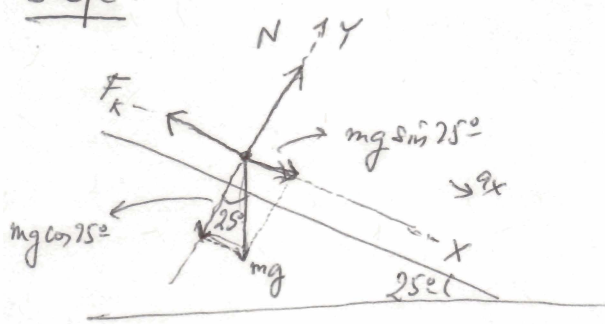
a constant deleleration from that initial speed to zero.

There is friction on slope and leveled path $\mu_K = 0.12$.

Slope: 2nd Newton's Law: $\vec{F}_{net} = m\vec{a}$

- → Free-body diagram ✓ (2)
- → Coordinate system XY ✓ (3)
- → Components of mg (4)
- → 2nd Newton's law along X & Y (5)



(5)
$$\begin{cases} F_{net\,x} = m a_x = mg\sin 25° - F_K = mg\sin 25° - \mu_K N & \text{(a)} \\ F_{net\,y} = m\,0 = N - mg\cos 25° & \text{(b)} \end{cases}$$

(b) $N = mg\cos 25°$ → (a) $\boxed{a_x = g\sin 25° - \mu_K g\cos 25°}$

$$a_x = 9.81\,(\sin 25° - 0.12\cos 25°) = 3.08\ m/s^2$$

→ Calculate final speed at bottom of slope (length = 41m)

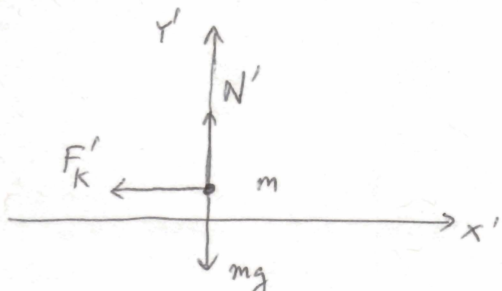
$x - x_0 = 41\,m$

$$\frac{v^2 - v_0^2}{x - x_0} = 2a_x \quad ; \quad v_0 = 0 \text{ (sled starts from rest)}$$

$$\rightarrow \quad v = \sqrt{2a_x(x - x_0)} = \sqrt{2 \times 3.08 \times 41} = 15.9\ m/s$$

Leveled path: 2nd Newton's Law: $\vec{F}_{net} = m\vec{a}$

- Free body diagram ✓ (2)
- Coord. system X'Y' ✓ (3)
- Components: not needed here ✓ (4)
- 2nd Newton's Law along X' & Y' (5)



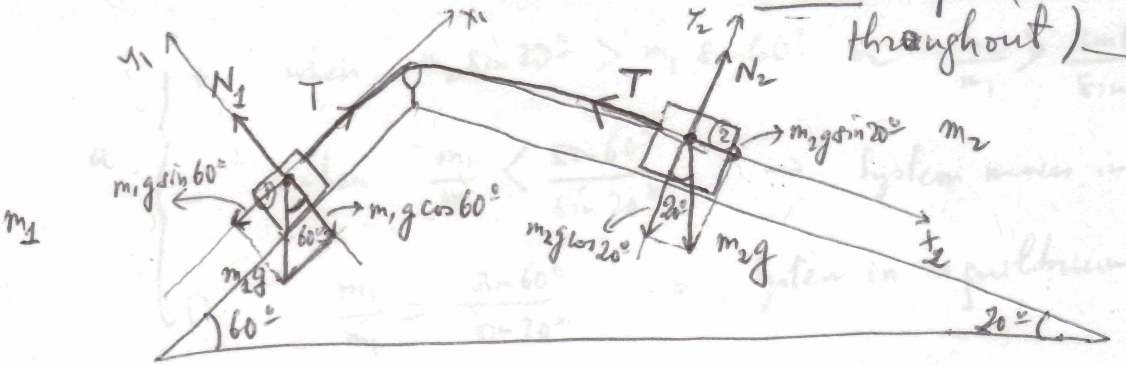
(5)
$$\begin{cases} F_{net\,x'} = -\mu_K N' = m a_{x'} \longrightarrow a_{x'} = \dfrac{-\mu_K m g}{m} = -0.12 \times 9.81 \\ F_{net\,y'} = N' - mg = m\,0 \rightarrow N' = mg \end{cases}$$

$$\boxed{a_{x'} = -1.18\ m/s^2}$$

→ constant deceleration from $v_0 = 15.9\ m/s$ to $v = 0$ → $x' - x_0' = \dfrac{0^2 - v_0^2}{2(-1.18)} = \boxed{107\,m}$

Ch5 (cont.) Method: 1) Sketch ✓ 2) Free-body diagram ✓ 3) Coord. for each object ✓ 4) components of force not along axes 5) Newton's 2nd Law

· Multiple objects: m_1 & m_2 connected together via a massless rope (same tension throughout) No friction



Massless rope:

T ← → T

same tension: $T - T = F_{net\ on\ rope} = 0 = m_{rope}\, a$

$0 \cdot a \rightarrow$ a could be non-zero.

(massless rope & same tension → non-violation of 2nd Newton's Law)

5) 2nd Newton's law for each object:

Object #1
$$\begin{cases} F_{net\,x_1} \overset{\circledast}{=} T - m_1 g \sin 60° = m_1 a_{1x} & (a) \\ F_{net\,y_1} = N_1 - m_1 g \cos 60° = 0 & (b) \end{cases}$$

⊛ We assume system move in CW direction (m_1 uphill & m_2 downhill)

Object #2
$$\begin{cases} F_{net\,x_2} = m_2 g \sin 20° - T = m_2 a_{2x} & (c) \\ F_{net\,y_2} = N_2 - m_2 g \cos 20° = 0 & (d) \end{cases}$$

6) Solve: for the common acceleration of the objects: $\boxed{a_{1x} = a_{2x}} \equiv a$

↳ (a) $T - m_1 g \sin 60° = m_1 a$

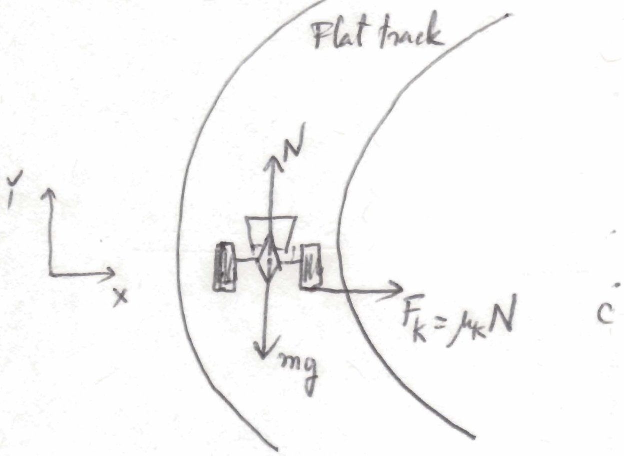
+ (c) $m_2 g \sin 20° - T = m_2 a$

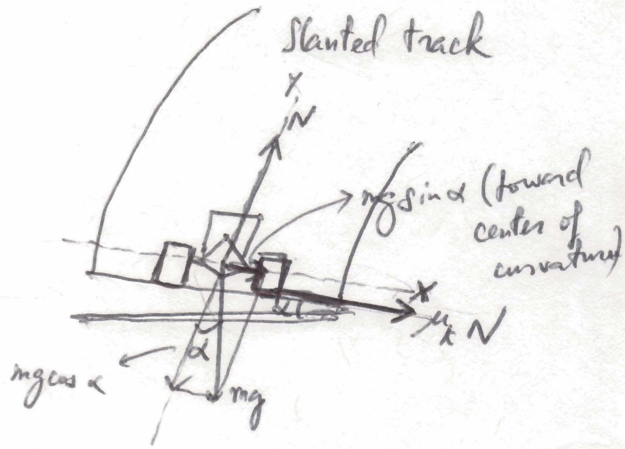
$g(m_2 \sin 20° - m_1 \sin 60°) = (m_1 + m_2) a \rightarrow \boxed{a = g \dfrac{m_2 \sin 20° - m_1 \sin 60°}{m_1 + m_2}}$

$$a \begin{cases} + & \text{when } m_2 \sin 20^\circ > m_1 \sin 60^\circ \text{ or } \dfrac{m_2}{m_1} > \dfrac{\sin 60^\circ}{\sin 20^\circ} \rightarrow \text{System moves in CW} \\ - & \text{when } \dfrac{m_2}{m_1} < \dfrac{\sin 60^\circ}{\sin 20^\circ} \rightarrow \text{System moves in CCW} \\ 0 & \dfrac{m_2}{m_1} = \dfrac{\sin 60^\circ}{\sin 20^\circ} \rightarrow \text{System in equilibrium} \end{cases}$$

<u>Circular Motion: (Uniform)</u> <u>Application of 2nd Newton's law to Car Racing.</u>

Turn: Radius of curvature is R





$$\text{2nd Newton Law} \begin{cases} F_{net_x} = \boxed{\mu_k N = m\dfrac{v^2}{R}} \\ F_{net_y} = N - mg = 0 \end{cases}$$

$$\text{2nd Newton law} \begin{cases} F_{net_x} = \boxed{mg\sin\alpha + \mu_k N = m\dfrac{v^2}{R}} \\ F_{net_y} = N - mg\cos\alpha = 0 \end{cases}$$

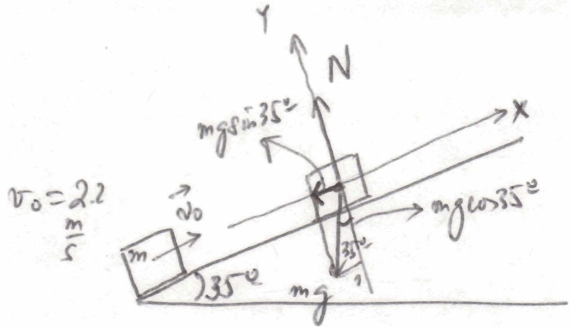
$\downarrow$

higher v during turn in slanted track

Possible scenarios: given m; μ_k; R what is v (max) for $\alpha = 0$ and $\alpha = 20^\circ$

5.33; 5.37

5.33



No friction

1) Sketch ✓ 2) FBD ✓ 3) X & Y 4) X & Y components for mg 5) 2nd Newton's law

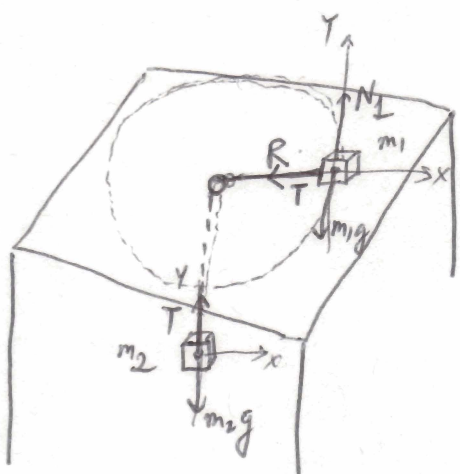
5) 2nd Newton's law
$$\begin{cases} F_{net_x} = -\cancel{m} g \sin 35° = \cancel{m} a_x & (\text{~~downward~~ down hill}) \\ F_{net_y} = N - mg\cos 35° = 0 \end{cases}$$

6) Solve for $x - x_0$ = 3rd Kinematic eq: $\dfrac{v^2 - v_0^2}{x - x_0} = 2a_x$

$$x - x_0 = \frac{v^2 - v_0^2}{2a_x} = \frac{-v_0^2}{2a_x} = \frac{v_0^2}{2g\sin 35°}$$

$$= \frac{2.2^2}{2 \times 9.81 \times \sin 35°} = \boxed{0.43\,m}$$

5.37



No friction

Massless string
↳ same tension

m_1: circular motion around hole at radius R
m_2: stationary ($a_2 = 0$)

m_1 & m_2 connected together

a) Tension in string T?

1) Sketch 2) FBD 3) Coord. XY 4) ✓ 5) 2nd Newton's law:

5) $m_1 \begin{cases} F_{net_x} = T = m_1 \dfrac{v^2}{R} & (i) \\ F_{net_y} = N_1 - m_1 g = 0 & (ii) \end{cases}$ $m_2 \begin{cases} F_{net_x} = 0 & (iii) \\ F_{net_y} = T - m_2 g = m_2 0 & (iv) \end{cases}$

$$\boxed{T = m_2 g}$$

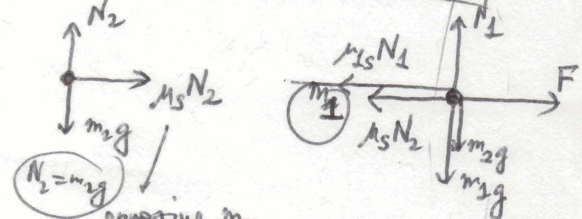
b) Period of circular motion: time to complete one turn $2\pi R$:

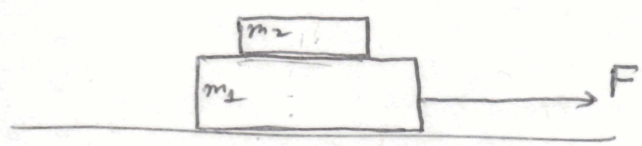
$$\hookrightarrow \frac{2\pi R}{v} = \boxed{2\pi\sqrt{\frac{m_1 R}{m_2 g}}}$$

2nd Newton's law: $v \overset{(i)}{=} \sqrt{\frac{TR}{m_1}} = \sqrt{\frac{m_2 g R}{m_1}}$

$$2\pi R \Big/ \sqrt{\frac{m_2 g R}{m_1}} = \frac{2\pi R}{\sqrt{\frac{m_2 g R}{m_1}}} = \frac{2\pi\sqrt{m_1 R}}{\sqrt{m_2 g}}$$

(5.48)





$m_1 = 1.2$ kg

$m_2 = 0.31$ kg

F applied on m_1

m_1 & m_2 accelerate together

$v_0 = 0 \rightarrow v = 0.96\frac{m}{s}$ in $\Delta t = 0.42$s

1) Sketch 2) FBD 3) Coord X & Y 4) ✓ 5) 2nd Newton's law

const. a: $v = v_0 + at \rightarrow a = \frac{v - v_0}{t} = \frac{0.96}{0.42}$

$\boxed{a = 2.28\, m/s^2}$

Then m_1 is brought to stop in 0.33s, m_2 slides off $\rightarrow$ estimate μ_s

(5) Focus on m_2: $\begin{cases} F_{net\,x} = \mu_s \cancel{m_2} g = \cancel{m_2} a \\ F_{net\,y} = N_2 - m_2 g = 0 \end{cases}$ $\rightarrow \mu_s = \frac{a}{g}$

$\downarrow$

min μ_s ($\mu_s \geq \frac{a}{g}$)

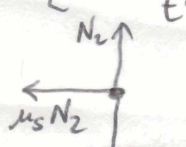
$$\rightarrow \boxed{\mu_s \geq \frac{a}{g} = \frac{2.28}{9.81} = 0.23}$$ for the two books to accelerate together

---

$\rightarrow$ constant deceleration: $\left[a' = \frac{v' - v_0'}{t'} = \frac{0 - 0.96}{0.33} = \boxed{-2.91\, m/s^2} \right.$

(leftward acceleration)

Focus on m_2:



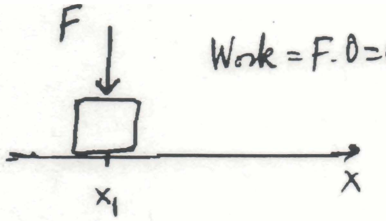
(under deceleration, top book tends to slide off to the right $\rightarrow$ static friction points left)

$F_{net\,x} = -\mu_s \cancel{m_2} g = \cancel{m_2} a' \rightarrow \mu_s = -\frac{a'}{g} = \frac{2.91}{9.81} = 0.3 \rightarrow \boxed{0.23 \leq \mu_s \leq 0.3}$

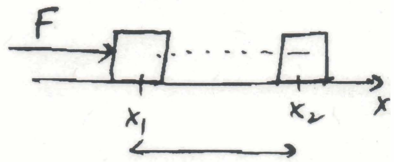
Ch 6 : Work, Energy and Power :

Work: $F \cdot \Delta x$ (unit : SI. $Nm = J$ (Joule))

Work $= F \cdot 0 = 0$

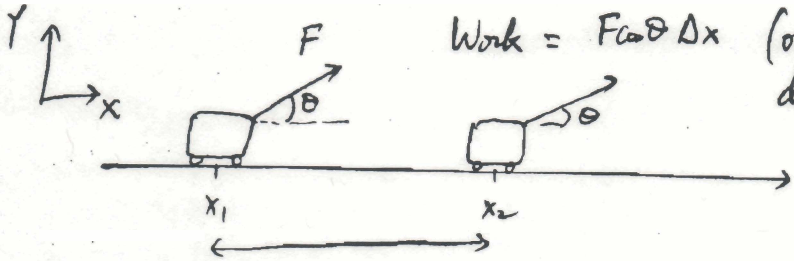


Work $= F \cdot \Delta x$



$\Delta x = x_2 - x_1$

Work $= F\cos\theta \, \Delta x$ (only force applied in the direction of the displacement did work)



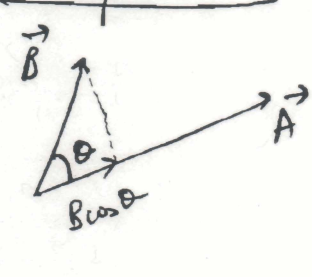
$\Delta x = x_2 - x_1$

Along y-direction : Work $= F\sin\theta \underbrace{\Delta y}_{0} = 0$

↳ Work done $= \vec{F} \cdot \Delta\vec{r}$

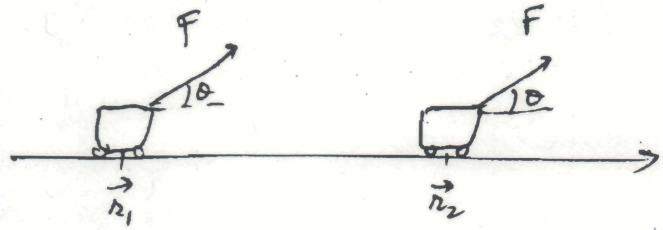
- $\vec{F}$ → Force applied (vector)
- $\cdot$ → scalar product (b/w 2 vectors produce a number)
- $\Delta\vec{r}$ → displacement

Scalar product: b/w two vectors $\vec{A}$ & $\vec{B}$



$$\vec{A} \cdot \vec{B} = A \underbrace{B\cos\theta}$$

projection of $\vec{B}$ onto the direction of $\vec{A}$

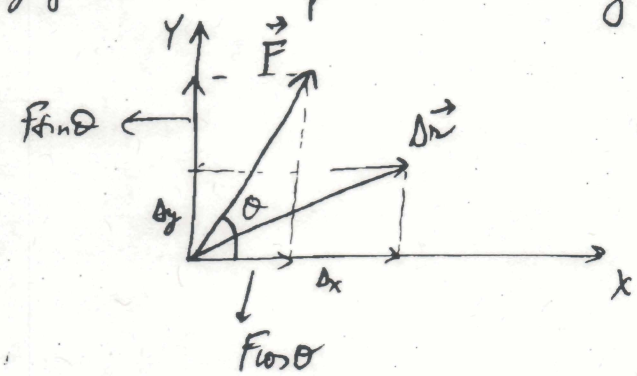


$$\vec{F}\cdot\Delta\vec{r} = F\,\Delta r\cos\theta$$

$$= \Delta r\,\underbrace{F\cos\theta}$$

projection of the force onto the direction of displacement

With the scaler product we can calulate the work done by forces and displacements along any direction:



$$\text{Work done} = \vec{F}\cdot\Delta\vec{r}$$

$$= \left(F\cos\theta\,\hat{i} + F\sin\theta\,\hat{j}\right)\cdot\left(\Delta x\,\hat{i} + \Delta y\,\hat{j}\right)$$

$$= F\cos\theta\,\Delta x + F\sin\theta\,\Delta y$$

$\hat{i}\cdot\hat{i} = 1$; $\hat{j}\cdot\hat{j} = 1$

$\hat{i}\cdot\hat{j} = 0$; $\hat{j}\cdot\hat{i} = 0$ $(\cos 90^\circ = 0)$

If the force applied is changing with the position:

$$\text{Work done} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

↳ infinitesimal displacement vector

↳ Work done in stretching a spring:

Hooke's law: $F = -kx$ (force applied by a person is kx)

↳ $\displaystyle\int_0^x F\,dx = \int_0^x kx\,dx = k\frac{x^2}{2}\Big]_0^x = \frac{1}{2}kx^2$

($\displaystyle\int^b x^n\,dx = \frac{x^{n+1}}{n+1}\Big]^b$)

Kinetic Energy:

2nd Newton's Law: $\vec{F}_{net} = \dfrac{d\vec{p}}{dt} \overset{m \text{ is constant}}{=} m\,\dfrac{d\vec{v}}{dt}$

$$\Delta K.E. = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net}\cdot d\vec{r} = m\int_{\vec{r}_1}^{\vec{r}_2} \frac{d\vec{v}}{dt}\cdot d\vec{r} = m\int_{\vec{r}_1}^{\vec{r}_2} d\vec{v}\cdot \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}}$$

$$= m\int_{\vec{r}_1}^{\vec{r}_2} v\,dv = \frac{1}{2}mv^2\Big]_{\vec{r}_1}^{\vec{r}_2} = \underbrace{\frac{1}{2}}mv_2^2 - \frac{1}{2}mv_1^2$$

linear motion: $\vec{v} \parallel d\vec{v} \rightarrow \vec{v}\cdot d\vec{v} = v\,dv\,\underbrace{\cos 0} = v\,dv$

Power: P Work or energy per unit time

Average power: $\bar{P} = \dfrac{\Delta \text{Work}}{\Delta t}$

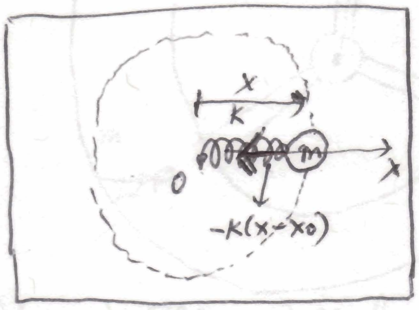
Instantaneous power: $P = \dfrac{d\,\text{Work}}{dt}$

Unit: $\dfrac{J}{s} = W$ (Watt)

Power & velocity: $P = \dfrac{d\,\text{Work}}{dt} = \dfrac{d}{dt}(\vec{F}\cdot\Delta\vec{r}) = \vec{F}\cdot\underbrace{\dfrac{d(\Delta\vec{r})}{dt}} = \vec{F}\cdot\vec{v}$

5.62 1) sketch: 2) FBD on mass m 3) coord X & Y 4) ✓ 5)

View from above:



No friction on air table.

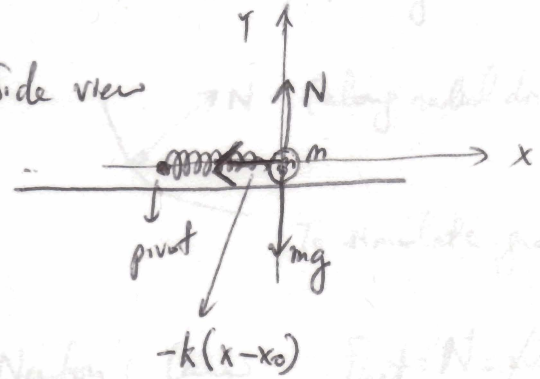
$m = 2.1\,kg$ → uniform circular motion $v = 1.4\, m/s$

$k = 150\,\frac{N}{m}$

$x_0 = 0.18\,m$ (unstretched length)

R?

Side view



5) 2nd Newton's Law:

$$\begin{cases} F_{net\,y} = N - mg = 0 \rightarrow N = mg \\ F_{net\,x} = \boxed{-k(x-x_0) = -m\frac{v^2}{(x)}} \end{cases}$$

separation b/w m & center of curvature

- Length of spring: x

~~change~~ displacement from unstretched length x_0 is $(x - x_0)$

$a = -\frac{v^2}{x}$ (toward center = toward negative x)

→ c) Solve for x:

$$kx - kx_0 = \frac{mv^2}{x}$$

$$kx^2 - kx_0 x = mv^2$$

$$kx^2 - kx_0 x - mv^2 = 0$$

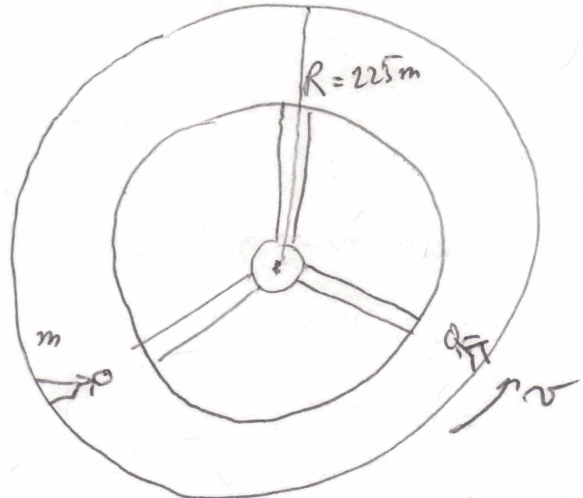
$$150x^2 - 27x - \underbrace{2.1\times1.4^2}_{4.12} = 0$$

$$\rightarrow x = \frac{27 \pm \sqrt{27^2 + 600\times4.12}}{300} \begin{cases} \boxed{x = 0.28\,m} \\ -0.097\,m \end{cases}$$

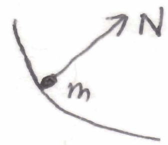
length of spring = radius of circular path.

5.56

Space station

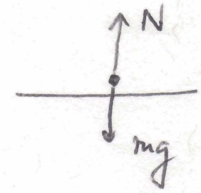


1) Sketch 2) FBD : 3) T along radial direction 4) v 5)



(along radial direction)

To simulate gravity : $N = mg$



on Earth.

5) 2nd Newton's Law : $F_{net} = N = \cancel{m}g = \cancel{m}\dfrac{v^2}{R}$

6) Solve for $v = \sqrt{gR} = \sqrt{9.81 \times 225}\ \dfrac{m}{s} = 46.98\ m/s$

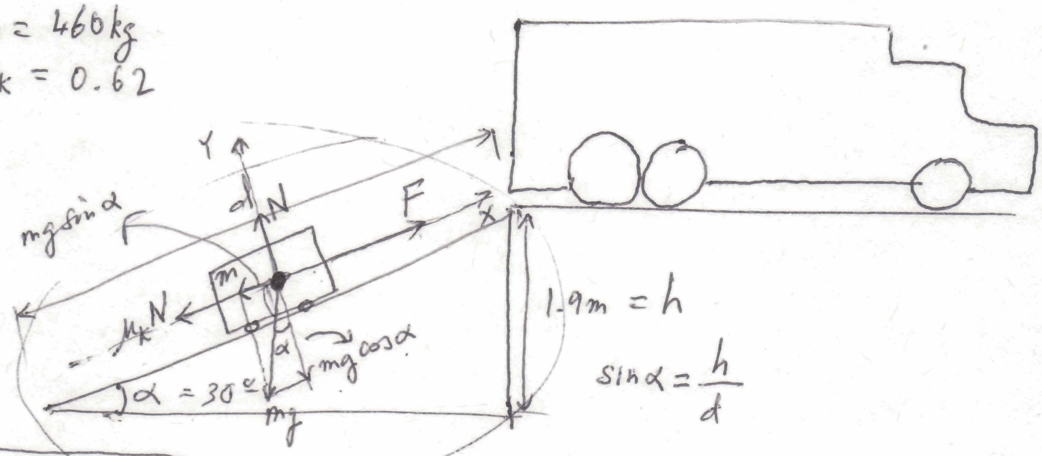
$$46.98\ \frac{\cancel{m}}{\cancel{s}} \cdot \frac{1\,rev}{2\pi \times 225\,\cancel{m}} \cdot \frac{60\,\cancel{s}}{1\,min} = 1.99\ \frac{rev}{min}$$

Ch 6 (Cont.) Work, Energy, and Power

Unit: SI
- Work : J (Joule)
- Energy: J (Joule)
- Power : $\frac{J}{s}$ = W (Watt)

Example:

$m = 460\,kg$
$\mu_k = 0.62$



$\sin\alpha = \frac{h}{d}$

$\boxed{\text{Work done}}$ in pushing the piano from bottom of ramp to top of ramp if $\alpha = 15°$ and if $\alpha = 30°$ (h is fixed) (in our sketch, d will be larger when $\alpha = 15°$).

$F \cdot d = F\frac{h}{\sin\alpha}$. What is F ? → force applied on piano.

1) Sketch ✓ 2) FBD ✓ 3) X & Y 4) components X & Y for mg 5) 2nd Newton's Law.

$$\begin{cases} F_{net_x} = \overbrace{\boxed{F - mg\sin\alpha - \mu_k N}}^{LHS} = \overbrace{\boxed{m a_x}}^{RHS} \\ F_{net_y} = N - mg\cos\alpha = 0 \end{cases}$$

$$\boxed{F = mg\sin\alpha + \mu_k mg\cos\alpha + m a_x}$$

Min. work required when $a_x = 0$ (constant speed)

$$\boxed{F = mg(\sin\alpha + \mu_k\cos\alpha)}$$

Work done : $W = F \cdot \frac{h}{\sin\alpha} = mg \frac{(\sin\alpha + \mu_k \cos\alpha)\, h}{\sin\alpha} = mgh\left(1 + \frac{\mu_k}{\tan\alpha}\right)$

$\hookrightarrow \begin{cases} \alpha = 15^\circ : W_{15^\circ} = 460 \times 9.81 \times 1.9 \left(1 + \frac{0.62}{\tan 15^\circ}\right) = 28.5\,kJ \\ \alpha = 30^\circ : W_{30^\circ} = 460 \times 9.81 \times 1.9 \left(1 + \frac{0.62}{\tan 30^\circ}\right) = 17.8\,kJ \end{cases}$

kilo = 1000

Force applied F

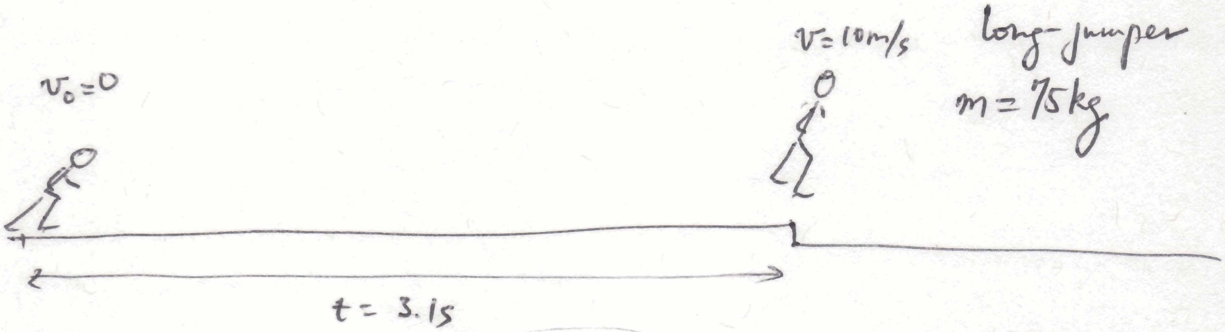
$\hookrightarrow \begin{cases} \alpha = 15^\circ \rightarrow F_{15^\circ} = 460 \times 9.81 \left(\sin 15^\circ + 0.62 \cos 15^\circ\right) = 3.88\,kN \\ \alpha = 30^\circ \rightarrow F_{30^\circ} = 460 \times 9.81 \left(\sin 30^\circ + 0.62 \cos 30^\circ\right) = 4.68\,kN \end{cases}$

Crew with ability to push up to 4kN $\rightarrow$ use 15° ramp

6.38
6.67;
6.80; 6.81;

6.38

long-jumper
$m = 75\,kg$



P (power output) $= \frac{\text{Work done } (0\,m/s \rightarrow 10\,m/s)}{\text{time } (3.1\,s)} = \frac{3750}{3.1} = 1.21\,kW$

Work done $(0\,m/s \rightarrow 10\,m/s) = \Delta KE = KE_{10\,m/s} - KE_{0\,m/s}$

$\boxed{KE = \frac{1}{2} m v^2}$

$= \frac{1}{2} 75 \times 10^2 - 0$

$= 3750\,J$

$1\,HP = 746\,W \rightarrow P = 1.21\,\cancel{kW}\, \frac{1\,HP}{0.746\,\cancel{kW}} = 1.6\,HP$

Kinetic energy:

$$\Delta KE = \text{Work done} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net} \cdot d\vec{r} = m \int_{\vec{r}_1}^{\vec{r}_2} \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

2nd Newton's Law

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

constant mass system.

$$= m \int_{\vec{r}_1}^{\vec{r}_2} d\vec{v} \cdot \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}} = m \int_{\vec{r}_1}^{\vec{r}_2} v\,dv = \frac{1}{2} m v^2 \Big]_{\vec{r}_1}^{\vec{r}_2} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

linear motion

$\vec{v} \parallel d\vec{v} \rightarrow \vec{v} \cdot d\vec{v} = v\,dv \cos 0 = v\,dv$

$$\rightarrow \boxed{KE = \frac{1}{2} m v^2}$$

---

w/o using $KE = \frac{1}{2}mv^2$

☆ calculate a, then $x - x_0$, find F_{net} applied by long-jumper

$a = 3.23\ m/s^2$; $d = x - x_0 = 15.5\ m$; $F_{net} = ma = 75\ kg \times 3.23\ \frac{m}{s^2}$

$= 241.94\ N$

$\rightarrow$ Work done $= F_{net} \cdot d = 3750\ J \rightarrow P = 1.6\ HP$

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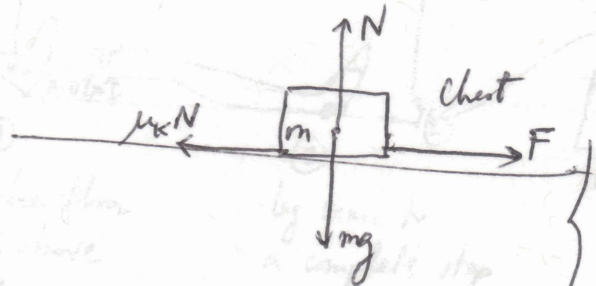
using $KE = \frac{1}{2}mv^2$

☆ $$F_{net} = \frac{\text{Work}}{d} = \frac{\frac{1}{2}mv^2 - 0}{d} = \frac{3750}{15.5} = 241.94\ N$$

6.67 a) P ? $m = 95\,kg$ @ $v = 0.62\,m/s$ (const.)

↳ $\frac{Work}{time}$ $\mu_k = 0.78$

$$= \frac{F \cdot d}{time} = F\, v = \mu_k m g v = 0.78 \times 95 \times 9.81 \times 0.62$$
?
$$= 450\,W$$



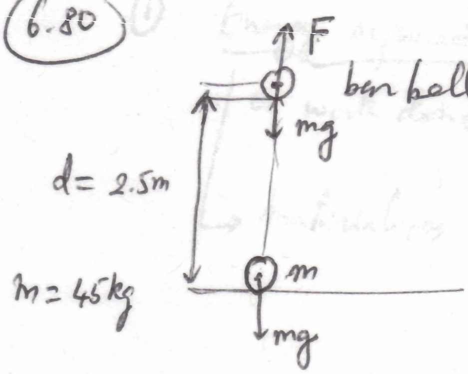
2nd Newton's law:

$F_{net\,x}$: $F - \mu_k N = ma = 0$ ↓ const. speed.

$F = \mu_k N = \mu_k mg$.

$F_{net\,y} = N - mg = 0 \rightarrow N = mg$.

6.80



$$230\,\cancel{cal}\ \frac{4.186\,kJ}{1\,\cancel{cal}} = 230 \times 4.186\,kJ$$

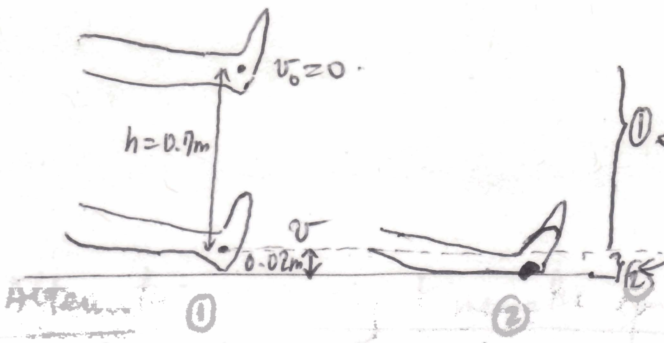
One lift would burn: $F.d = mgd$
$$= 45 \times 9.81 \times 2.5$$
$$= 1.1\,kJ$$

$F - mg = ma \rightarrow F = mg$.
↳ least economy: $a = 0$

$$230 \times 4.186\,\cancel{kJ}\ \frac{1\ lift}{1.1\,\cancel{kJ}} = 873\ lifts$$

How long ?

6.81



Leg touches floor CM still above the floor

Leg came to a complete stop w/ CM on floor

Leg: $m = 8$kg

Free fall : 0.7m

Stopping distance : 0.02m

Force on leg ? against claim of 80N

1^{st} alternative : using work / energy

1. Energy acquired by leg during free fall : $F \cdot d = F \cdot h = mgh$
   or work done on leg
   → materializes into K.E of leg = $\frac{1}{2} m v^2 = mgh$
   speed of leg as it touchs the floor (CM still above floor)

2. During stopping distance of 0.02m leg would loose all the KE acquired during 1 : by applying a force F_{stop} on floor such that:

$$F_{stop} \cdot d_{stop} = \frac{1}{2} m v^2 = mgh$$

$$F_{stop} = \frac{mgh}{d_{stop}} = \frac{8 \times 9.81 \times 0.7}{0.02} = 2744N$$

By 3^{rd} Newton's Law (Action & reaction) : floor exerts equal and opposite force on leg = 2744N

2nd Alternative: using kinematic equations & 2nd Newton's law

① Free fall: calculate v: $\frac{v^2 - 0^2}{h} = 2g \rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.7}$
$= 3.706 \text{ m/s}$

② Stopping distance: → calculate deceleration a, then F_{stop}

$$\frac{0^2 - 3.7^2}{0.02} = 2a \rightarrow a = \frac{-3.7^2}{0.04} = -343.35 \text{ m/s}^2$$

$$F_{stop} = ma = 8 \times (+342) = +2747 \text{ N}$$

↳ 2nd Newton's Law

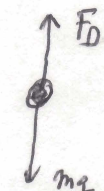
↳ Floor exerts equal & opposite force on leg!
(Not 80N)

Drag forces (air resistance) & terminal speed

↳ $F_D = \frac{1}{2} C \rho_{fluid} A_{body} v_{body}^2$

C: constant
ρ: fluid density
A: surface area of body
v: speed of body

Why?



$F_{net} = mg - F_D < mg$

Is F_D constant during a free fall? No: $F_D \propto v^2$
as $v \uparrow \rightarrow F_D \uparrow$ at some point $F_D = mg \rightarrow F_{net} = 0$
→ object has no longer any acceleration → continue to go down at const speed or terminal speed.

During a free fall do you want to get a terminal speed earlier or later?

$$F_D = mg \rightarrow \boxed{v_T = \sqrt{\frac{2mg}{C \rho_{fluid} A_{Body}}}}$$