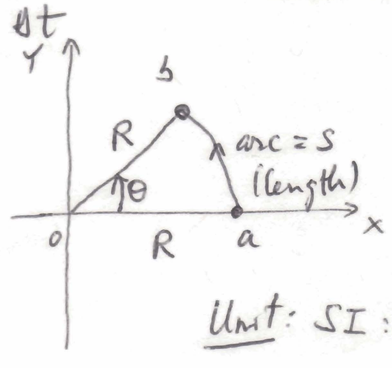
θ : "theta" : angle

τ : "tau" : torque

I : Moment of inertia

Angular velocity : ω

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \rightarrow \text{instantaneous angular velocity} : \omega = \frac{d\theta}{dt}$$



$$\theta = \frac{arc}{R} = \frac{s}{R}$$

linear speed along the circular path

$$\omega = \frac{d}{dt}\theta = \frac{d}{dt}\left(\frac{s}{R}\right) = \frac{1}{R}\frac{ds}{dt} = \frac{v}{R}$$

Unit: SI : $\frac{1}{s}$ or $\frac{rad}{s}$ or s^{-1} ; alternative : $\frac{°}{s}$; rpm (rev. per minute)

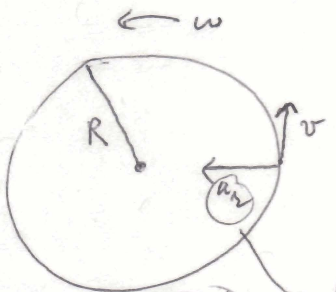
Angular acceleration:

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} \quad \rightarrow \quad \alpha = \frac{d\omega}{dt}$$

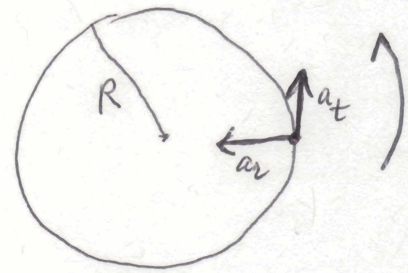
Unit: SI: $\frac{1}{s^2}$; $\frac{rad}{s^2}$

Alternative: $\frac{°}{s^2}$ or $\frac{rev}{min^2}$, ...

2D



More generally:



changing direction of v

$v = R\omega$

UCM

non-UCM

a_r = radial acceleration

a_t = tangential acceleration

$$\boxed{\begin{aligned} a_r &= \frac{v^2}{R} = \frac{R^2\omega^2}{R} = R\omega^2 \\ a_t &= \frac{dv}{dt} = \frac{d(R\omega)}{dt} = R\frac{d\omega}{dt} = R\alpha \end{aligned}}$$

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F} \equiv rF\sin\theta \ \hat{\tau}$$

θ: angle b/w $\vec{r}$ & $\vec{F}$

$\vec{r}$: from pivot to force application point

$\vec{F}$: Force applied

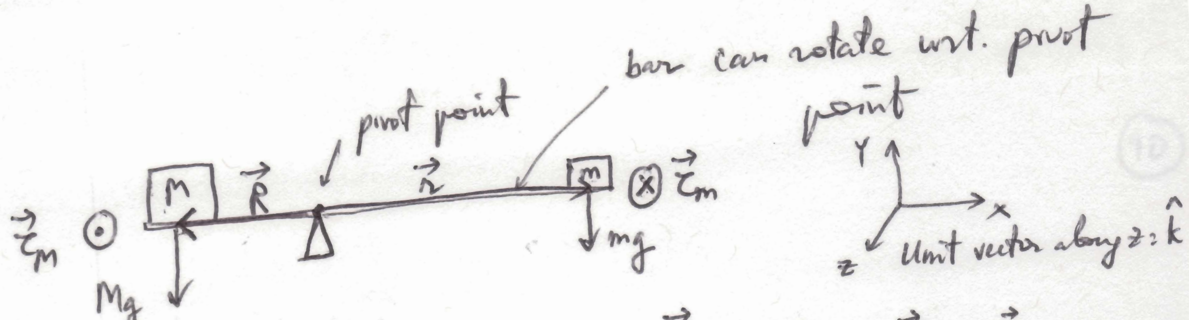
$\times$: "$\boxed{\text{cross-product}}$": a product b/w two vectors that is another vector

$\hat{\tau}$ = a unit vector normal (perpendicular) to the plane formed by $\vec{r}$ & $\vec{F}$

Torque is a vector that is perpendicular to both $\vec{r}$ & $\vec{F}$.

Unit: SI: Nm

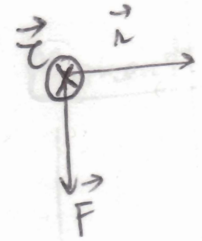
Example :



On bar: $\vec{\tau}_{Total} = \vec{\tau}_M + \vec{\tau}_m = \vec{R} \times M\vec{g} + \vec{r} \times m\vec{g}$
$= RMg\,\hat{k} - rmg\,\hat{k} = (RM - rm)g\,\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F}$: is $\perp$ to both $\vec{r}$ & $\vec{F}$

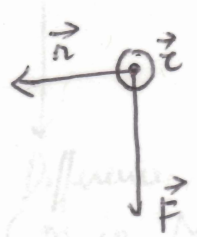
e.g.:



$\vec{r}$ & $\vec{F}$ in plane of screen: $\vec{\tau}$ direction (or direction of $\hat{\tau}$) is given by <u>right hand rule</u>

RHR: right hand fingers along 1st vector ($\vec{r}$), as you turn these fingers toward $\vec{F}$ ~~your palm~~, thumb indicates direction of torque.

$\rightarrow$ <u>here torque is into the screen.</u> $\otimes$



$\rightarrow$ <u>here torque is out of screen.</u> $\odot$

Total torque on bar is 0 when $RM = rm \Rightarrow \boxed{\dfrac{R}{r} = \dfrac{m}{M}}$

Analog of 2nd Newton's Law:

$$F = m a \longrightarrow \tau = I \alpha$$

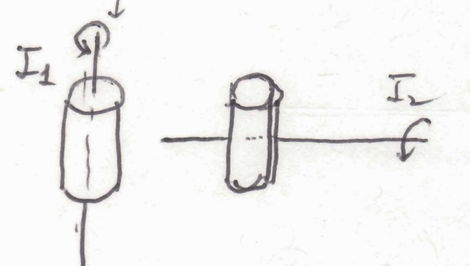
τ: Torque I: Moment of Inertia α: angular acceleration

Moment of inertia:

$$I = \sum_i m_i r_i^2$$

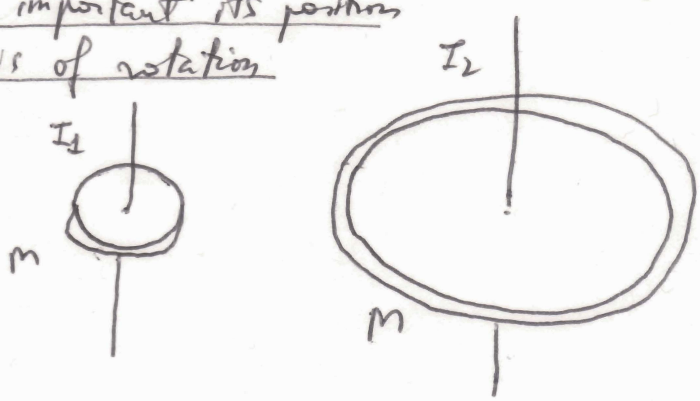
r_i: $\boxed{\text{position of component } i \text{ wrt axis of rotation}}$

$$I = \int dm\, r^2$$



Different I's depending on axis of rotation.

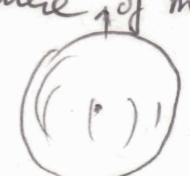
Difference wrt. mass inertia (m in Newton's Law): <u>also is important its position wrt axis of rotation</u>



$$I_1 < I_2$$

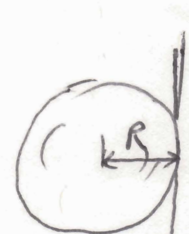
Parallel axis theorem:

Solid sphere of mass M; radius R



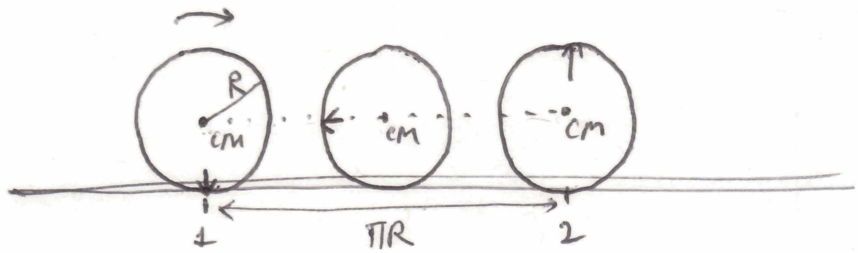
$$I_1 = \frac{2}{5} M R^2$$

R: sep. b/w axes



$$I_2 = I_1 + MR^2 = \frac{7}{5} MR^2$$

Rolling Motion: → non skidding e.g. car moves under rolling motion of wheels under normal condition.



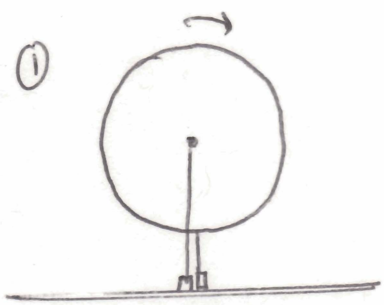
For rolling (non skidding) motion: sep. b/w 1 & 2 is πR!

CM moving in linear motion b/w 1 & 2:

$$\left\{ \begin{aligned} v_{CM} &= \frac{\pi R}{\Delta t} \\ \omega &= \frac{\Delta\theta}{\Delta t} = \frac{\pi}{\Delta t} \end{aligned} \right\} \quad \boxed{v_{CM} = \omega R} \text{ For rolling motion.}$$

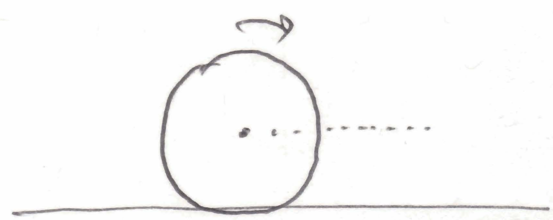
v_{CM} → translation, ωR → rotation

①



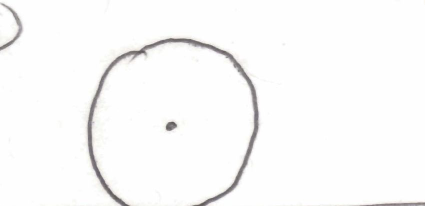
just rotation

③



Rolling motion: – Rotation wrt CM; – linear motion / translation of CM

②



skidding w/ no rotation

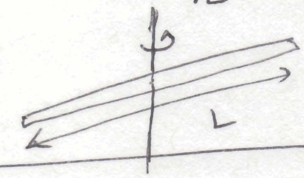
$$KE = \begin{cases} \text{linear motion ②} : \frac{1}{2} m v_{cm}^2 \\ \text{rolling motion ③} : \boxed{\frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2} \end{cases}$$

$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \left(\frac{v_{CM}}{R}\right)^2 = \frac{1}{2}\left(m + \frac{I}{R^2}\right) v_{cm}^2$$

compared to ② (skidding only) → rolling motion: extra inertia due rotation

Moment of inertia : $I = \alpha M R^2$

sphere wrt center axis : $\alpha = \frac{2}{5}$
cylinder wrt center axis : $\alpha = \frac{1}{2}$
thin rod of length L
wrt center axis : $\alpha = \frac{1}{12}$
↳ $I = \frac{1}{12} M L^2$



↳ Round and symmetrical object :

↳ Rolling motion of round & symmetrical object :

$$KE = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} \frac{I}{R^2} v_{cm}^2$$

$$\omega = \frac{v_{cm}}{R}$$

$$= \frac{1}{2}\left(M + \frac{I}{R^2}\right) v_{cm}^2 = \frac{1}{2}(M + \alpha M) v_{cm}^2$$

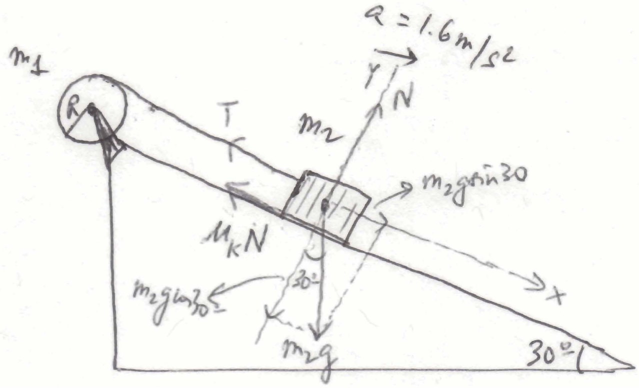
$$\frac{I}{R^2} = \alpha M$$

$$KE = \frac{1}{2} M \underbrace{(1+\alpha)} v_{cm}^2$$

Rolling (moving forward with rotation) → $KE = \frac{1}{2} M (1+\alpha) v_{cm}^2$
Skidding (moving forward without rotation) → $KE = \frac{1}{2} M v_{cm}^2$

↳ For a same Total KE → rolling → lower v_{cm} ! → ABS Braking System.
skidding

10.57



$m_2 = 2.4\,kg$
$m_1 = 0.85\,kg$
$R = 0.05\,m$
μ_k ? b/w block & slope

Two objects :

m_2 : FBD; Coord X&; Components for $m_2 g$

$$\begin{cases} F_{net_x} = m_2 g \sin 30° - T - \mu_k N = ma \\ F_{net_y} = N - m_2 g \cos 30° = 0 \end{cases}$$

$$\boxed{m_2 g (\sin 30°) - \mu_k m_2 g \cos 30° - T = m_2 a}$$

To get μ_k ; I need to find T.

m_1 : solid drum (cylinder): rotation wrt its center

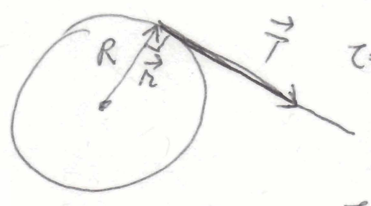
$I = \frac{1}{2} m_1 R^2$

accelerated : $\alpha = \frac{a_t}{R}$

$\alpha = \frac{a}{R}$

Analogy of 2nd Newton Law :

$$\boxed{\tau = I \alpha}$$



$\tau = rT \sin 90 = RT$

$$RT = I\alpha = \frac{1}{2} m_1 R^{\cancel{2}} \frac{a}{\cancel{R}}$$

$$\boxed{T = \frac{1}{2} m_1 a}$$

$$\mu_k = \frac{m_2 g \sin 30° - m_2 a - T}{m_2 g \cos 30°} = \frac{m_2 g \sin 30° - m_2 a - \frac{1}{2} m_1 a}{m_2 g \cos 30°}$$

$$= \frac{2.4 \times 9.81 \sin 30° - 2.4 \times 1.6 - \frac{1}{2} 0.85 \times 1.6}{2.4 \times 9.81 \times \cos 30°} = 0.36.$$

Ch 11: Rotational Vectors & Angular Momentum.

Linear

$$\boxed{\vec{F} = m\vec{a}}$$

↳ More generally:

$$\vec{F}_{net,\ external} = \frac{d\vec{p}}{dt}$$

$\vec{p}$: linear momentum

$\vec{p} = m\vec{v}$

$\longleftrightarrow$

Rotational

$$\vec{\tau} = \vec{r} \times \vec{F} \rightarrow \boxed{\vec{\tau} = I\vec{\alpha}}$$

More generally:

$$\vec{\tau}_{net,\ external} = \frac{d\vec{L}}{dt}$$

$\vec{L}$: angular momentum

$\vec{L} = \vec{r} \times \vec{p}$

↓

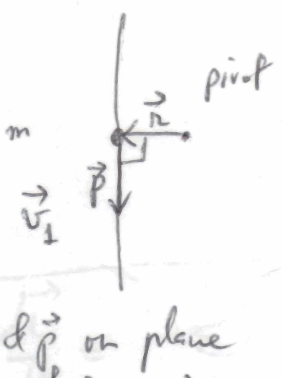
cross product.

$\vec{L}$ is a vector that is perpendicular to both $\vec{r}$ & $\vec{p}$

Angular momentum $\vec{L}$:

$L = rp\sin\theta$ (θ: angle b/w $\vec{r}$ & $\vec{p}$)

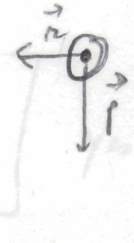
Direction is given by the RHR: right hand fingers along 1st vector of the cross-product: $\vec{r}$, as these fingers turn toward the 2nd vector $\vec{p}$, thumb indicates direction of $\vec{L}$.



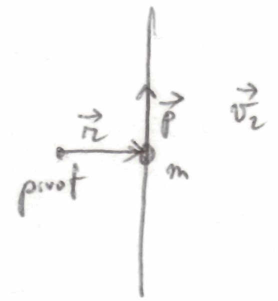
($\vec{r}$ & $\vec{p}$ on plane of screen)

What is the angular momentum of m wrt pivot?

$$\vec{L} = \vec{r} \times \vec{p} = \begin{cases} L = rp \\ \text{Direction:} \end{cases}$$

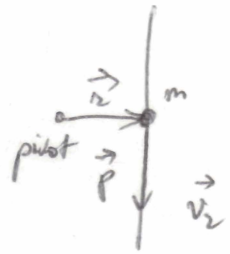


≡ out of screen.

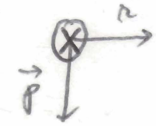


$$\vec{L} = \vec{r} \times \vec{p} = \begin{cases} L = rp \\ \text{Direction:} \end{cases}$$

out of screen



$$\vec{L} = \begin{cases} L = rp \\ \text{Direction} \end{cases}$$



into page

Do we have a good reason to believe in $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$?

$$\vec{L} = \sum_{i} \vec{L}_i = \sum_i \vec{r}_i \times \vec{p}_i$$

i → components of a system

$$\frac{d\vec{L}}{dt} = \sum_i \frac{d}{dt}\left(\vec{r}_i \times \vec{p}_i\right) = \sum_i \left(\frac{d\vec{r}_i}{dt} \times \vec{p}_i + \vec{r}_i \times \frac{d\vec{p}_i}{dt} \right)$$

$$\frac{d\vec{r}_i}{dt} \times \vec{p}_i = \vec{v}_i \times m_i \vec{v}_i = m_i \underbrace{(\vec{v}_i \times \vec{v}_i)}_{0}$$

$$\underbrace{\vec{r}_i \times \vec{F}_i}_{\vec{\tau}_i} \leftarrow \text{2nd Newton's Law}$$

$$\boxed{\frac{d\vec{L}}{dt} = \underbrace{\sum_i \vec{\tau}_i}_{\vec{\tau}_{net}}} \Rightarrow \boxed{\vec{\tau}_{net} = \frac{d\vec{L}}{dt}} \text{ yes.}$$

Importance consequence:

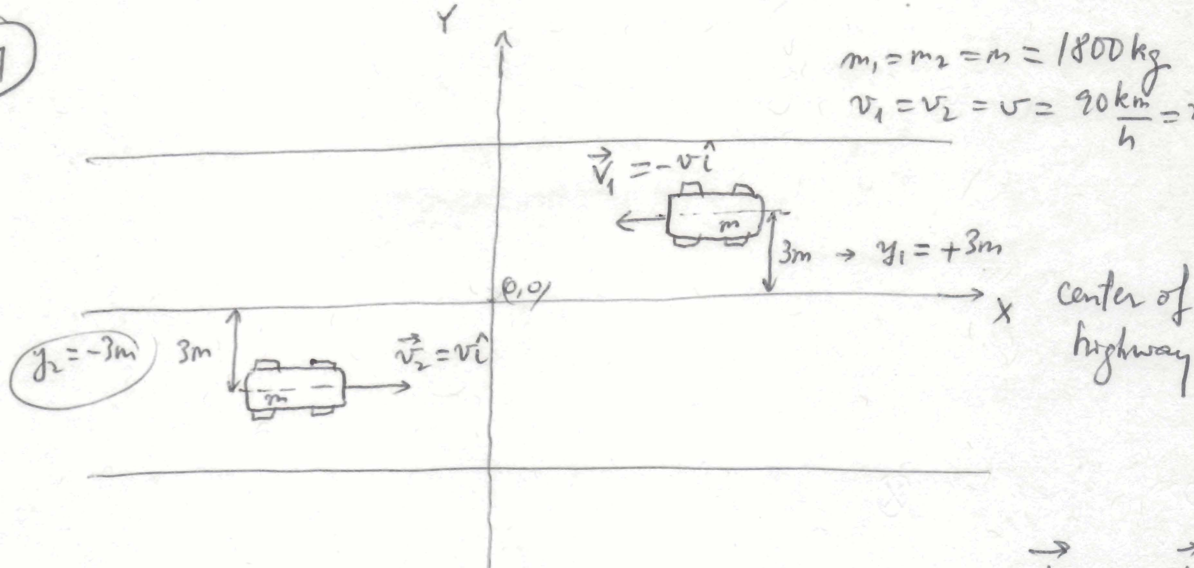
if $\vec{\tau}_{net} = 0 \rightarrow \vec{L}$ is conserved

$$\boxed{\vec{L}_i = \vec{L}_f}$$

$$\boxed{\text{Summary} \begin{cases} \vec{F}_{net,\,external} = 0 \rightarrow \vec{P}_i = \vec{P}_f \\ \vec{\tau}_{net,\,external} = 0 \rightarrow \vec{L}_i = \vec{L}_f \end{cases}}$$

11.37

$m_1 = m_2 = m = 1800\,kg$
$v_1 = v_2 = v = 90\frac{km}{h} = 25\,m/s$



System with 2 components: car 1 & car 2 → $\vec{L}_{Total} = \vec{L}_1 + \vec{L}_2$

$$\vec{L}_{Total} = \underbrace{\vec{r}_1 \times \vec{p}_1}_{\text{on XY plane}} + \underbrace{\vec{r}_2 \times \vec{p}_2}_{\text{on XY plane}}$$

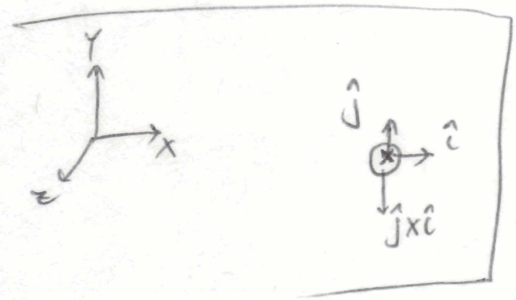
$(\vec{p}_i = m_i \vec{v}_i)$
$\vec{p}_i \parallel \vec{v}_i$

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$ || $\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$

$\vec{p}_1 = m_1\vec{v}_1 = -mv\,\hat{i}$ || $\vec{p}_2 = mv\,\hat{i}$

$$\vec{L}_{total} = (x_1\hat{i} + y_1\hat{j}) \times (-mv\hat{i}) + (x_2\hat{i} + y_2\hat{j}) \times (mv\hat{i})$$

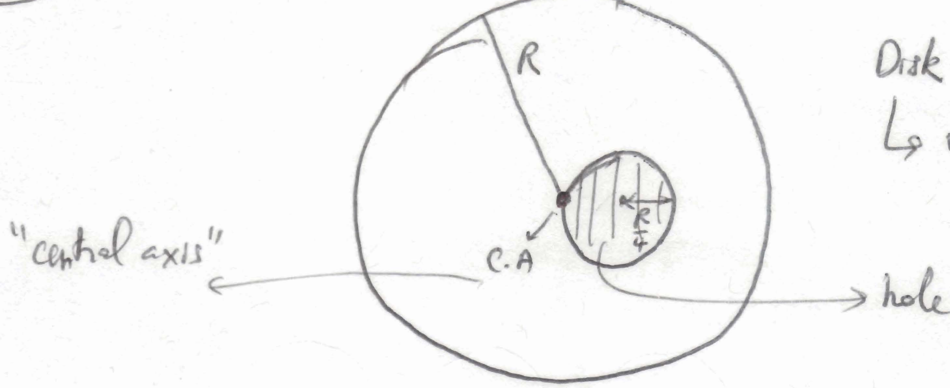
$$= -mvy_1\underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}} + mvy_2\underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}}$$



$$\Rightarrow \vec{L}_{Total} = 3mv\,\hat{k} + 3mv\,\hat{k}$$

$$= 6mv\,\hat{k} = 6 \times 1800 \times 25\,\hat{k}$$

$$= 27 \times 10^4\ Js \text{ or } \frac{kg\,m^2}{s}$$

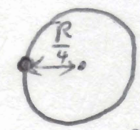


Disk : R , M

↳ wrt central axis $I = \frac{1}{2} MR^2$

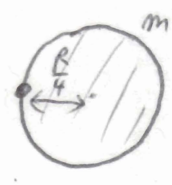
Find I' with the hole ; wrt central axis

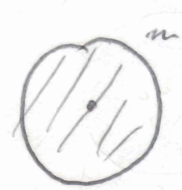
$$I' = I - I_{Hole} = \underbrace{\frac{1}{2}MR^2}_{\text{wrt c.a}} - \underbrace{I_{Hole}}_{\text{wrt. c.a.}}$$

↳ 

Mom. of inertia for a disk wrt axis at the edge :

Parallel axis theorem :



=


$+ \; m\left(\frac{R}{4}\right)^2$

$$\underset{\text{wrt c.a.}}{I_{Hole}} = \frac{1}{2}m\left(\frac{R}{4}\right)^2 + m\left(\frac{R}{4}\right)^2 = \frac{3}{2}m\frac{R^2}{16}$$

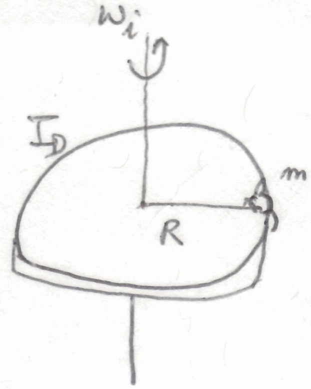
$$= \frac{3}{32}mR^2$$

Find m : mass a disk of radius $\frac{R}{4}$:

$$m = \frac{\pi\left(\frac{R}{4}\right)^2}{\pi R^2}M = \frac{M}{16} \longrightarrow \underset{\text{wrt C.a}}{I_{Hole}} = \frac{3}{32}\cdot\frac{M}{16}R^2$$

$$I' = \frac{1}{2}MR^2 - \frac{1}{2}M\frac{3R^2}{16^2} = \frac{1}{2}mR^2\underbrace{\left(1 - \frac{3}{16^2}\right)}_{0.988} = 0.494\,MR^2$$

11.40



$R = 0.25 m$

$I_D = 0.0154 \ kg m^2$

$w_i = 22 \ rpm$ (rev. per min)

$m = 19.5 g = 0.0195 \ kg$

a We expect $w_f > w_i$: with the mouse at the axis → it no longer contribute to the total moment of inertia:

$\vec{\tau}_{net, extend} = 0 \rightarrow \vec{L}_i = \vec{L}_f$

System of disk + mouse $\boxed{I_i w_i = I_f w_f}$

$\left(\tau = I\alpha \right.$

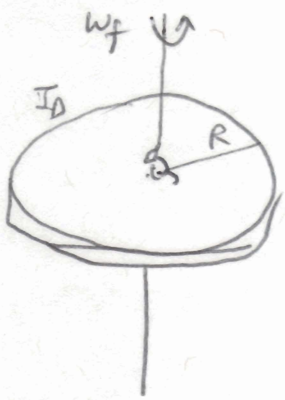
$\tau = \frac{dL}{dt}$

$\frac{dL}{dt} = I\alpha = I\frac{dw}{dt}$

$\boxed{\text{const } I}$

$\frac{dL}{dt} = \frac{d}{dt}(Iw)$

$\left. \boxed{L = Iw} \right)$



$$w_f = \frac{I_i}{I_f} w_i = \left| \frac{(I_D + mR^2)}{I_D} \right| 22 \ rpm$$

$$w_f = \frac{0.0154 + 0.0195 \times 0.25^2}{0.0154} 22 \ rpm$$

$$= 23.7 \ rpm.$$

b Work done by mouse: $= KE_f - KE_i$

$\boxed{I_i = I_f \frac{w_f}{w_i}}$

~~$I_i = I_f + mR^2$~~

$$= \frac{1}{2} I_f w_f^2 - \frac{1}{2} I_i w_i^2 = \frac{1}{2} I_f \left[w_f^2 - \frac{w_f}{\cancel{w_i}} w_i^{\cancel{2}} \right]$$

$$= \frac{1}{2} I_f w_f^2 \left[1 - \frac{w_i}{w_f} \right]$$

in SI:

$23.7 \frac{\cancel{rev}}{\cancel{min}} \cdot \frac{\cancel{min}}{60 s} \cdot \frac{2\pi \times 0.25}{\cancel{rev}} = [illegible] \ s^{-1}$
0.62

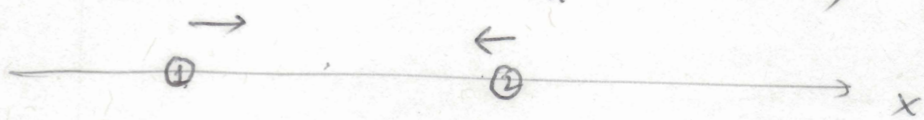
$$= \frac{1}{2} 0.0154 \times 0.62^2 \left[1 - \frac{22}{23.7} \right] = 2 \times 10^{-4} J$$

9.39 10.58

9.33 ; 9.57 ; 11.47 ; 11.51 ;

9.33 1D Elastic collision b/w 2 protons $m_1 = m_2 = m$

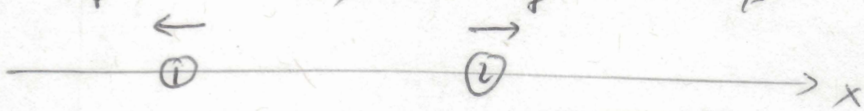
$\vec{v}_{1i} = 6.9\times10^6\, m/s\ \hat{i}$ $\vec{v}_{2i} = 11\times10^6\, m/s\ (-\hat{i})$



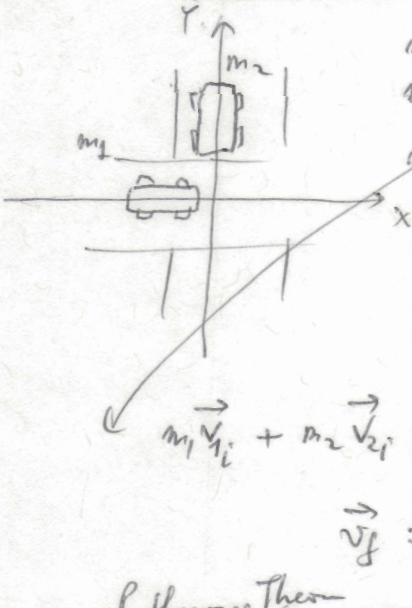
$$V_{1f} = \frac{m_1 - m_2}{m_1+m_2} V_{1i} + \frac{2m_2}{m_1+m_2} V_{2i} \overset{m_1=m_2}{=} \frac{2\cancel{m}}{2m} v_{2i}$$

$$V_{2f} = \frac{2m_1}{m_1+m_2} V_{1i} + \frac{m_2 - m_1}{m_1+m_2} V_{2i} = \frac{2\cancel{m}}{2\cancel{m}} v_{1i}$$

$\vec{v}_{1f} = 11\times10^6\, m/s\ (-\hat{i})$ $\vec{v}_{2f} = 6.9\times10^6\, m/s\ \hat{i}$



9.57 2D inelastic collision : $\boxed{\vec{P}_i = \vec{P}_f}$



$m_1 = 1200kg$
$m_2 = 2200kg$
moving together after collision, $\underbrace{\text{skid } 22m}$

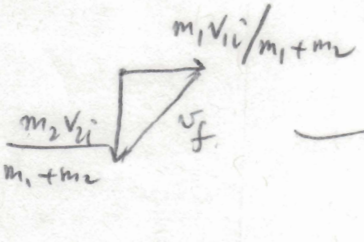
↳ come to a stop at the end of 22m due to friction. $\mu_K = 0.91$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1+m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1+m_2} = \frac{m_1 v_{1i}\hat{i} + m_2 v_{2i}\hat{j}}{m_1+m_2}$$

Pythagoras Theorem

$$\boxed{v_f^2 = \frac{m_1^2 v_{1i}^2 + m_2^2 v_{2i}^2}{(m_1+m_2)^2}} \quad (1)$$



Both cars use up their total KE after collision by skidding 22m:

with friction : $F_k = \mu_k N$ → work done on friction: $F_k d$

$= \mu_k N d = \mu_k (m_1+m_2) g d$

$$\boxed{\frac{1}{2}\cancel{(m_1+m_2)} v_f^2 = \mu_k \cancel{(m_1+m_2)} g d} \quad (2)$$

$$\boxed{v_f^2 = 2\mu_k g d} \quad (2)$$

(1) & (2) → $$\frac{m_1^2 v_{1i}^2 + m_2^2 v_{2i}^2}{(m_1+m_2)^2} = 2\mu_k g d$$

$$\boxed{v_{1i} = \frac{1}{m_1}\sqrt{2\mu_k g d (m_1+m_2)^2 - m_2^2 v_{2i}^2}}$$

<u>Check:</u> if at least one of the cars exceeded limit of 25 km/h

a) Assume car #2 (Buick) going at speed limit of 25 km/h $= 6.94 \frac{m}{s}$

$$\hookrightarrow v_{1i} = \frac{1}{1200}\sqrt{2\times 0.91\times 9.81\times 22\,(3400)^2 - 2200^2\times 6.94^2} = 54.66\, m/s$$
$$= 197\, km/h$$

b) Now switch: $m_2 = 1200\, kg$ (Toyota)
$m_1 = 2200\, kg$ (Buick)

Assume car #2 (Toyota) going at speed limit into intersection at 6.94 m/s

$$v_{1i} = \frac{1}{2200}\sqrt{2\times 0.91\times 9.81\times 22\times (3400)^2 - 1200^2\times 6.94^2} = 30.4\, m/s$$

$= 109\, km/h$

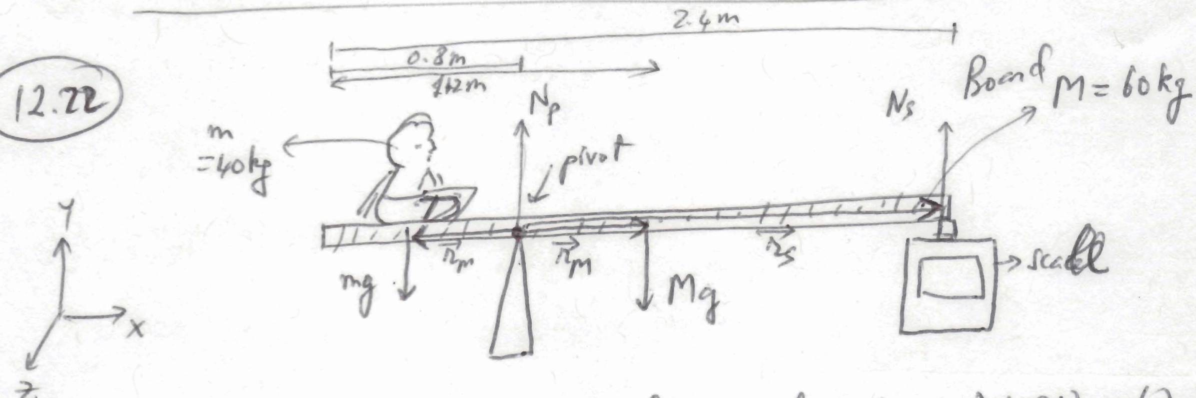
Ch12: Static Equilibrium:

→ Application of force & torque balance:

$$1)\ \sum_i \vec{F}_i = 0 \qquad 2)\ \sum_i \vec{\tau}_i = 0$$

System will not move nor rotate under these two conditions. → static equilibrium.

12.22



Location of child so scale reading is a) 100N b) 300N

a) Board is in static equilibrium: → 4 forces acting on board:

$mg,\ Mg,\ N_p,\ N_s \rightarrow \boxed{N_p + N_s - mg - Mg = 0}$

Pivot → 3 torques acting on board: no torque from $\vec{N}_p$ since $\vec{r}_p = 0$ ($\vec{r}$: pivot to application point)

$$\boxed{\vec{\tau}_m + \vec{\tau}_M + \vec{\tau}_s = 0}$$

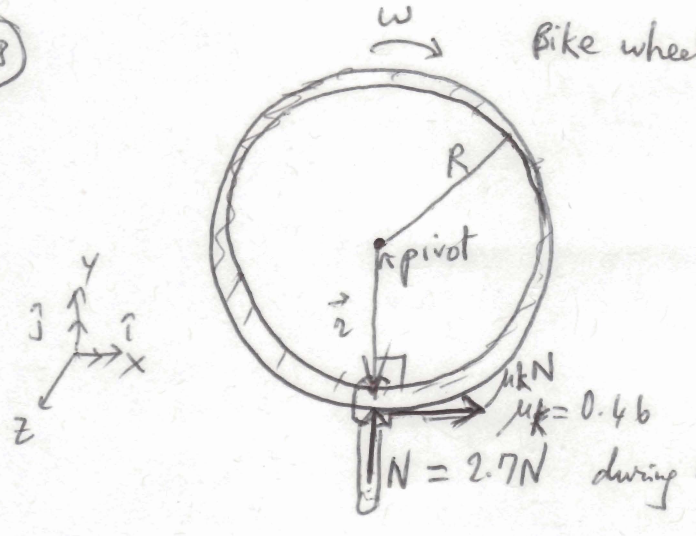
$$r_m mg\hat{k} - r_M Mg\hat{k} + r_s N_s\hat{k} = 0 \rightarrow (r_m \underset{40kg}{m}g - \underbrace{r_M}_{0.4m} \underset{60kg}{M}g + \underbrace{r_s}_{1.6} \underset{100N}{N_s}) = 0$$

$$r_m = \frac{0.4 \times 60 \times 9.81 - 1.6 \times 100}{40 \times 9.81} = 0.19m \text{ (from pivot)}$$

→ from left: loc. of child is $0.8 - 0.19 = \boxed{0.61m}$

b) → $r_m = -0.62m$ (from pivot) → loc. child from left → $0.8 + 0.62 = 1.42m$.

(10.58)



Bike wheel $\left\{ \begin{array}{l} M = 1.9\,kg. \\ R = 0.33\,m \end{array} \right.$

$\omega_i = 230\,rpm$

$\boxed{\omega_f\ ?} \rightarrow < 230\,rpm.$ b/c of friction b/w wheel & wrench.

$\mu_k = 0.46$

$N = 2.7N$ during $\Delt = 3.1s$

$\rightarrow$ Friction $\cdot\ \mu_k N \longrightarrow \quad \vec{\tau}_k = \vec{r} \times \vec{F}_k = -R\hat{j} \times \mu_k N \hat{i}$

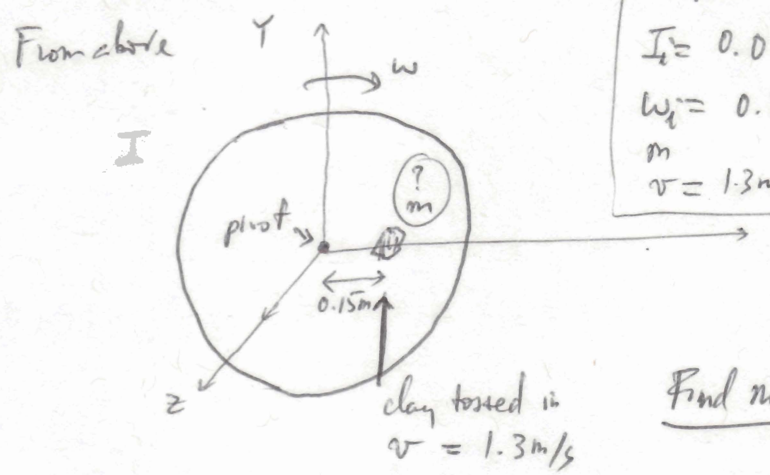
$$= R\mu_k N \underbrace{\left(-\hat{j} \times \hat{i}\right)}_{\hat{k}}$$

$\rightarrow \quad \tau = I\alpha \ \rightarrow \ \alpha = \dfrac{\tau}{I_{wheel}} = \dfrac{\cancel{R}\mu_k N}{MR^{\cancel{2}}}$ deceleration during $\Delta t = 3.1s$

$$\rightarrow \quad \omega_f = \omega_i - \alpha\Delta t = 230\,rpm - \underbrace{\underbrace{\frac{0.46 \times 2.7}{1.9 \times 0.33}}_{\frac{rad}{s}}\, 3.1\, \frac{60s}{min}\, \frac{1rev}{2\pi rad}}_{58.6\ rpm}$$

$$\boxed{\omega_f = 171\ rpm.}$$

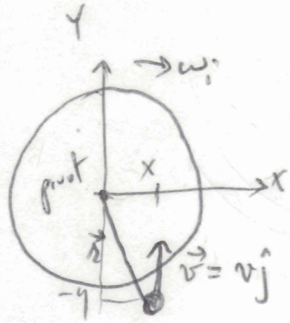
(11.47) From above

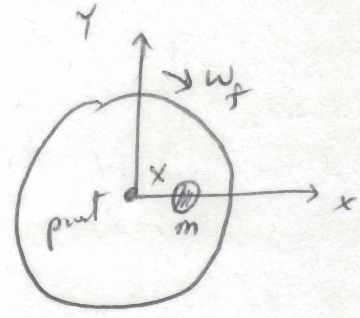


Before (clay hits disk)	After (clay on disk)
$I_i = 0.021\,kgm^2$	$I_f =$
$\omega_i = 0.29\,rad/s$	$\omega_f = 0.085\,rad/s$
m	
$v = 1.3\,m/s.$	

Find mass of clay

system disk + clay: $\vec{\tau}_{net,\,external} = 0 \rightarrow \vec{L}_i = \vec{L}_f$





initial

final

$$\vec{L}_i = I\vec{\omega}_i + \underbrace{\vec{r} \times m\vec{v}}_{\text{only x-component of } \vec{r} \text{ contributes to cross product}}$$

$I\vec{\omega}_i$: cw $\rightarrow \omega_i(-\hat{k})$

$$= -I\omega_i\hat{k} + mxv\,\underset{(\hat{i}\times\hat{j})}{\hat{k}}$$

$$= (-I\omega_i + mxv)\,\hat{k}$$

$$\vec{L}_f = I_f\,\vec{\omega}_f$$

$\vec{\omega}_f$: cw $\rightarrow \omega_f(-\hat{k})$

$$= -(I + mx^2)\,\omega_f\hat{k}$$

$$\vec{L}_i = \vec{L}_f \rightarrow \quad -I\omega_i + mxv = -(I + mx^2)\,\omega_f$$

$$mxv + mx^2\omega_f = I\omega_i - I\omega_f$$

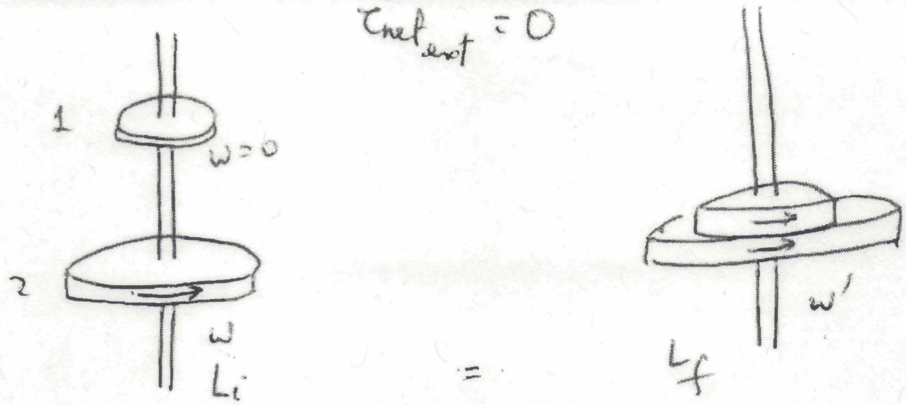
$$m = \frac{I(\omega_i - \omega_f)}{x(v + x\omega_f)}$$

$$= \frac{0.021(0.29 - 0.085)}{0.15(1.3 + 0.15 \times 0.085)}$$

$$= 0.0218\,\text{kg} \text{ or } \boxed{21.8\,\text{g}}$$

11.51

$\vec{\tau}_{net_{ext}} = 0$



stick together ("inelastic collision")

$R_1 = 2.3\,cm$; $m_1 = 0.27\,kg$
$R_2 = 3.5\,cm$; $m_2 = 0.44\,kg$
$\omega_i = 180\,rpm$

Disks $= I_{\substack{central\\axis}} = \frac{1}{2} m R^2$

(a) $L_i = I_2 \omega \quad = \quad L_f = (I_1 + I_2)\omega'$

$$\omega' = \frac{I_2}{I_1 + I_2}\,\omega = \frac{1}{\frac{I_1}{I_2} + 1}\,\omega = \frac{1}{\frac{m_1 R_1^2}{m_2 R_2^2} + 1}\,180\,rpm$$

$\omega' = 142\,rpm.$

(b) Fraction of initial KE lost into friction.

$$\frac{KE_i - KE_f}{KE_i} = \frac{\frac{1}{2}\left(\frac{1}{2} m_2 R_2^2\right)\omega^2 - \frac{1}{2}\left[\frac{1}{2} m_1 R_1^2 + \frac{1}{2} m_2 R_2^2\right]\omega'^2}{\frac{1}{2}\frac{1}{2} m_2 R_2^2 \omega^2}$$

Here $KE = \frac{1}{2} I \omega^2$ (just rotation, no translation of CM for neither (i) nor (f) situations)

$$= 1 - \left[\frac{m_1 R_1^2}{m_2 R_2^2} + 1\right]\left(\frac{\omega'}{\omega}\right)^2$$

$$= 1 - \left[\frac{0.27 \times 2.3^2}{0.44 \times 3.5^2} + 1\right]\left(\frac{142}{180}\right)^2 = 0.21 \text{ or } 21\%$$