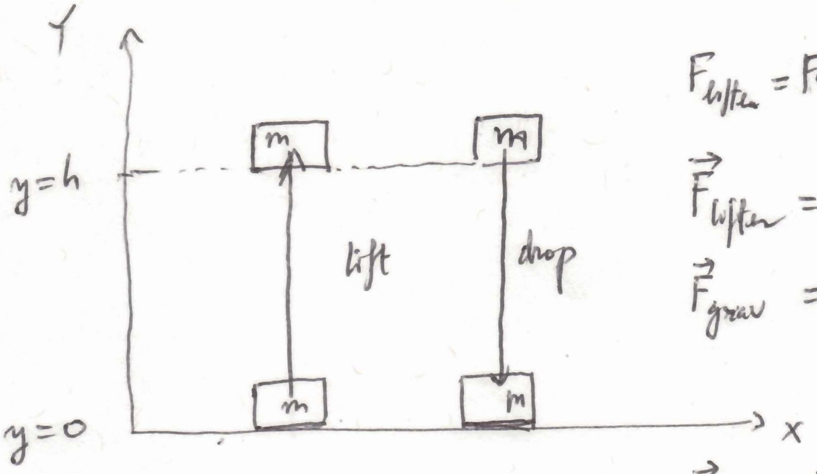
Ch 7: Conservation of Energy



F_{lifter} = Force by lifter = mg (no upward acceleration)

$\vec{F}_{lifter} = mg\,\hat{j}$; $\vec{\Delta}_y = h\,\hat{j}$

$\vec{F}_{grav} = \boxed{-}mg\,\hat{j}$; $\vec{\Delta}_y = h\,\hat{j}$

$\vec{\Delta}_{y\,drop} = -h\,\hat{j}$

Lifting : { Work done by lifter : $\vec{F}_{lifter} \cdot \vec{\Delta y} = +mgh$

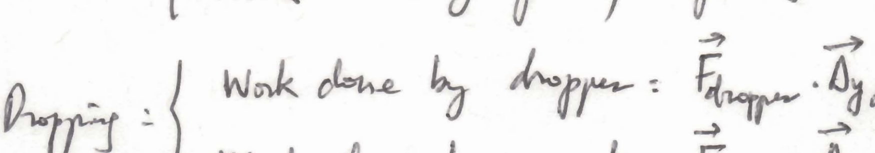
Work done by gravity : $\vec{F}_{grav} \cdot \vec{\Delta y} = -mgh$

Dropping : { Work done by dropper : $\vec{F}_{dropper} \cdot \vec{\Delta}_{y\,drop} = 0 \cdot (-h\hat{j}) = 0$

Work done by gravity : $\vec{F}_{grav} \cdot \vec{\Delta}_{y\,drop} = +mgh$

→ Work done by gravity or gravitational potential energy is conserved; or gravitation is a conservative force.

Kinetic friction : μ_K



Moving box at constant speed :

$0 \to l$ { $\vec{F}_{app} = \mu_K mg\,\hat{i}$

$\vec{F}_f = -\mu_K mg\,\hat{i}$

$\vec{\Delta}_x = l\,\hat{i}$

$0 \to l$ { work done by pusher : $\vec{F}_{app} \cdot \vec{\Delta x} = \mu_K mgl$

work done by friction : $\vec{F}_f \cdot \vec{\Delta x} = -\mu_K mgl$

$\vec{F}_{app} = -\mu_K mg\,\hat{i}$

$\vec{F}_f = \mu_K mg\,\hat{i}$ $l \to 0$ { work done by pusher : $\vec{F}_{app} \cdot \vec{\Delta x} = \mu_K mg\,l$

work done by friction : $\vec{F}_f \cdot \vec{\Delta x} = -\mu_K mgl$

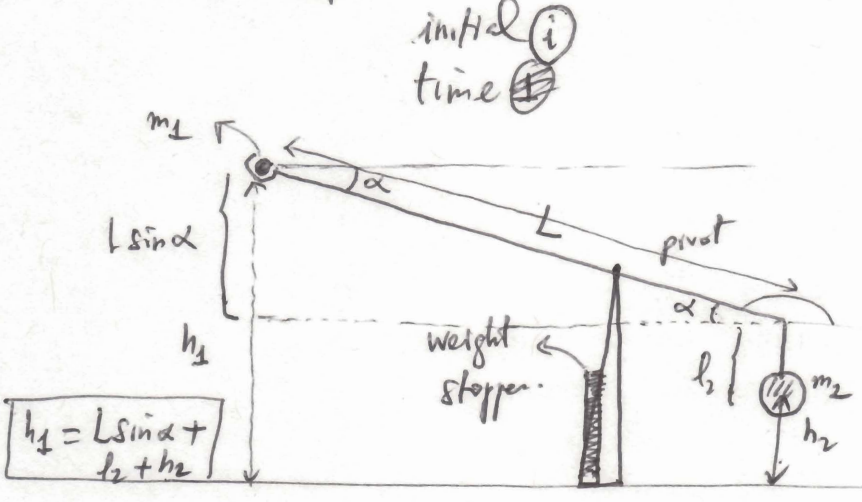
energy is not conserved → friction is not a conservative force.

Conservation of Mechanical Energy:

Mechanical energy: sum of kinetic energy and gravitational potential energy: $\frac{1}{2}mv^2 + mgh$

$$\boxed{\left(\frac{1}{2}mv_i^2 + mgh_i\right) = \left(\frac{1}{2}mv_f^2 + mgh_f\right)}$$

P.P.3 q.7.1: Trebuchet or catapult:



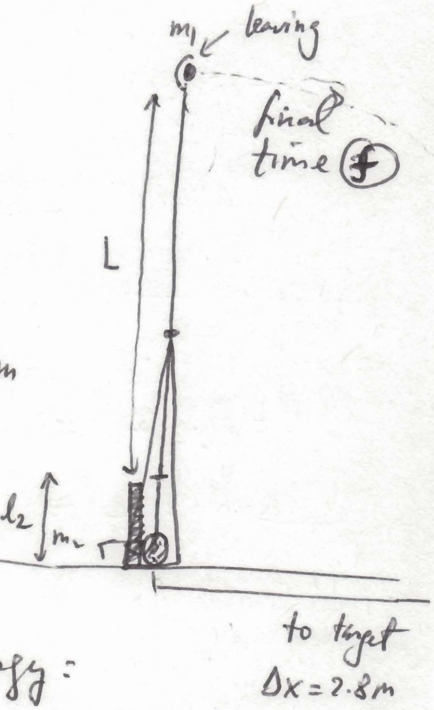
$L + l_2 = 1.1m$

$m_2 = 1kg$

$m_2 = 0.1kg$

$\boxed{h_2\,?}$ →

$\Delta x_{Range} = 2.8m$



→ System based on conservation of mechanical energy:

$M.E_{i}$ of m_1 & m_2 = ME_{f} of m_1 & m_2

ST.
$$\underbrace{\frac{1}{2}m_1 \underset{0}{\cancel{v_{1i}}}^{2} + \frac{1}{2}m_2 \underset{0}{\cancel{v_{2i}}}^{2}}_{\underset{0}{\cancel{KE_i}}} + \underbrace{m_1 g h_1 + m_2 g \boxed{h_2}}_{P.E_i} = \underbrace{\boxed{\frac{1}{2}m_1 v_{1f}^2} + \frac{1}{2}m_2 \underset{\substack{0\\ \text{due to weight stopper}}}{v_{2f}}^{2}}_{K.E_f} + \underbrace{m_1 g\,1.1 + \cancel{m_2 g\cdot 0}}_{P.E_f}$$

$$m_2 g h_2 = \frac{1}{2}m_1 v_{1f}^2 + \underbrace{m_1 g(1.1 - h_1)}_{m_1 g(1.1 - L\sin\alpha - l_2 - h_2)}$$

$$\boxed{(m_1+m_2)gh_2 = \frac{1}{2}m_1 v_{1f}^2} + m_1 g\,(\underbrace{1.1 - L\sin\alpha - l_2}_{\text{estimate } \alpha = \dots})$$

$$\boxed{h_2 = \frac{\frac{1}{2} m_1 v_{1f}^2}{(m_1 + m_2) g}}$$

Find v_{1f}: from projectile motion of m_1:

horizontal: uniform motion: $\Delta x = v_{1f} \underset{\uparrow \text{need.}}{\Delta t} \Rightarrow v_{1f} = \frac{\Delta x}{\Delta t} = \frac{2.8}{\sqrt{\frac{2.2}{9.81}}} = 5.9 \frac{m}{s}$

vertical: constant deceleration: $\Delta y = \frac{1}{2} g t^2 \rightarrow t = \sqrt{\frac{2 \Delta y}{g}} = \sqrt{\frac{2 \times 1.1}{9.81}}$

Conserv. M.E: $h_2 = \frac{\frac{1}{2} 0.1 \times 5.9^2}{(0.1+1) \times 9.81} \simeq \quad m$

Ch8: Gravity:

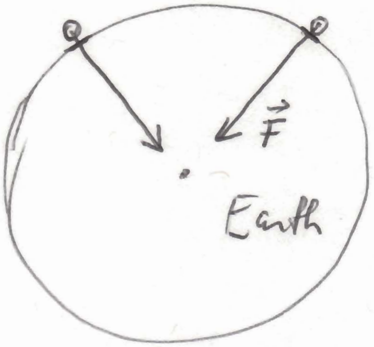
$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

"Universal": it works for marble size objects up to planets & stars

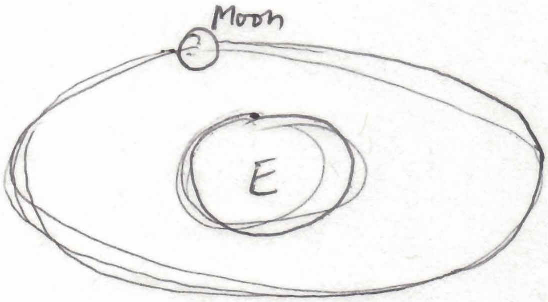
$\hookrightarrow$ Force of gravitational attraction b/w two objects of masses m_1 & m_2; separated by r (distance)

→ G: universal gravitational constant: $\boxed{6.67\times 10^{-11}\frac{Nm^2}{kg^2}}$

→ Direction of force is center to center, toward the more massive object



For us ground is "flat" → F is vertical. In reality F is toward the center of the Earth



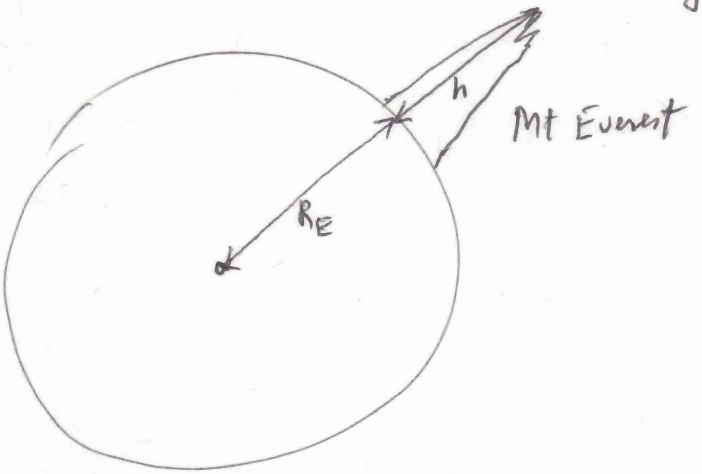
$\boxed{\text{Why } g = 9.81\,m/s^2?}$

$M_{Earth} = 5.97\times 10^{24}\ kg$

$R_E = 6.37\times 10^{6}\ m$

For an object of mass $m_2 = m$ on the surface of the Earth: $r = R_E$ → $F = G\frac{M_E}{R_E^2}\cancel{m} \underset{\substack{\downarrow \\ \text{Force of gravity}}}{=} \cancel{m}g \rightarrow \boxed{g = G\frac{M_E}{R_E^2}}$

$$g = \frac{6.67\times10^{-11} \times 5.97\times10^{24}}{(6.37\times10^{6})^2} = 9.8123\ m/s^2 \simeq 9.81\ m/s^2 .$$

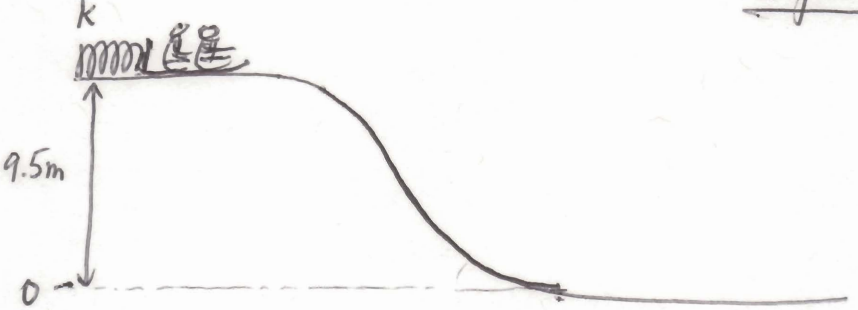


$$\longrightarrow g' = \frac{GM_E}{\underset{\substack{\downarrow \\ 6.37\times10^{6}}}{(R_E+h)^2}} < g$$

$$\begin{cases} h \simeq 10m \rightarrow g' = g \\ h \simeq 8000m \rightarrow g' < g \end{cases}$$

(7.43)

No friction



$k = 890\,\frac{N}{m}$

$m = 80\,kg$
(2 kids + toboggan)

$\Delta x = 2.6m \rightarrow v?$ final speed at bottom of hill?

→ Difference wrt other problems before conservation of energy?

↳ 1) Curve slope (unknown shape) || Before:
2) Angle is not known.

Flat slope, known θ

a) → Conservation of Mechanical energy: determine
- initial time: kids & toboggan at top
- final time: kids & toboggan at bottom

$$KE_i + PE_i + EPE_i = KE_f + PE_f$$

PE_i ↓ grav. potential energy

EPE_i ↓ Elastic Potential Energy from compressed spring

$$\frac{1}{2}m\underset{\downarrow\atop 0}{\cancel{v_0}}^2 + mgh + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m\,\underset{\downarrow\text{ speed at bottom of hill}}{v}^2 + mg0$$

$$v = \sqrt{2gh + \frac{k}{m}(\Delta x)^2} = \sqrt{2\times 9.81\times 9.5 - \frac{890}{80}(2.6)^2}$$

$$\boxed{v = 16.1\,m/s}$$

b) Fraction of final KE stored in the compression of spring $\frac{\frac{1}{2}mv^2}{\frac{1}{2}k(\Delta x)^2} = \frac{80\times 16.1^2}{890\,(2.6)^2}$

$$\frac{1}{2}mv^2 = mgh + \frac{1}{2}k(\Delta x)^2 \quad \text{or} \quad \frac{1}{2}mv^2 > \frac{1}{2}k(\Delta x)^2$$

$$\to \frac{\frac{1}{2}k(\Delta x)^2}{\frac{1}{2}mv^2} = \frac{1}{3.44} = 0.29 \quad \text{or} \quad 29\%$$

29% of the final KE comes from the elastic potential energy of compressed spring ; the remaining 71% comes from the grav. pot. energy

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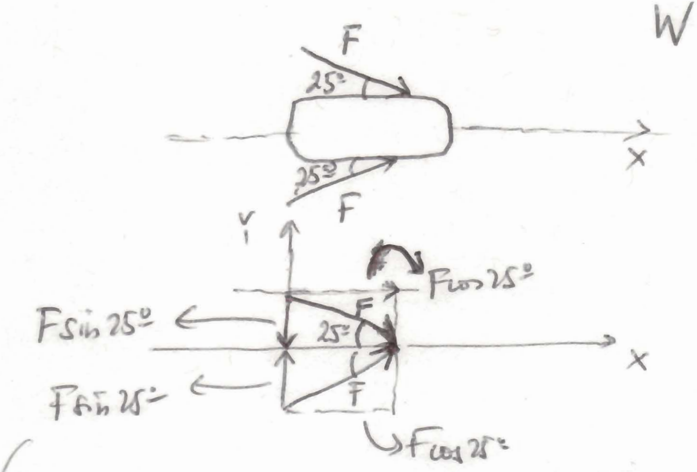
6.44 ; 7.64 ; 7.27 ; 7.43.

8.42

6.44

View from above:



scalar product

$$W = \vec{F} \odot \vec{\Delta x}$$

$$\boxed{\vec{\Delta x} = \Delta x\,\hat{i} = 5.6\,\hat{i}\ (m)}\ ⊛$$

$$\vec{F} = F\cos 25°\,\hat{i} + F\sin 25°\,\hat{j}$$

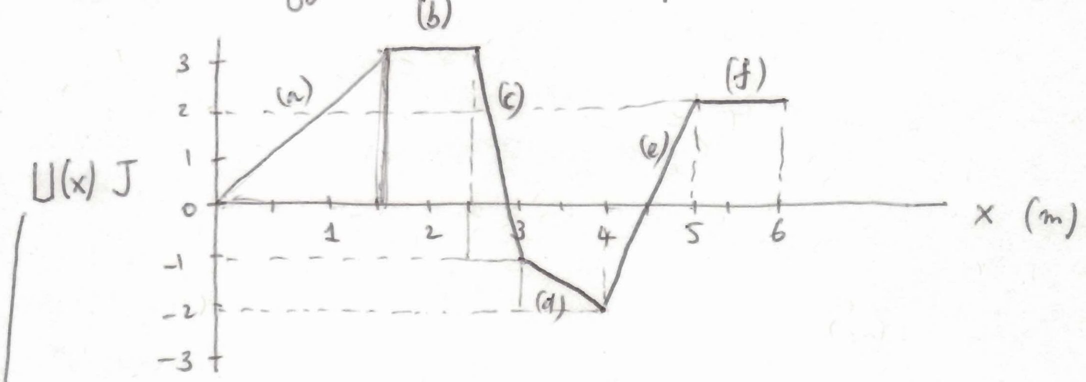
F = 280N

$$\boxed{\vec{F} = 280\cos 25°\,\hat{i} + 280\sin 25°\,\hat{j}}$$

$$W = 280\cos 25°\ 5.6\ \underbrace{\hat{i}\cdot\hat{i}}_{1.1\cos(0) = 1} = 1421\,J = 1.421\,kJ$$

⊛ Since $F_{net_y} = 0$, car will not move sideway → $\Delta y = 0$.

(7.27) Potential energy curve for certain particle



Find force on particle:

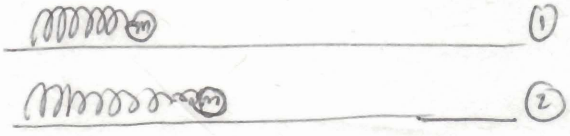
derivative vector or "gradient"

Potential energy = $\underbrace{\boxed{\Delta U_{AB} = -\int_A^B \vec{F}\cdot d\vec{r}}}_{}$ → $\vec{F} = -\vec{\nabla} U$

in 1D: $\boxed{F = \ominus \frac{\partial U}{\partial x}}$ → slope of U

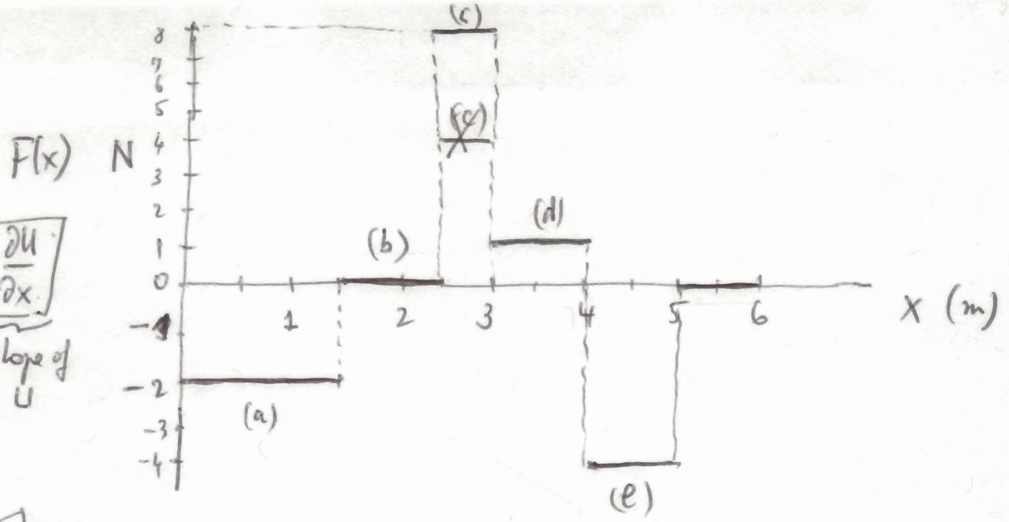
Change of potential energy is minus the work done

(when work is done, potential energy is reduced)

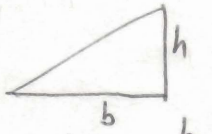


(spring has more elastic potential energy than ①)

If I release the mass, spring would do work since it pulls the mass certain distance Δx applying force $-K\Delta x$. As it does work it looses elastic potential energy.



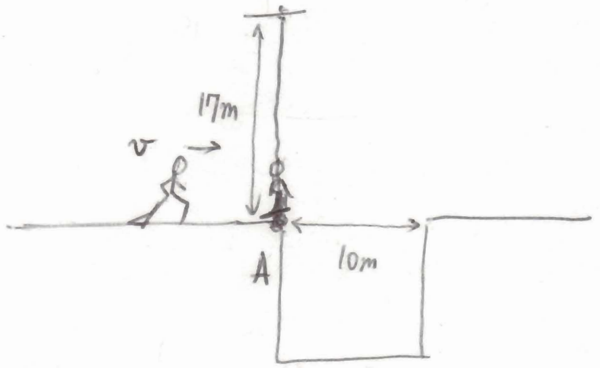
$$\boxed{F = -\underbrace{\frac{\partial U}{\partial x}}_{\text{slope of } U}}$$



$\text{slope} = \frac{h}{b}$

7.43 did in pg 66.

7.64

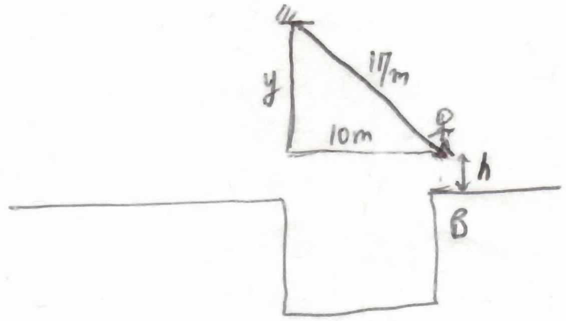


Conservation of energy:

use ground as ref. for grav. potential energy: mgh

→ initial @ A : { P.E = 0, KE = $\frac{1}{2}mv^2$

→ final @ B : { PE = mgh, KE = 0



$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

$$v = \sqrt{2g\underset{\uparrow}{h}}$$

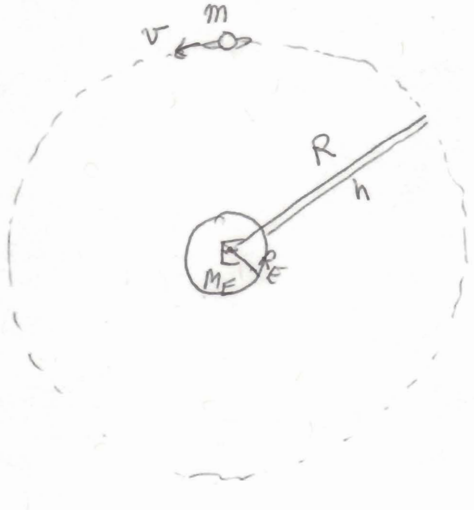
$$y^2+10^2 = 17^2 \rightarrow y = \sqrt{17^2-10^2} = 13.75m \rightarrow h = 17 - y = 3.25m$$

$$\rightarrow v = \sqrt{2\times 9.81\times 3.25}\ m/s = 7.99\ m/s$$

(min. speed)

Orbital motion :

Starting with spherical symmetry → circular orbit :



satellite under UCM (uniform circular motion) at constant speed v

$R = R_E + h$

→ Force providing change of direction (radial acceleration) for satellite is : $F = G\frac{M_E\, m}{R^2}$

→ 2nd Newton's law : $F = ma = m\frac{v^2}{R}$

$$\rightarrow \quad G\frac{M_E \cancel{m}}{R^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{R}} \quad \rightarrow \quad v = \sqrt{\frac{GM_E}{R}}$$

<u>Period of orbital motion</u>:

$$\boxed{T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM_E}{R}}} = \frac{2\pi}{\sqrt{GM_E}} R^{3/2}}$$

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right) R^3$$

3rd Kepler's law → period squared is proportional to the cube of the <u>semimajor</u> axis

(is radius in circular orbit)

↓

Elliptical orbits

<u>Orbital Period</u> for a satellite of any mass at $h = 250$ km :

How long to make one turn around Earth.

$$T = \frac{2\pi}{\sqrt{G.M_E}} (R_E + h)^{3/2} = 5400s = 1.5h$$

8.42

Pasachoff orbits Sun w/ $T = 1417$ days.

What is the semimajor axis of its orbit? $\underbrace{\text{semimajor axis}}_{a}$

Earth's orbital radius and period as units for distance & time:

$$T^2 \propto a^3 \quad \text{or} \quad \left\{ \begin{array}{l} T_E^2 = \boxed{C}\, a_E^3 \\ T_p^2 = \boxed{C}\, a_p^3 \end{array} \right.$$

same given they are both under grav. attraction of the SUN

$$\frac{a_p^3}{a_E^3} = \frac{T_p^2}{T_E^2}$$

$$\hookrightarrow a_p^3 = \frac{T_p^2}{T_E^2}\, a_E^3 \Rightarrow a_p = \left(\frac{T_p}{T_E}\right)^{2/3} a_E$$

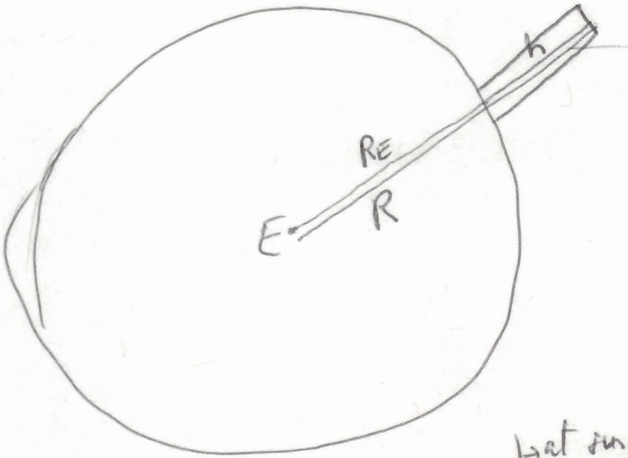
orb radius of Earth wrt SUN

$$= \underbrace{\left(\frac{1417}{365}\right)^{2/3}}_{2.47} a_E$$

$$\boxed{a_p = 2.47\, a_E}$$

8.18

(8.18)



→ Sear's Tower in Chicago

$$R = R_E + h$$

$$F = G\frac{M_E m}{R^2} \longrightarrow$$

→ at surface $R = R_E$ $\quad g = \frac{GM_E}{R_E^2}$

→ top of Sear's Tower : $g' = \frac{GM_E}{(R_E+h)^2}$

$$\Delta g = 0.00136\frac{m}{s^2} = g - g' = GM_E\left(\underbrace{\frac{1}{R_E^2} - \frac{1}{(R_E+h)^2}}\right)$$

$$\frac{(R_E+h)^2 - R_E^2}{R_E^2(R_E+h)^2} = \frac{+2R_Eh + h^2}{R_E^2(R_E+h)^2}$$

$$= \frac{h(h+2R_E)}{R_E^2(R_E+h)^2}$$

$$\Delta g = \frac{GM_E}{R_E^2}\frac{h(h+2R_E)}{(R_E+h)^2}$$

Need h

To get a good approxi to h = $\begin{cases} 2R_E + h \simeq 2R_E \\ R_E + h \simeq R_E \end{cases}$

$\downarrow$

6370000m

$$\Delta g = \underbrace{\frac{GM_E}{R_E^2}}_{g}\frac{h\,2\cancel{R_E}}{R_E^{\cancel{2}}} \rightarrow \frac{\Delta g}{g} = \frac{2h}{R_E} \rightarrow h = \frac{\Delta g}{g}\frac{R_E}{2}$$

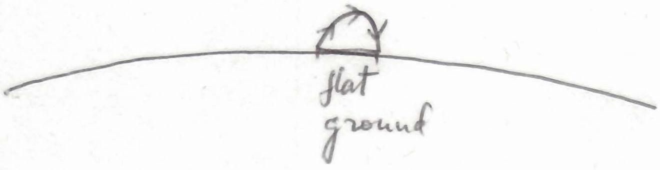
$$= \frac{0.00136}{9.81}\frac{6370000}{2}$$

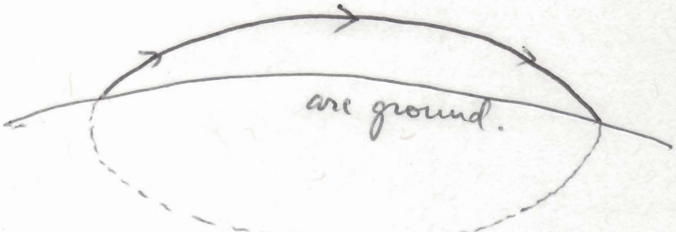
$$\boxed{h = 441.5m}$$

Ch 8 Gravitation (Cont.)

Projectile motion:

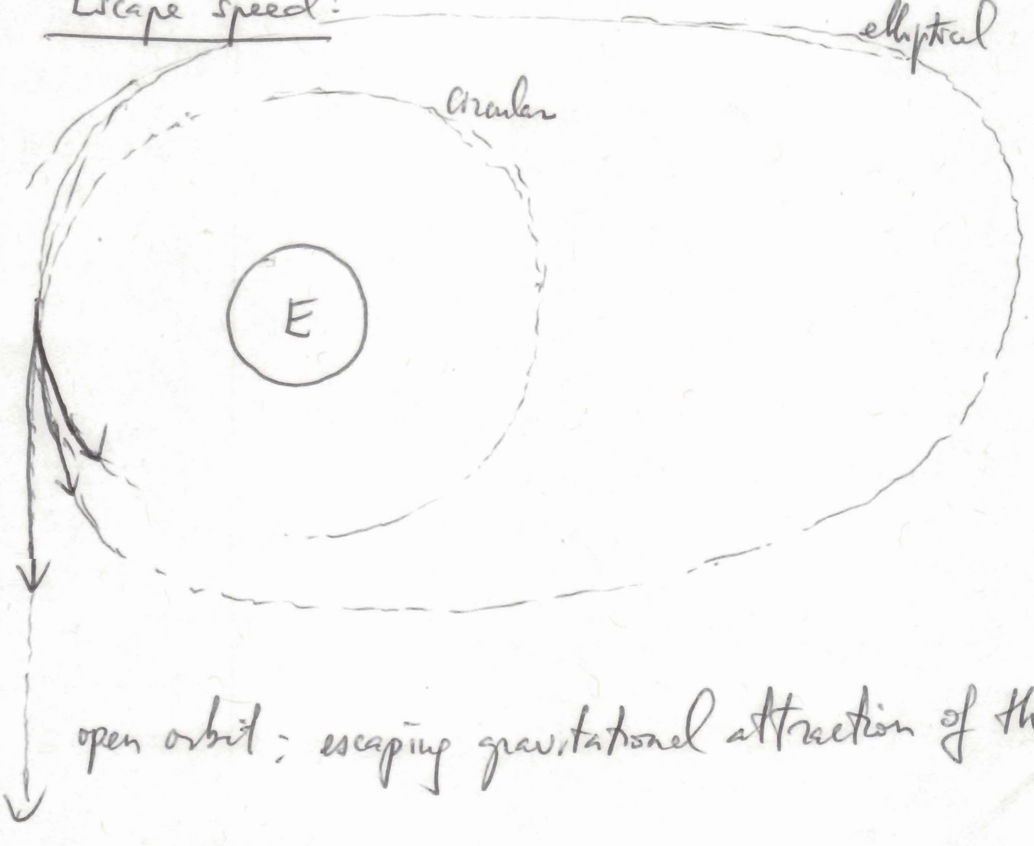
So far : inverted parabola : good for small range → surface is flat.





long range ballistic motion
→ projectile motion (under influence of gravity) → is more generally part of an elliptical orbit.

Escape speed:



open orbit : escaping gravitational attraction of the Earth

An object can escape grav. attraction of a larger object when it total energy is not negative (at least zero)

$$\underbrace{KE}_{\frac{1}{2} m v_{esc}^2} + \underbrace{PE}_{\boxed{-G\frac{M_E m}{r}}} = 0 \rightarrow v_{esc} = \sqrt{\frac{2GM_E}{r}}$$

Escape from surface: $r = R_E \rightarrow v_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.37 \times 10^{6}}}$

$$= 11.2 \frac{km}{s} = \boxed{40320 \frac{km}{h}}$$

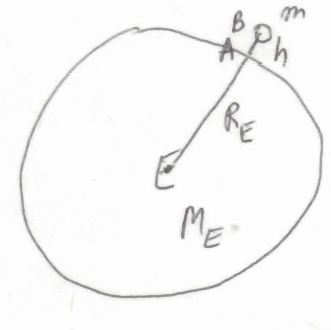
$$\Delta U = -\int_A^B \vec{F} \cdot d\vec{r} = -\int_A^B G\frac{M_E m}{r^2} dr = GM_E m \left(\frac{1}{r}\right)_A^B$$

$$\boxed{\Delta U = GM_E m \left(\frac{1}{\infty} - \frac{1}{r}\right) = -\frac{GM_E m}{r}}$$ Gravitational potential energy.

$\boxed{\text{Ref at } \infty}$ $\begin{cases} @B : r = \infty \\ @A : r = r \end{cases}$

$\boxed{\text{Related to } mgh \text{ (will see why?)}}$

For objects close to surface of the Earth: $h \ll R_E$



$$\Delta U_{\underset{\swarrow\ \ \searrow}{(AB)}} = U_B - U_A = \frac{-GM_E m}{R_E + h} - \frac{-GM_E m}{R_E}$$

A: ground

B: object location at height h from ground.

$$= GM_E m \left(-\frac{1}{R_E + h} + \frac{1}{R_E}\right)$$

$$\boxed{U = -\frac{GM_E m}{r}} \rightarrow \Delta U_{AB} = GM_E m \left(\frac{-\cancel{R_E} + \cancel{R_E} + h}{(R_E + h) R_E} \right) = GM_E m \frac{h}{(R_E + h) R_E}$$

$$= \underbrace{\frac{GM_E m}{R_E^2}}_{g} h = \boxed{mgh}$$

$$\boxed{\begin{array}{c} h \ll R_E \\ R_E + h \approx R_E \end{array}}$$

$\downarrow$

6370000 m

approx to arrive at this alternative expression for grav. potential energy.

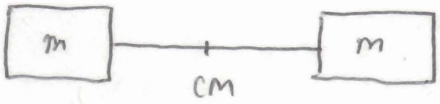
Ch 9: System of Particles

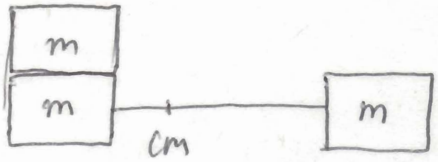
So far: FBD: reducing our object to a point → Center of Mass

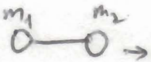
Center of Mass: average position of components (of a system) weighted by their masses.

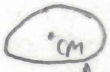
↓

$\vec{R}$









$$\left\{ \boxed{\begin{aligned} \vec{R} &\equiv \frac{\sum_i m_i \vec{r}_i}{M} \\ \vec{R} &= \frac{1}{M}\int \vec{r}\, dm \end{aligned}} \right. \quad ; \quad M = \sum_i m_i \quad \text{(Total mass)}$$

Any change to 2nd Newton's Law when components of a system are considered: not the equation only a subtlety:

$$\underbrace{\vec{F}_{Net}} = M \frac{d^2\vec{R}}{dt^2}$$

Net force on an object is its total mass time the acceleration of the center of mass

↓

Subtlety: Only coming from external forces.
(Forces b/w components of the system are internal, by the Newton 3rd Law of action & reaction: they cancel in pairs)

Linear Momentum of an object:

position of component i of system

$$\vec{P} = \underset{\substack{\downarrow\\ \text{Total mass}\\ \text{of object}}}{M}\underset{\substack{\searrow\\ \text{Velocity of}\\ \text{its center of mass}}}{\vec{V}} = M\frac{d\vec{R}}{dt} = \cancel{M}\frac{d}{dt}\left(\frac{\sum_i m_i \vec{r}_i}{\cancel{M}}\right) = \sum_i m_i \underbrace{\frac{d\vec{r}_i}{dt}}_{\substack{\text{Velocity of}\\ \text{component}\\ i \to \vec{v}_i}}$$

$$\vec{P} = \sum_i \underbrace{m_i \vec{v}_i}_{\vec{p}_i}$$

$\vec{p}_i$: linear momentum of component i

Very important consequence of 2nd Newton's Law:

$$\underbrace{\vec{F}_{Net}}_{\substack{\text{Net force}\\ \text{on system}}} = \frac{d\vec{P}}{dt}$$

$\vec{P}$ → Total momentum of system

$$\text{If } \underbrace{\vec{F}_{net}}_{\substack{\text{External}\\ \text{Net force}}} = 0 \longrightarrow \boxed{\begin{array}{l}\vec{P} \text{ is constant}\\ \text{Conservation of Linear Momentum}\end{array}}$$

(Different Law than Conservation of Mechanical Energy!)

PP4 q 9.1 → example of

$\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f$

→ <u>Inelastic Collision</u> : two objects <u>stick together</u> after collision → Total KE of the two is <u>not conserved</u> : some initial KE is used in changing internal structure. E.g: throwing a sticky ball on a running kid.

Only linear momentum is conserved:

$$\boxed{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f}$$

→ <u>Elastic collision</u> : Total KE of the two colliding objects is <u>also conserved</u>

$$\boxed{\begin{array}{l} m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \\ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \end{array}}$$

<u>1D</u>: 2 equations $\left\{ \begin{array}{l} 1 \text{ CLM} \\ 1 \text{ CKE} \end{array} \right.$ $\gg$ can find 2 unknowns: e.g.: v_{1f} & v_{2f} (when we are given v_{1i} ; v_{2i} & m_1, m_2)

<u>2D</u>: 3 equations $\left\{ \begin{array}{l} 2 \text{ CLM's (one for x ; one for y)} \\ 1 \text{ CKE} \end{array} \right.$

→ can't find $\vec{v}_{1f}$ & $\vec{v}_{2f}$ (they are 4 unknowns) unless extra information is provided (look for this info when solving 2D elastic collision problem)

<u>Results</u>.

<u>1D elastic</u>

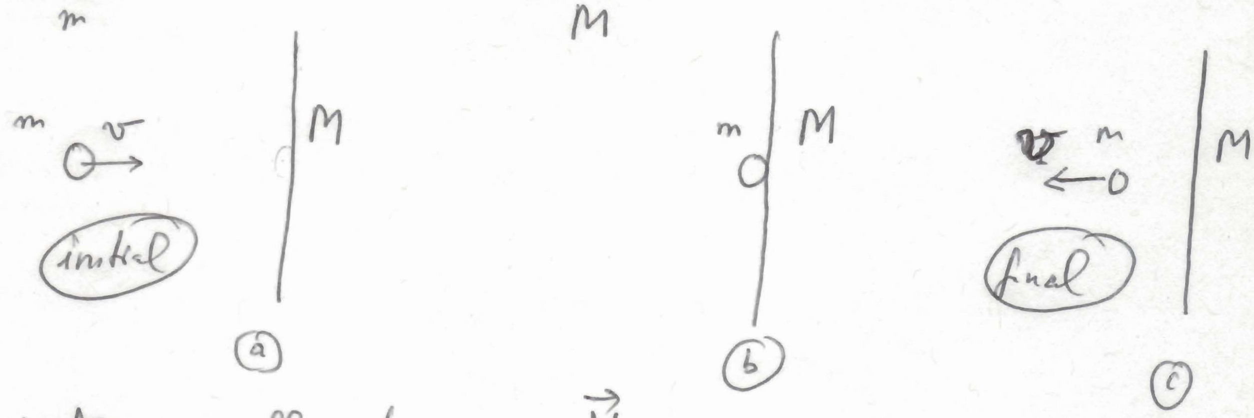
$$\left\{ \begin{array}{l} v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i} + \dfrac{2m_2}{m_1 + m_2} v_{2i} \\ v_{2f} = \dfrac{2m_1}{m_1 + m_2} v_{1i} + \dfrac{m_2 - m_1}{m_1 + m_2} v_{2i} \end{array} \right.$$

<u>1D elastic</u>: $v_{1i} + v_{1f} = v_{2i} + v_{2f}$

pool ([illegible])

A ball colliding with a wall of pool table → $m \ll M$

$\underline{\text{A ball}}_{m}$ $\underline{\text{wall}}_{M}$



→ Assume wall so heavy → $\vec{V}_{wall} \simeq 0$

→ System ball – wall : $\boxed{\vec{F}_{Net_{ext}} = 0 \rightarrow \vec{P}_i = \vec{P}_f}$

$$\boxed{\vec{P}_i = \vec{P}_f}$$

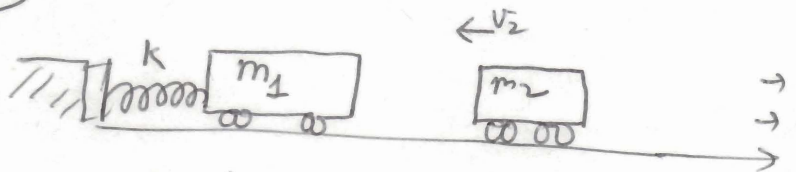
$$mv = m(-v) + \underbrace{2mv}$$

Momentum transferred to wall

$$2mv = \underline{M}V \rightarrow V = \frac{2mv}{M} \simeq 0$$

HW4 → Ch9: $\underline{4}, \underline{6}$; $\underline{39}$; $\underline{41}$; $\underline{43}$; $\underline{48}$; $\underline{52}$ (rest for HW5).

(9.41)



→ Spring attached to car #1.
→ cars couple together after collision.

$m_1 = 11000$ kg
$k = 3.2 \times 10^5 \frac{N}{m}$
$m_2 = 9400$ kg
$\vec{v}_2 = -8.5\,\hat{i}$ $\frac{m}{s}$

(a) Max. compression of spring?

System of components: ① & ②

$\vec{F}_{Net,ext} = 0 \rightarrow \boxed{\vec{P}_i = \vec{P}_f}$

initial: $\begin{cases} \vec{v}_1 = 0 \\ \vec{v}_2 = -8.5\,\hat{i} \text{ m/s} \end{cases}$

final: $\begin{cases} v_{1f} = v_{2f} = v_f \end{cases}$

$$m_2 \vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_2}{m_1 + m_2}\vec{v}_2 = \frac{9400}{20400}(-8.5\,\hat{i}) \text{ m/s}$$

$$= -3.92\,\hat{i} \text{ m/s}$$

The two

Max compression of spring when all this KE from the two cars after collision has been transfered into elastic P.E:

$$\cancel{\tfrac{1}{2}} k \Delta x^2 = \cancel{\tfrac{1}{2}}(m_1+m_2)\, v_f^2 \rightarrow \Delta x = \sqrt{\frac{m_1+m_2}{k}}\,|v_f|$$

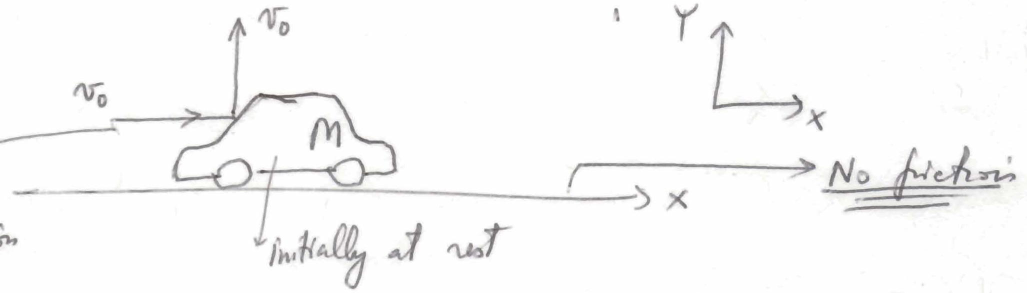
$$= \sqrt{\frac{20400}{320\,000}}\; 3.92 \;\; m$$

$$= 0.989\, m$$

b) Speed of two cars when rebound from spring?

$$\vec{v} = +3.92\hat{\imath}\ (m/s)$$

(9.43)



→ water jet

velocity changes direction not magnitude v_0

initially at rest

↳ provide a forward acceleration to the car: a_x

a) Expression for a_x?

System: water jet & car → No friction → $\vec{F}_{net,ext} = 0$

$$\underbrace{\vec{P}_i = \vec{P}_f}$$

initial: water jet horizontal: $\vec{v}_W = v_0\hat{\imath}$
car at rest: $\vec{v}_{car} = 0$

final: $\vec{v}_{W_f} = v_0\hat{\jmath}$
$\vec{v}_{car_f}$

$$\frac{d}{dt}:\left(m\vec{v}_W + M\cdot 0 = m\vec{v}_{W_f} + M\vec{v}_{car_f}\right) \rightarrow \text{Need } \vec{a}_{car} = \frac{d\vec{v}_{car,f}}{dt}$$

$$\frac{dm}{dt}\vec{v}_W = \frac{dm}{dt}\vec{v}_{W_f} + M\boxed{\vec{a}_{car_f}}$$

$$\frac{dm}{dt}v_0\hat{\imath} = \frac{dm}{dt}v_0\hat{\jmath} + M\vec{a}_{car} \rightarrow \boxed{\vec{a}_{car} = \frac{1}{M}\frac{dm}{dt}v_0(\hat{\imath}-\hat{\jmath})}$$

Forward acceleration of car is $\boxed{\frac{1}{M}\frac{dm}{dt}v_0 = a_x}$

b) Max. speed reach by car ?

↳ Car starts from rest, accelerates at $\frac{1}{M}\frac{dm}{dt}v_0$; when it reaches the speed v_0 → no further pushing since that is also the speed of water → no further acceleration

→ max speed for car is v_0 !

(9.48) Three component system : { astronaut : $M_1 = 60kg$
oxigen tank : $m_2 = 14kg$
Camera : $m_3 = 5.8kg$

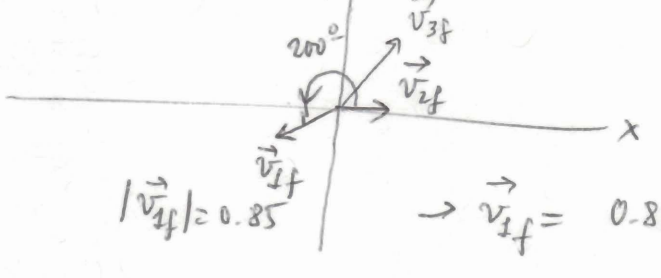
↓

In space : $\vec{F}_{net, external} = 0 \rightarrow \vec{P}_i = \vec{P}_f$

$$\text{initial} \left\{ \begin{array}{lll} m_1 & : & \vec{V}_{1i} = 0 \\ m_2 & : & \vec{V}_{2i} = 0 \\ m_3 & : & \vec{V}_{3i} = 0 \end{array} \right\rangle \vec{P}_i = 0$$

$$\text{final} \left\{ \begin{array}{lll} m_1 & : & \longrightarrow \vec{v_1} \\ m_2 & : & \vec{v}_{2f} = 1.6\ \hat{i}\ (m/s) \\ m_3 & : & \vec{v}_{3f} = ? \end{array} \right\rangle \vec{P}_f = -60\times0.8\,\hat{i} - 60\times0.3\,\hat{j} + 14\times1.6\hat{i} + 5.8\,v_{3x}\,\hat{i} + 5.8\,V_{3y}\,\hat{j}$$

↓



$|\vec{v}_{1f}| = 0.85$

$$\rightarrow \vec{v}_{1f} = 0.85\cos 200°\,\hat{i} + 0.85\sin 200°\,\hat{j}$$

$$= -0.8\,\hat{i} - 0.3\hat{j} \quad m/s$$

$$0 = (-60\times0.8 + 14\times1.6 + 5.8V_{3x})\hat{i} + (-60\times0.3 + 5.8V_{3y})\hat{j}$$

$$\Rightarrow \left\{ \begin{array}{ll} -48 + 22.4 + 5.8v_{3x} = 0 & \rightarrow v_{3x} = \frac{48 - 22.4}{5.8}\frac{m}{s} = 4.41\,m/s \\ -18 + 5.8\,V_{3y} = 0 & \longrightarrow v_{3y} = \frac{18}{5.8}\,m/s = 3.1\,m/s \end{array} \right.$$

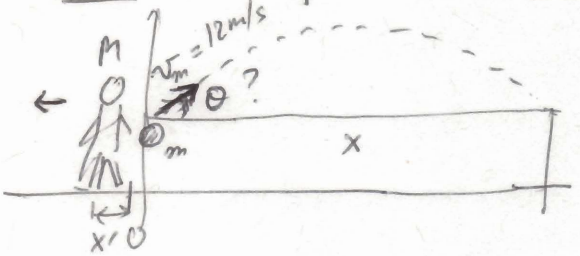
$\vec{v}_{3f} = 4.41\,\hat{i} + 3.1\hat{j}$ m/s or $v_{3f} = 5.33\,m/s$ at 34.3° CCW from X axis

(9.52) System: you (M) & a rock (m) on frictionless ice

$\rightarrow \vec{F}_{net,ext} = 0 \rightarrow \boxed{\vec{P}_i = \vec{P}_f}$

initial: $\vec{P}_i = 0$

final: $\vec{P}_f$



M : $\vec{v}_M = 0$

m : $\vec{v}_m = 0$

$x + x' = 15.2m$

$$0 = m\vec{v}_m + M\vec{v}_M \quad \begin{cases} x \Rightarrow \; 0 = m v_{mx} + M v_{Mx} \rightarrow \boxed{v_{Mx} = -\frac{m}{M} v_{mx}} \\ y \rightarrow \; 0 = m v_{my} + M v_{My} \end{cases}$$

$$V_{Mx} = -\frac{4.5}{65} 12\cos\theta \rightarrow x' = V_{Mx} \cdot \underset{\downarrow\, 2t_{up}}{t} \rightarrow \boxed{x' = -\frac{4.5}{65} 12\cos\theta \frac{24\sin\theta}{g}}$$

Projectile motion.

Vertical motion of rock: $0 = 12\sin\theta - g\, t_{up} \rightarrow \boxed{t_{up} = \frac{12\sin\theta}{g}}$

Horizontal motion of rock: $x = v_{mx}\, 2t_{up} = 12\cos\theta \frac{24\sin\theta}{g}$

$x - x' = 15.2$

$$12^2 \frac{\overset{\rightarrow \sin 2\theta}{2\cos\theta\sin\theta}}{g} + \frac{4.5}{65} 12^2 \frac{\overset{\sin 2\theta}{2\cos\theta\sin\theta}}{g} = 15.2$$

$$\sin 2\theta \left[\frac{144}{9.81}\left(1 + \frac{4.5}{65}\right) \right] = 15.2$$

a) $$\sin 2\theta = \frac{15.2}{\frac{144}{9.8}\left(1 + \frac{4.5}{65}\right)} = 0.968$$

$2\theta = \sin^{-1} 0.968 \rightarrow \boxed{\theta = 37.78°}$

b) How fast are you moving?

$$v_{M_x} = -\frac{4.5}{65} 12 \cos(37.78^{\circ}) = -0.658 \, m/s$$