Ch 3 (cont.) Projectile Motion

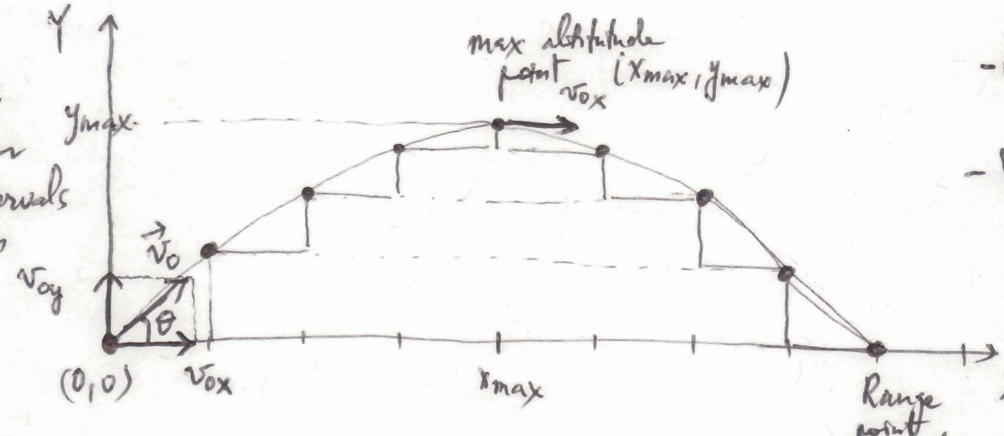
Kinematic equations (constant acceleration)

$$\begin{cases} (1) & \vec{v} = \vec{v}_0 + \vec{a}t \\ (2) & \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \end{cases}$$

↳ Assumption: motions along perpendicular directions are independent of each other.

Example: marble going along a horizontal track at a constant velocity, then it gets launched up vertically, the combined motion is a projectile motion

Snapshots at regular time intervals of position of particle



- Horizontal motion: uniform or const. vel
- Vertical motion: constant acceleration of gravity (up: deceleration; down: acceleration)

$$1)\begin{cases} v_x = v_{0x} = v_0\cos\theta \\ v_y = v_{0y} - gt = v_0\sin\theta - gt \end{cases}$$

$$2)\begin{cases} x = x_0 + v_0\cos\theta\, t \\ y = y_0 + v_0\sin\theta\, t - \frac{1}{2}gt^2 \end{cases}$$

$a = -g$ (deceleration) on the way up to the maximum altitude point. (x_{max}, y_{max})

Graphical description:

- Separation of different positions along x: 1) increasing 2) decreasing 3) same ✓
- " " " " " y: 1) increasing (down) 2) decreasing (up) 3) same

In our sketch $(x_0, y_0) = (0, 0)$

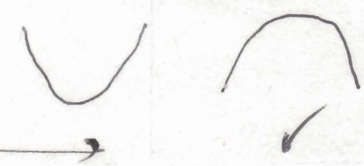
Eliminate time t to arrive at the "trajectory equation":

2) $x = v_0 \cos\theta \, t \rightarrow t = \dfrac{x}{v_0 \cos\theta}$

$$y = \cancel{v_0} \sin\theta \frac{x}{\cancel{v_0} \cos\theta} - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta}$$

$$\boxed{y = x \tan\theta - \frac{g}{2 v_0^2 \cos^2\theta} x^2}$$

Projectile motion is described with a parabola in the XY plane



Maximum altitude point: $(x_{max}, y_{max}) = \left(\dfrac{v_0^2 \sin 2\theta}{2g}, \dfrac{v_0^2 \sin^2\theta}{2g} \right)$

1) $v_y = v_0 \sin\theta - g t$

at $(x_{max}, y_{max}) \rightarrow v_y = 0 \rightarrow t_{max} = \dfrac{v_0 \sin\theta}{g}$

2) $x_{max} = v_0 \cos\theta \, t_{max} = \dfrac{v_0^2 \cos\theta \sin\theta}{g} = \dfrac{v_0^2 \sin 2\theta}{2g}$

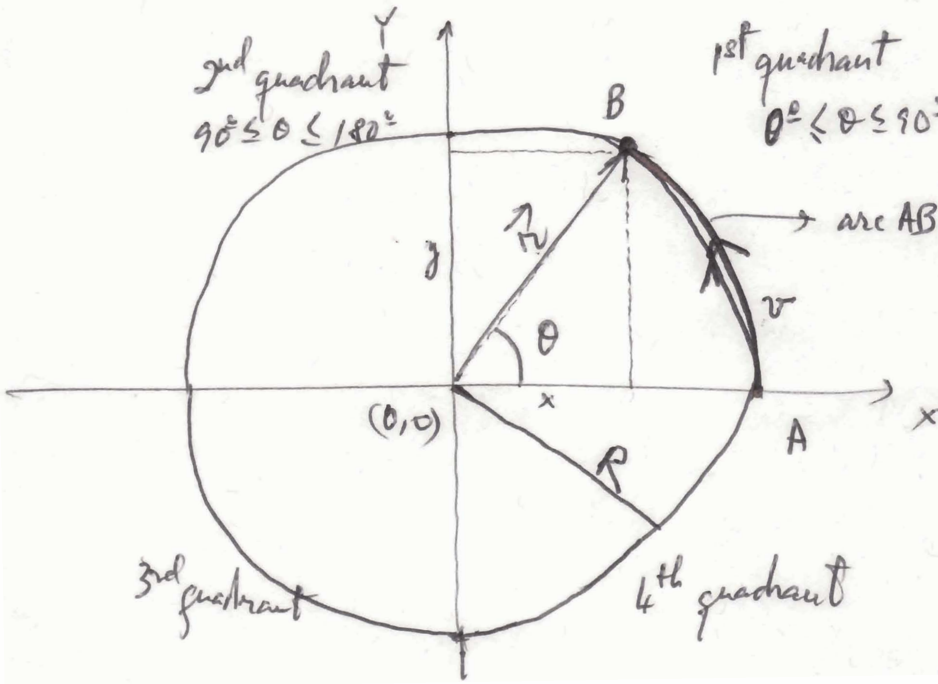
Trig: $2 \cos\theta \sin\theta = \sin 2\theta$

$y_{max} = v_0 \sin\theta \dfrac{v_0 \sin\theta}{g} - \dfrac{1}{2} g \dfrac{v_0^2 \sin^2\theta}{g^{\cancel{2}}} = \dfrac{v_0^2 \sin^2\theta}{2g}$

Range point: $(2x_{max}, 0) = \left(\dfrac{v_0^2 \sin 2\theta}{g}, 0 \right)$

Uniform Circular Motion:

↳ constant speed along a circular trajectory (not constant velocity!)



Assume it takes time t to go from A to B: $\theta = \frac{\text{arc}AB}{R} = \frac{vt}{R}$

(θ is increasing with t)

$$\vec{r} = x\hat{i} + y\hat{j} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j} = R\left[\cos\left(\frac{vt}{R}\right)\hat{i} + \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

(position vector $\vec{r}$ has fixed length R, but varying orientation)

$$\vec{v} = \frac{d\vec{r}}{dt} = R\left[-\frac{v}{R}\sin\left(\frac{vt}{R}\right)\hat{i} + \frac{v}{R}\cos\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$\rightarrow \vec{v} = v\left[-\sin\left(\frac{vt}{R}\right)\hat{i} + \cos\left(\frac{vt}{R}\right)\hat{j}\right]$$

(velocity vector $\vec{v}$ has $\underbrace{\text{fixed length } v}$, but varying direction)

$$|\vec{v}| = \sqrt{v^2\left(-\sin\frac{vt}{R}\right)^2 + v^2\left(\cos\frac{vt}{R}\right)^2} = v\sqrt{\sin^2() + \cos^2()} = v$$

→ uniform motion.

since $\vec{v}$ changes with time

$$\vec{a} = \frac{d\vec{v}}{dt} = v\left[-\frac{v}{R}\cos\left(\frac{vt}{R}\right)\hat{i} - \frac{v}{R}\sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$= -\frac{v^2}{R}\left[\cos\left(\frac{vt}{R}\right)\hat{i} + \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

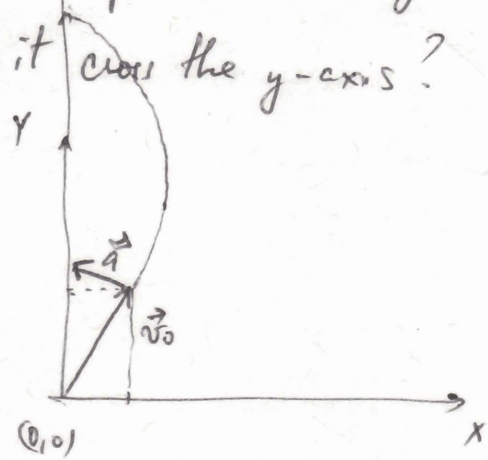
(acceleration vector has fixed length $\frac{v^2}{R}$, direction radially inward)

(3.60) $\vec{v}_0 = 11\,\hat{i} + 14\,\hat{j}\ \frac{m}{s}$; $\vec{a} = -1.2\,\hat{i} + 0.26\,\hat{j}\ \frac{m}{s^2}$

(const. acceleration)

a) When does particle cross y axis?

Will it cross the y-axis? Yes



$$2D \begin{cases} 1)\ \vec{v} = \vec{v}_0 + \vec{a}t \\ 2)\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \end{cases}$$

$$1) \begin{cases} v_x = v_{0x} + a_x t \\ v_y = v_{0y} + a_y t \end{cases} \qquad 2) \begin{cases} \boxed{x = 0 + v_{0x}t + \frac{1}{2}a_x t^2} \\ y = 0 + v_{0y}t + \frac{1}{2}a_y t^2 \end{cases}$$

When particle crosses y-axis: $x = 0 \rightarrow 0 = 11\cancel{t} - \frac{1.2}{2}t^{\cancel{2}}$

$$\rightarrow t = \frac{11}{0.6} = 18.3\,s$$

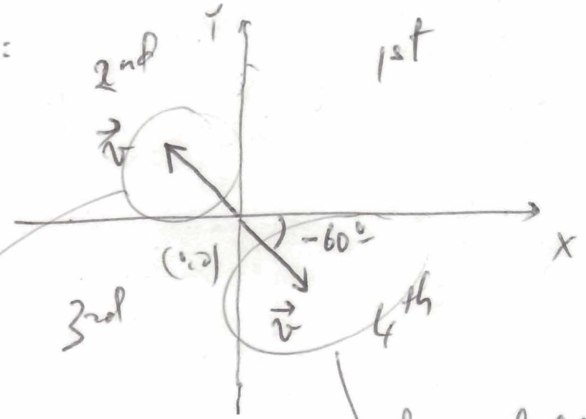
b) What is y at the crossing? $y = 14\,(18.3) + \frac{0.26}{2}(18.3)^2$

$$= 300\,m$$

c) How fast & in what direction it goes at crossing with Y axis?

$$\left.\begin{aligned} v_x &= 11 - 1.2\,(18.3) = -10.96\ m/s \\ v_y &= 14 + 0.26\,(18.3) = 18.8\ m/s \end{aligned}\right\} \rightarrow \begin{aligned} v &= \sqrt{10.96^2 + 18.8^2} = 21.7\,m/s \\ \theta_v &= \tan^{-1}\frac{18.8}{-10.96} = -60^\circ \end{aligned}$$

Quadrant check:



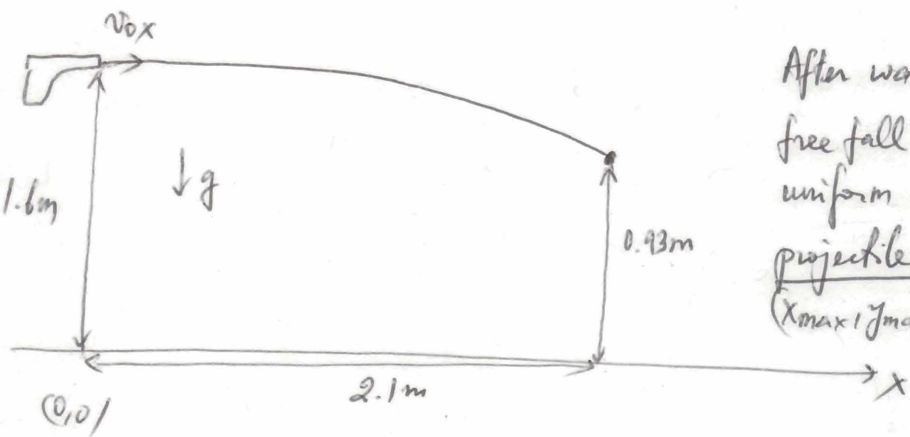
correct direction

$= \theta_{v\,(\text{calculator})} + 180° = -60+180°$

$= 120°$

$\rightarrow \quad \vec{v} = \begin{cases} v = 21.7 \text{ m/s} \\ \theta_v = 120° \end{cases}$

3.61



After water leaves gun: free fall along y & uniform motion along x = projectile motion from (x_{max}, y_{max}) to ~~(x_{Range}, y_{Range})~~ (x, y)

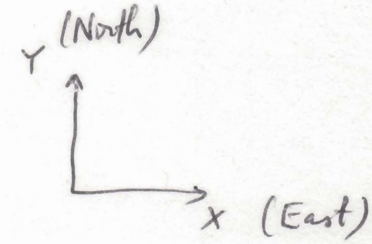
What was v_{0x}? $(v_{0y} = 0)$

$$2D \begin{cases} 1) \ \vec{v} = \vec{v}_0 + \vec{a}t \longrightarrow \begin{cases} v_y = 0 + gt \\ v_x = v_{0x} \end{cases} \\ 2) \ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \longrightarrow \begin{cases} x - x_0 = v_{0x}t \\ y - y_0 = -\frac{1}{2}gt^2 \rightarrow \end{cases} \end{cases}$$

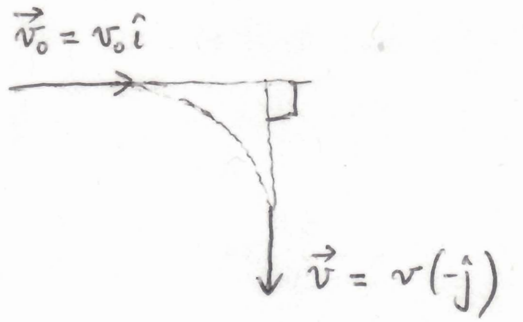
$\underbrace{0.93 - 1.6}_{-0.67} = -\frac{9.81}{2}t^2 \rightarrow t = \sqrt{\frac{2 \times 0.67}{9.81}}$

$\rightarrow x - x_0 = v_{0x}t \rightarrow \boxed{v_{0x} = \frac{x - x_0}{t} = \frac{2.1}{0.37} = 5.68 \text{ m/s}} \rightarrow \boxed{t = 0.37\text{s}}$

3.66 ; 3.29; 3.47; 3.60V



3.29

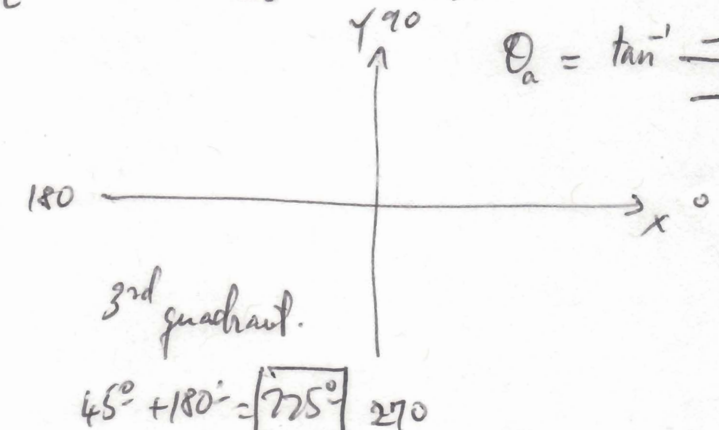


Speedometer reading remains constant → $\boxed{v = v_0}$

Average acceleration vector ? $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v} - \vec{v}_0}{\Delta t} =$

$\dfrac{-v_0 \hat{j} - v_0 \hat{i}}{\Delta t} = \dfrac{v_0}{\Delta t}\left(-\hat{i} - \hat{j}\right)$ → direction of $\vec{a}$ is

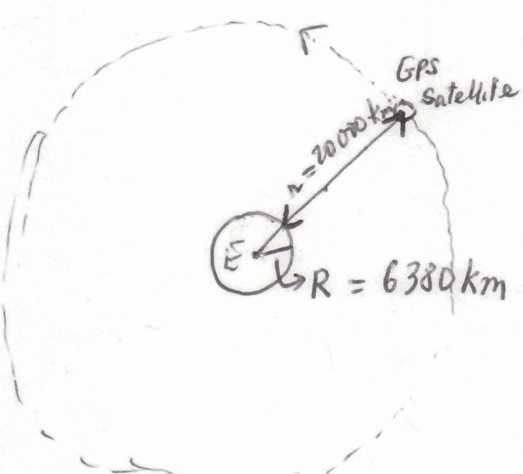
$\theta_a = \tan^{-1} \dfrac{-1}{-1} = 45°$



$45° + 180° = \boxed{225°}$

South west.

3.47



Orbital motion = uniform circular motion. (UCM)

Orbital period ?

$T = \dfrac{2\pi (R+r)}{v} = \dfrac{2\pi (R+r)}{\sqrt{0.058 \times 9.81 \times (R+r)}}$

$a = 0.058 \times g$

in UCM $a = \dfrac{v^2}{(R+r)} \rightarrow v = \sqrt{0.058 g \, (R+r)}$

$$\rightarrow T = \frac{2\pi\sqrt{26380000}}{\sqrt{0.058 \times 9.81}} \cancel{s} \cdot \frac{1h}{3600\cancel{s}} = 11.88 \text{ hrs} \simeq 12 \text{ hours.}$$

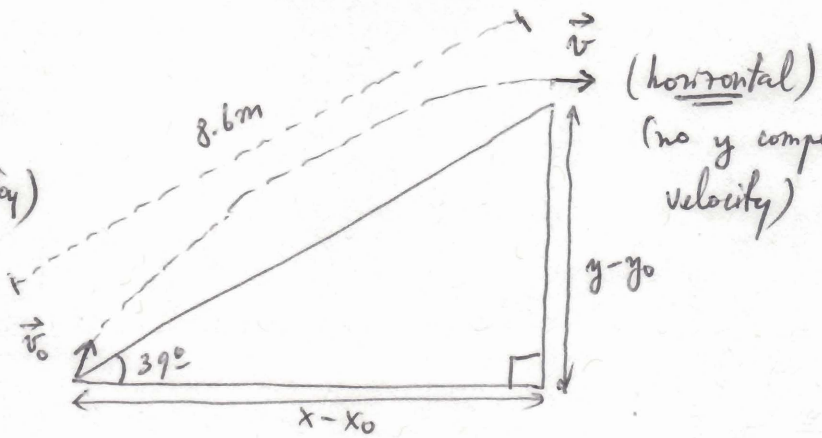
3.66

y, x

$\vec{v}_0$? (v_{0x}, v_{0y})

$\boxed{\text{Projectile motion}}$

$\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$



(1) $\vec{v} = \vec{v}_0 + \vec{a}t$ $\begin{cases} v_x = v_{0x} \\ v_y = 0 = v_{0y} - gt \end{cases}$ → Need t to continue.

We also have $\begin{cases} x - x_0 = 8.6 \cos 39° \\ y - y_0 = 8.6 \sin 39° \end{cases}$

(2) $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ $\begin{cases} x - x_0 = v_{0x} t \\ y - y_0 = v_{0y} t - \frac{1}{2} g t^2 \end{cases}$

$$\begin{cases} 8.6 \cos 39° = v_{0x} t & (a) \\ 8.6 \sin 39° = v_{0y} t - \frac{9.81}{2} t^2 & (b) \\ 0 = v_{0y} - 9.81 t & (c) \end{cases}$$

(c) $\rightarrow t = \frac{v_{0y}}{9.81}$

(c) in (b) $\rightarrow$ $8.6 \sin 39° = \frac{v_{0y}^2}{9.81} - \frac{\cancel{9.81}}{2} \frac{v_{0y}^2}{(9.81)^{\cancel{2}}} = \frac{1}{2} \frac{v_{0y}^2}{9.81}$

$$\rightarrow v_{0y} = \sqrt{2 \times 9.81 \times 8.6 \times \sin 39°} = 10.3 \text{ m/s}$$

(a) $v_{0x} = \frac{8.6 \cos 39°}{t} = \frac{9.81 \times 8.6 \times \cos 39°}{10.3} = 6.36 \text{ m/s}$

$$\vec{v}_0 = v_{ox}\hat{i} + v_{oy}\hat{j} = \underbrace{6.36\,\hat{i} + 10.3\,\hat{j}}_{1^{st}\ \text{quadrant}} \ \frac{m}{s} \rightarrow \text{polar} \begin{cases} \sqrt{6.36^2 + 10.3^2} \\ 12.1\,m/s \\ \theta_{v_0} = \tan^{-1}\frac{10.3}{6.36} \\ = 58.3^\circ \end{cases}$$

Alternative solution : observation : have $x-x_0$; $y-y_0$; a_x; a_y;
don't have time information $\rightarrow$ third equation :

$$\underbrace{\frac{v^2 - v_0^2}{\Delta x} = 2a}_{1D} \rightarrow \underbrace{\begin{cases} \dfrac{v_x^2 - v_{ox}^2}{x - x_0} = 2a_x & \text{(i) Not useful} \\ \dfrac{v_y^2 - v_{oy}^2}{y - y_0} = 2a_y & \text{(ii)} \end{cases}}_{2D}$$

(ii) $$v_{oy}^2 = -2a_y(y - y_0) \Rightarrow v_{oy} = \sqrt{-2a_y(y - y_0)}$$

$$= \sqrt{2 \times 9.81 \times 8.6 \sin 39^\circ}$$

$$= 10.3\,m/s$$

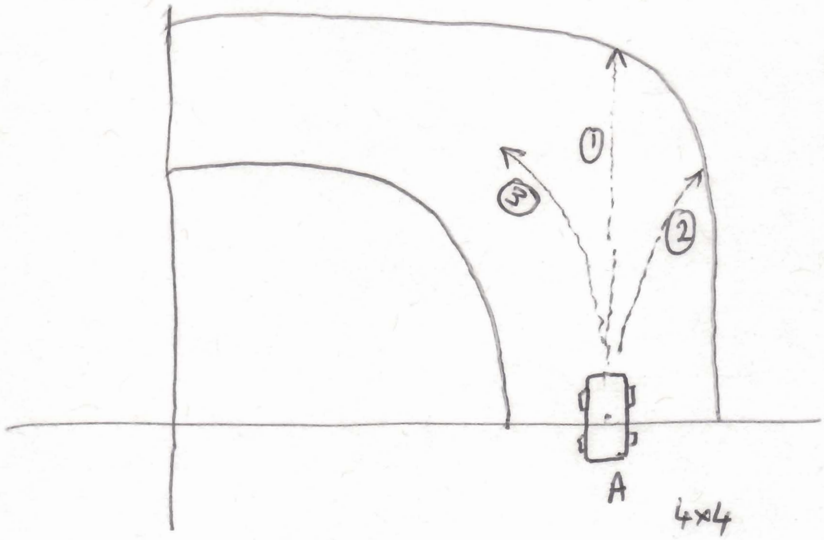
$$v_{ox} = \frac{x - x_0}{t} = \frac{8.6 \cos 39^\circ}{\frac{10.3}{9.81}} = 6.36\,m/s$$

$$v_y = v_{oy} - gt$$

$$0 = v_{oy} - gt \rightarrow t = \frac{v_{oy}}{g}$$

$$\rightarrow \vec{v}_0 = 6.36\,\hat{i} + 10.3\,\hat{j} \ \ m/s \rightarrow \begin{cases} 12.1\,m/s \\ 58.3^\circ \end{cases}$$

Ch. 4 Force & Motion:



icy road, down slope at A (entering a 90° turn at constant speed)

Very icy: 1 likely happens Why? a force (friction) that would change direction of motion under normal conditions is absent. Conclusion: a force is needed to change a motion → change a velocity $\vec{v} = (v, \theta)$

Even if v stays constant, changing its direction requires a force. Direction is important for force → Force is a vector. Sometimes there could be more than one force involved; in that case, the net force is the one that will change the motion.

1st Newton's Law: a body in uniform motion will stay in uniform motion, a body at rest will stay at rest, unless there is a net force acting on the body. (Law of inertia)

2nd Newton's Law: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\left\{ \vec{p} \equiv \text{linear momentum} \equiv m\vec{v} \right.$

$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt} = \underbrace{\frac{dm}{dt}\vec{v}} + m\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}$$

If the body's mass is not changing with time : $\frac{dm}{dt} = 0$

$\rightarrow \vec{F}_{net} = m\vec{a}$

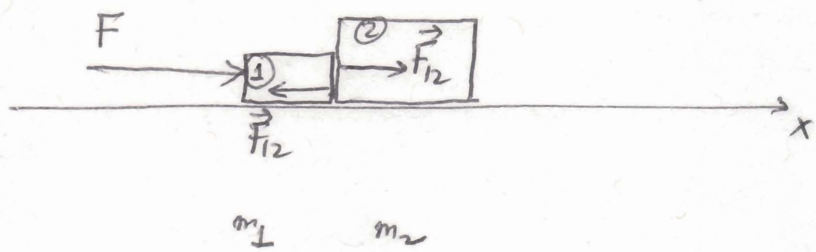
If the body's mass changes significantly with time : as in a rocket ship where lots fuel is burned. → 1st term should be included.

. Dimension of the force : $[F] = \frac{[p]}{[t]} = \frac{[m][v]}{[t]} = \frac{M\frac{L}{T}}{T}$

$$= \frac{ML}{T^2}$$

. Unit in SI : $kg\frac{m}{s^2} = N$ (Newton)

<u>3rd Newton's Law</u> : If A exerts a force on B, B exerts an equal and opposite force on A (Law of action & reaction)



No friction

. F is applied on box ① ; the two move ; what force is changing motion of ② ?

$\left\{ \begin{array}{l} \cancel{F} \\ \tilde{F} = F_{12} \end{array} \right.$

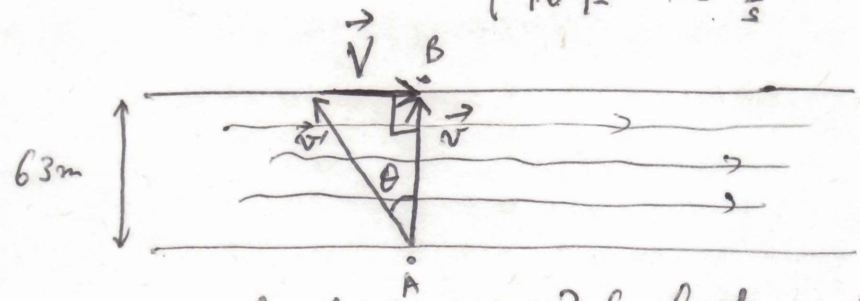
Net force on ① $\left\{ F - F_{12} \right.$

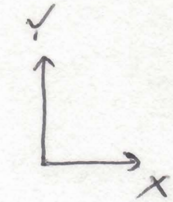
3.73 ✓.

3.34; 3.49 ; 4.53 ; 4.16 ; 4.38

3.34 | 2D relative motion

$$\begin{cases} \vec{V} = 0.57\,\hat{i}\ \left(\frac{m}{s}\right) \\ |\vec{v}'| = 1.3\ \frac{m}{s} \end{cases}$$





a) What is the direction of $\vec{v}'$? (velocity vector of boat wrt. water)

→ We want $\vec{v} = \vec{v}' + \vec{V}$, which the velocity vector of boat wrt ground, to point along AB (straight across river)

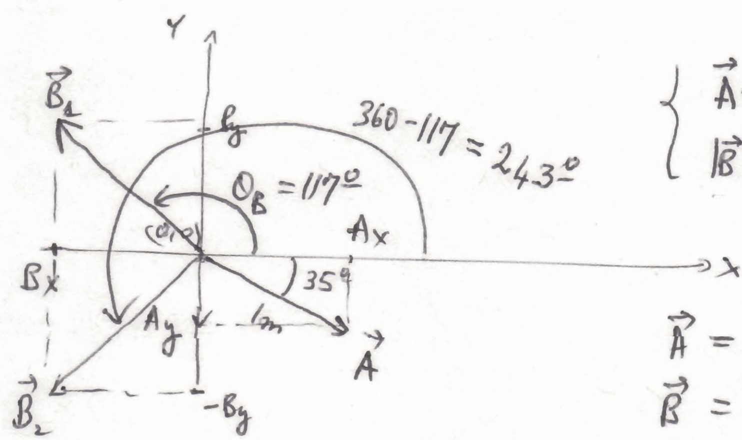
→ So $\vec{v}'$ needs to point to left of AB, forming an angle θ:

$$\theta = \sin^{-1}\frac{|\vec{V}|}{|\vec{v}'|} = \sin^{-1}\frac{0.57}{1.3} = 26^\circ$$

b) How long will it take you to cross the river?

$$t = \frac{63\,m}{|\vec{v}|} = \frac{63}{\sqrt{v'^2 - V^2}} = \frac{63}{\sqrt{1.3^2 - 0.57^2}} = \frac{63}{1.17} = 53.9\,s$$

3.49 |



$$\begin{cases} \vec{A} + \vec{B} = z\,\hat{j} \\ |\vec{B}| = 1.8\,m \end{cases}$$

$$\vec{A} = 1\cos 35\,\hat{i} - 1\sin 35^\circ\,\hat{j}$$

$$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j}$$

$$\vec{A} + \vec{B} = \underbrace{(B_x + \cos 35)}_{0}\,\hat{i} + (B_y - \sin 35^\circ)\,\hat{j}$$

→ $B_x = -\cos 35^\circ = -0.82\,m$

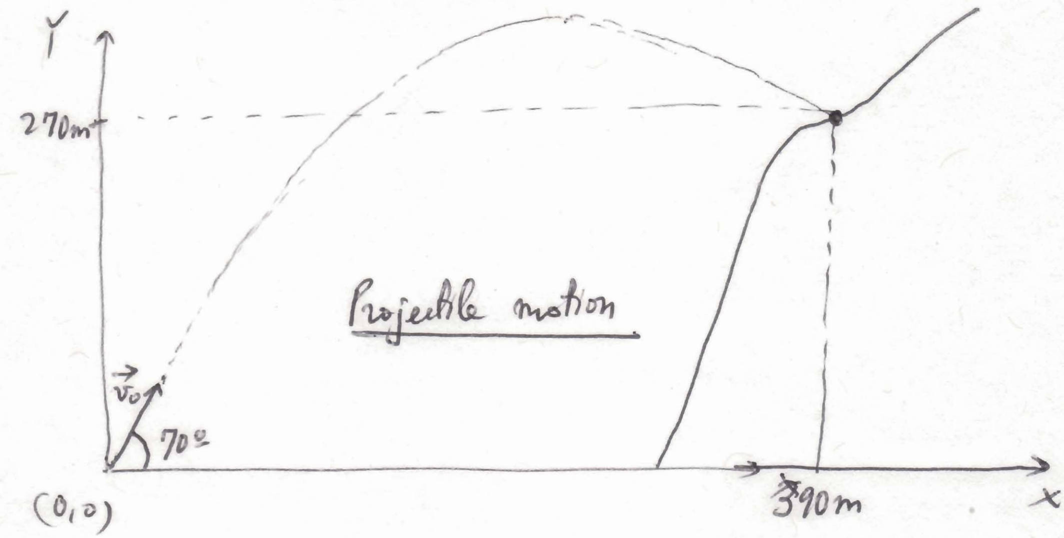
$B_y = \pm\sqrt{1.8^2 - (-0.82)^2} = 1.6\,m$ } $\vec{B}_1 = -0.82\,\hat{i} + 1.6\,\hat{j}$ (m)

$\theta_B = \tan^{-1}\frac{1.6}{-0.82} = -62.9^\circ + 180^\circ = 117^\circ$

Also $\vec{B}_2$ would satisfy $\vec{A} + \vec{B}_2 \parallel \hat{y}$-axis.

$$\begin{cases} \vec{B}_1 = -0.82\,\hat{i} + 1.6\,\hat{j} & (\theta_{B_1} = 117^\circ \text{ CCW from X-axis}) \\ \vec{B}_2 = -0.82\,\hat{i} - 1.6\,\hat{j} & (\theta_{B_2} = 243^\circ \text{ CCW from X-axis}) \end{cases}$$

3.73



v_0 ?

Trajectory equation : $$y = x \tan\theta - \frac{g}{2 v_0^2 \cos^2\theta} x^2$$

(initial point at origin of coordinates $x_0 = y_0 = 0$ & θ the angle of the initial velocity)

Can I find v_0 from this equation? $\left\{ \begin{array}{l} \theta \checkmark \\ x = 390\,m;\ y = 270\,m \checkmark \\ g = 9.81\,m/s^2 \end{array} \right.$

$$\rightarrow \frac{1}{v_0^2} = (x\tan\theta - y)\frac{2\cos^2\theta}{g x^2} \Rightarrow v_0 = \frac{x}{\cos\theta}\sqrt{\frac{g}{2}(x\tan\theta - y)}$$

$$\rightarrow v_0 = \frac{390}{\cos 70}\sqrt{\frac{9.81}{2}(390\tan 70^\circ - 270)} = 89.2\ m/s$$

<u>Alternative solution:</u>

1) $v_x = v_{0x} \quad (a_x = 0)$
$v_y = v_{0y} - gt \quad (a_y = -g)$ } Projectile motion

2) $x - x_0 = v_{0x} t \rightarrow 390 = v_{0x} t$ (a)

$y - y_0 = v_{0y} t - \frac{1}{2} g t^2 \rightarrow 270 = v_{0y} t - \frac{1}{2} g t^2$ (b)

$\tan 70° = \frac{v_{0y}}{v_{0x}} \rightarrow v_{0y} = v_{0x} \tan 70°$

(a) $390 = v_{0x} t \longrightarrow t = \frac{390}{v_{0x}}$

(b) $270 = v_{0x} \tan 70° \, t - \frac{1}{2} g t^2$

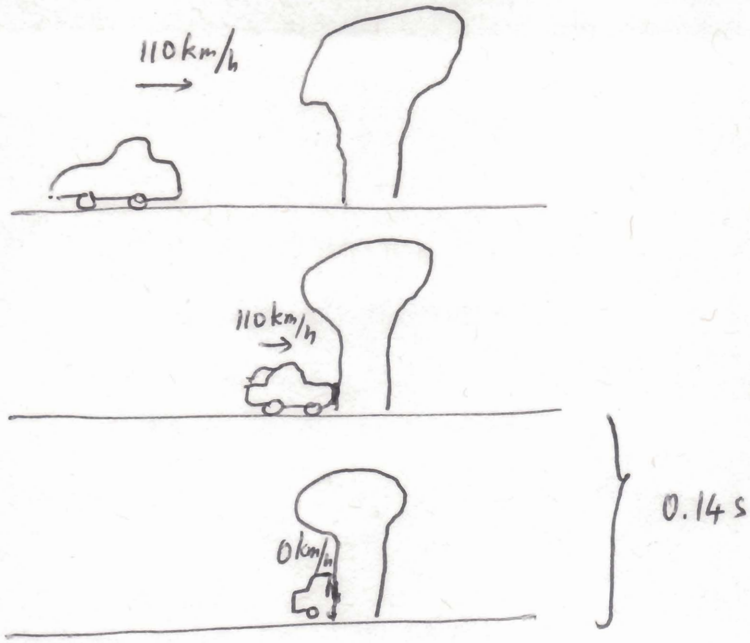
$270 = 390 \tan 70° - \frac{1}{2} 9.81 \frac{390^2}{v_{0x}^2}$

$\rightarrow v_{0x} = \sqrt{\frac{9.81 \times 390^2}{2(390 \tan 70° - 270)}} = 30.5 \, m/s$

$\rightarrow v_{0y} = v_{0x} \tan 70° = 83.5 \, m/s.$

$\rightarrow v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{30.5^2 + 83.5^2} = 89.2 \, m/s$

4.16



$m = 60\,kg$
passenger w/ seat belt
$v_0 = 110\,km/h$.

$v = 0\,km/h$
seat belt brought passenger from v_0 to 0
→ exerting a stopping force

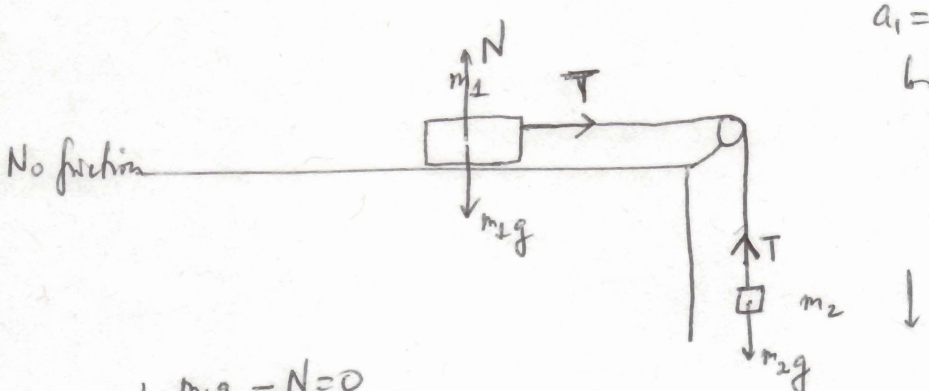
$$v = v_0 - at$$
$$0 = v_0 - at \rightarrow a = \frac{v_0}{t}$$

$$v_0 = 110\,\frac{km}{h} \cdot \frac{1h}{3600s} \cdot \frac{1000m}{1km}$$
$$= 30.56\,m/s$$

$$F = m \cdot a$$
$$= 60kg \cdot \frac{30.56\,\frac{m}{s}}{0.14s}$$
$$= 13095\,N$$
(like under ~20 people)

4.

$a_1 = a_2$ same as both objects are connected together.



$$F_{net_2} = m_2 g - T = m_2 a$$

$$m_1 \begin{cases} m_1 g - N = 0 \\ F_{net_1} = T = m_1 a \end{cases}$$

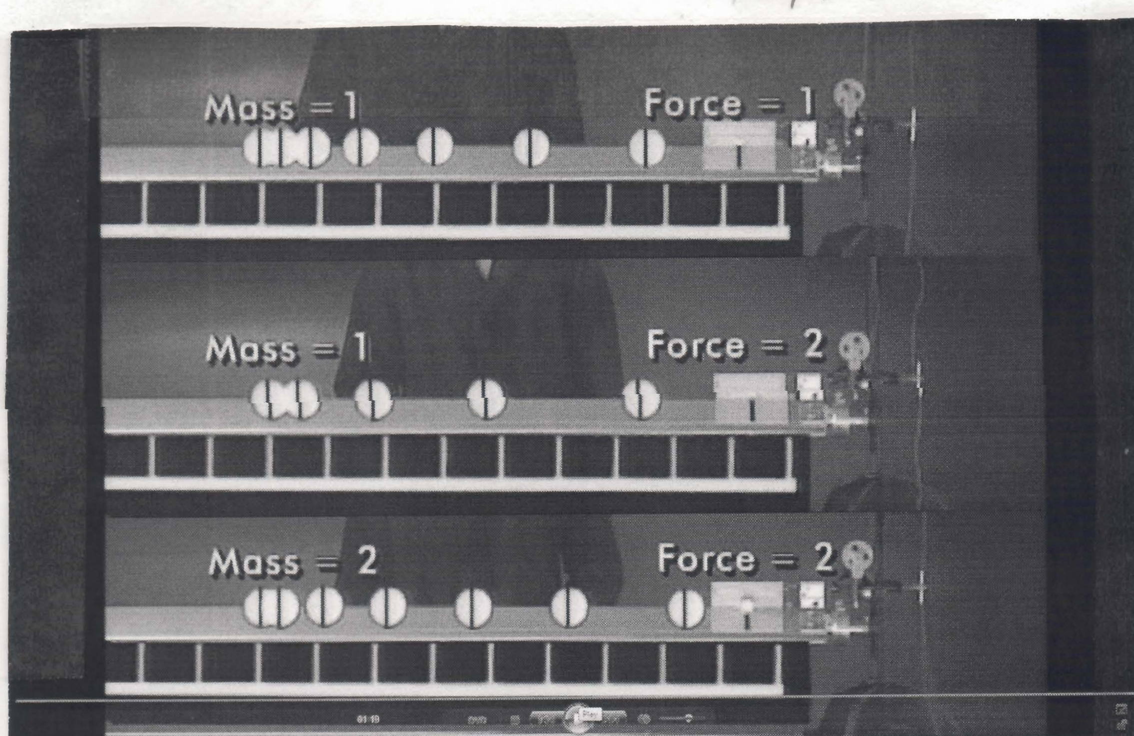
$$m_2 g - m_1 a = m_2 a$$

$$m_2 g = (m_1 + m_2) a$$

$$\boxed{a = \frac{m_2}{m_1 + m_2} g}$$

#2) Double m_2 $\rightarrow$ $a' = \dfrac{2m_2}{m_1 + 2m_2} g$: $\boxed{2a > a' > a}$.

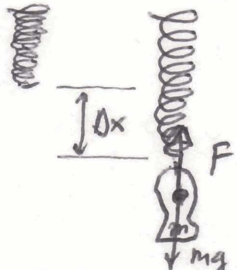
#3) Double m_2 & m_1 $\rightarrow$ $a'' = \dfrac{\cancel{2}m_2}{\cancel{2}m_1 + \cancel{2}m_2} g = a$



4.38

$k = 340\ N/m$ spring : Hooke's law: $F = -k\Delta x$ stretch retracting

$m = 6.7kg$



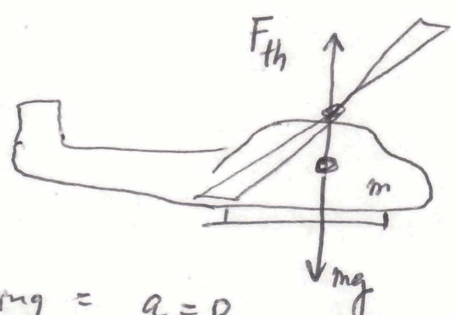
What is Δx ? Looking at F_{net} on fish :

$$F_{net} = 0 = mg - F = mg - k\Delta x$$

$$\rightarrow \Delta x = \frac{mg}{k} = \frac{6.7 \times 9.81}{340}\ m = 0.193\ m$$

4.53

$m = 4300kg$



a) Hovering = $a = 0$

F_{th}: thrust force by air on blades ; $-F_{th}$ is force exerted by blades on air (Law of action & reaction or Newton's third law)

$$\rightarrow F_{net} = ma = 0 = mg - F_{th} \rightarrow -F_{th} = -mg = -4300 \times 9.81$$

$$\boxed{F_{th} = -\ 42 \times 10^3\ N}$$

pointing downward.

b) helicopter dropping at 21 m/s with $a = -3.2\ m/s^2$

(upward acceleration of $3.2\ m/s^2$)

$$F_{net} = ma = mg - F_{th} \rightarrow -F_{th} = ma - mg = m(a-g)$$

$$= -4300(-3.2 - 9.81)$$

$$\boxed{-F_{th} = -55.9 \times 10^3\ N}$$

c) Helicopter rising at 17 m/s , speed increasing at $a = 3.2 m/s^2$

$\downarrow$

$F_{net\ upward} = ma = F_{th} - mg$

$\rightarrow \quad -F_{th} = -ma - mg = -m(a+g)$

$= -4300(3.2 + 9.81)$

$= \boxed{-55.9 \times 10^3\ N}$

d) Helicopter rising at steady 15 m/s $\rightarrow a = 0$

$\downarrow$

$F_{net\ upward} = ma = 0 = F_{th} - mg$

$-F_{th} = -mg = -4300 \times 9.81 = \boxed{-42 \times 10^3 N}$

e) Helicopter rising at 15 m/s w/ speed decreasing at $3.2 m/s^2$

$\hookrightarrow a_{up} = -3.2 m/s^2$

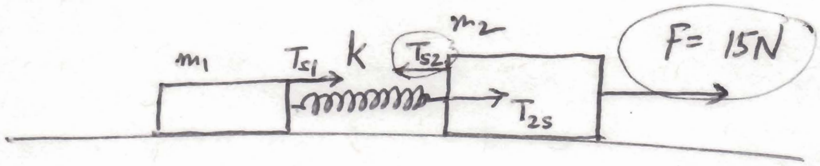
$F_{net\ upward} = ma = F_{th} - mg$

$-F_{th} = -ma - mg = -m(a+g)$

$= -m(-3.2 + 9.81)$

$= \boxed{-28.4 \times 10^3\ N}$

4.51



No friction

$m_1 = 2kg$
$m_2 = 3kg$
$k = 140 N/m$

Both are going at same acceleration a since they are connected together

What is Δx for spring? $\Delta x = \frac{T_{2s}}{k}$; T_{2s}: tension force of m_2 on spring

→ by Newton's third Law: T_{s2} tension force of spring on m_2 : $T_{2s} = T_{s2}$

We only add forces applying on a same object!

Similar happens b/w spring & m_1 $\left\{ T_{s1} = T_{s2} \equiv T \right.$

Looking at $\begin{cases} m_1 = \text{Net on } m_1 \; F_{net_1} = T_{s_1} = T_{s2} = \boxed{T = m_1 a} \\ m_2 : \text{Net force on } m_2 : F_{net_2} = F - T_{s2} = \boxed{F - T = m_2 a} \end{cases}$

$$\begin{cases} T = m_1 a \rightarrow a = \frac{T}{m_1} \\ F - T = m_2 a \longrightarrow F - T = \frac{m_2}{m_1} T \rightarrow T = \left[\frac{F}{\left(1 + \frac{m_2}{m_1}\right)}\right] \end{cases}$$

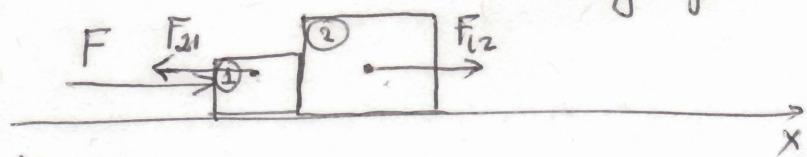
$$= \frac{15}{1 + \frac{3}{2}}$$

$$T = \frac{15}{\frac{5}{2}} = 6N$$

$$\boxed{\Delta x = \frac{T}{k} = \frac{6N}{140 \frac{N}{m}} = 0.0429 m}$$

Ch4 (cont.)

The two boxes are moving together → same acceleration a



No friction b/w boxes & surface they're laying on

m_1
m_2

Box #2 : Net force is F_{12} : exerted by 1 on 2
(The weight and normal forces are not relevant to horizontal motion along x)

2nd Newton's Law: $F_{12} = m_2 a$

Box #1: Net force is $F - F_{21} = m_1 a$ (2nd Newton's law)

$$F_{21} = F_{12} \quad \text{(3rd Newton's law)}$$

$$\rightarrow \quad F - m_2 a = m_1 a$$

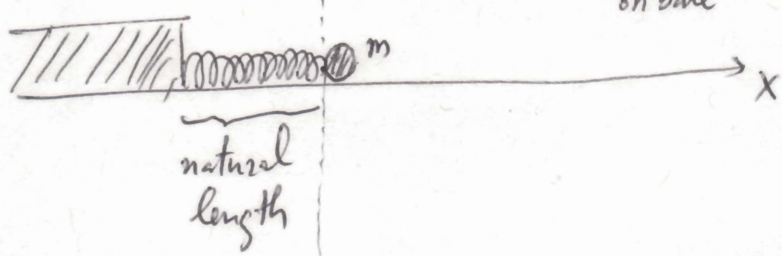
$$F = (m_1 + m_2) a$$

$$a = \frac{F}{m_1 + m_2}$$

Measuring forces:

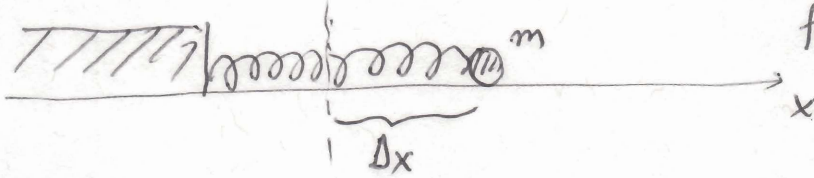
Spring balance : Hooke's law : $F = -K \Delta x$ $\begin{cases} K: \text{spring constant} \\ \Delta x: \text{stretch or displacement w.r.t. the natural length} \end{cases}$

$$F_{\text{on ball}} = -k\Delta x = 0$$



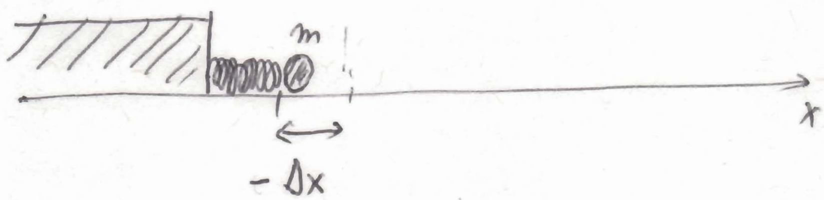
$$F_{\text{on ball}} = -k\Delta x$$

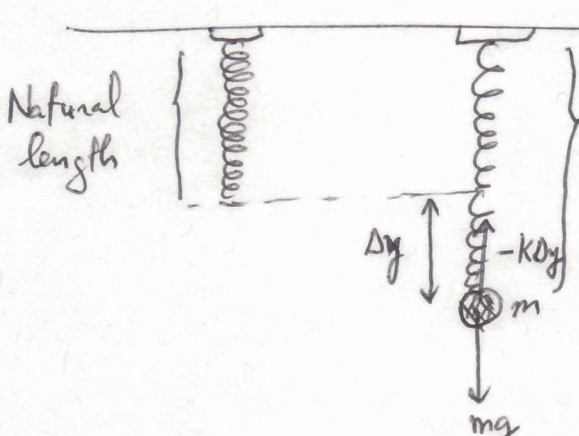
force is opposite to displacement of the ball



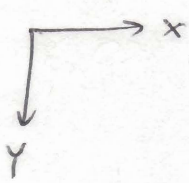
$$F_{\text{on ball}} = -k(-\Delta x) = +k\Delta x$$

force is opposite to displacement of the ball





New equilibrium length



→ Net force on m: $mg - k\Delta y$

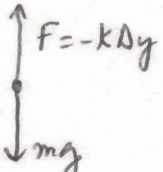
If the mass is not moving → $a = 0$

$$\rightarrow mg - k\Delta y = ma = 0$$

$$\boxed{\Delta y = \frac{mg}{k}}$$

<u>Free-body diagram</u>:

Forces on mass m: now described as a point.



Fritional forces: when a body is in contact with a surface

Static friction:

- $F_s = \underbrace{\mu_s}_{} \tilde{N}$ — Normal force exerted by surface on body

  "mu sub s" : coefficient of static friction

- Threshold force for some object to start moving

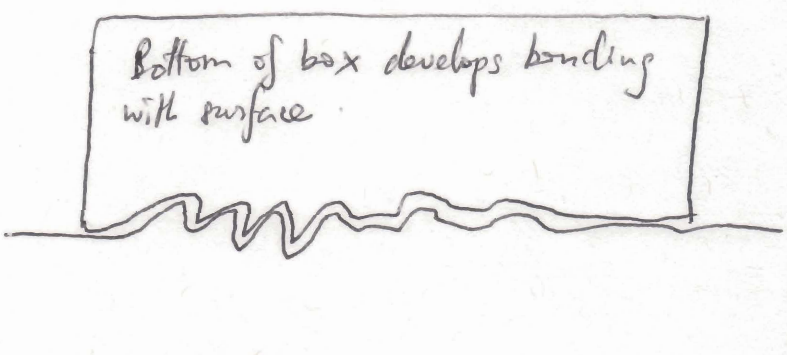
Kinetic friction:

- $F_k = \mu_k N$

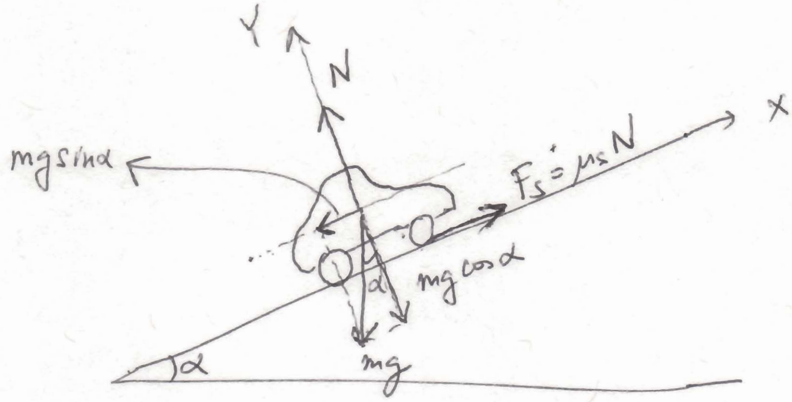
- while object is moving on a surface

Observation: when trying to push a heavy box, we apply increasing force, suddenly it starts moving (we reached the threshold to overcome static friction). As we continue without increasing the force applied the object acquires good acceleration ⇒ kinetic friction is lower than static friction.

Explanation:



Static friction:



→ Car is parked on a slope of angle α

$\boxed{\text{Forces on car:}}$

- mg (vertical) ✓
- N ✓

$\boxed{-mg\cos\alpha + N = 0}$ (no motion perpendicular to slope)

$mg\sin\alpha$

If there is no friction car would slide down

- $F_s = \mu_s N$

Friction has no unique direction like weight (always downward, toward Earth's center): it always tends to oppose motion.
In this case car tends to slide down → F_s points up hill.
with magnitude $\mu_s N = \mu_s mg\cos\alpha$.

A minimum of μ_s is needed : $\mu_s N \geq mg\sin\alpha$

$$\mu_s \cancel{mg}\cos\alpha \geq \cancel{mg}\sin\alpha$$

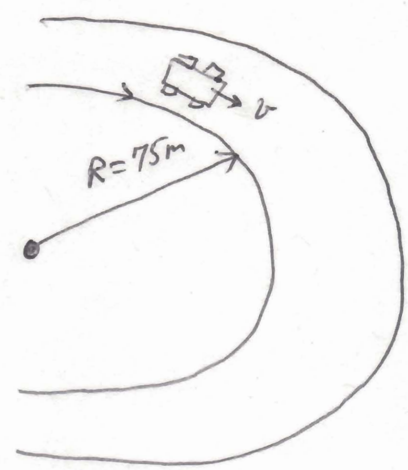
$$\boxed{\mu_s \geq \tan\alpha}$$

(if $\mu_s < \tan\alpha$ → car will slide down hill)

μ_s & μ_K depends on surface area in contact & roughness of materials.

3.45

$\boxed{\text{Uniform Circular Motion}}$: car rounding a turn of $R = 75m$ at constant speed.



v? so $a = g$ toward center of curvature

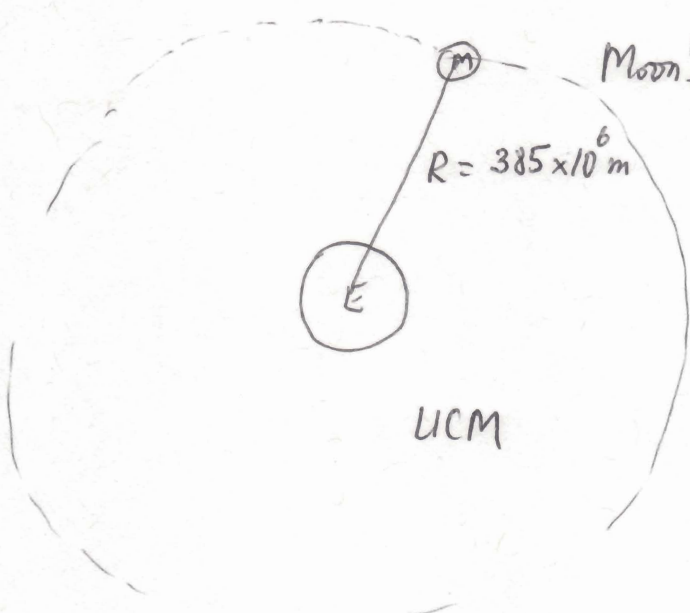
$$a = \frac{v^2}{R} \longrightarrow v = \sqrt{aR}$$

$$= \sqrt{9.81 \times 75} =$$

$$= 27.1 \frac{m}{s}$$

$$27.1 \frac{\cancel{m}}{\cancel{s}} \cdot \frac{1km}{1000 \cancel{m}} \cdot \frac{3600 \cancel{s}}{1h} = 97.6 \frac{km}{h}$$

3.46



Moon's Orbital period is 27 days.

$$27 \cancel{days} \cdot \frac{24 \cancel{hr}}{\cancel{day}} \cdot \frac{3600s}{\cancel{h}} = 2332800s$$

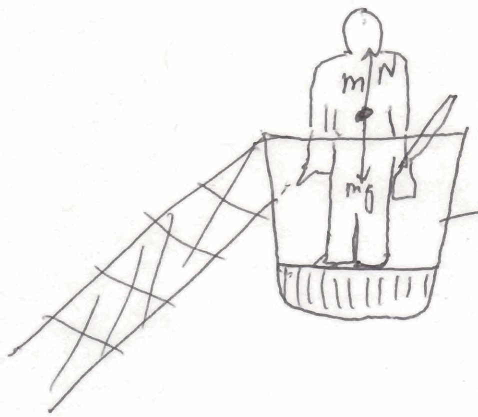
$$a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R}$$

$$= \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 \times 385 \times 10^6}{2332800^2}$$

$$= 0.00279 \; m/s^2$$

$$= 2.79 \times 10^{-3} \; m/s^2$$

4.41



$m = 74\,kg$

cherry picker or "elevator" bucket

Free-body diagram of forces on surgeon:

a) Bucket at rest : $a = 0$

$$F_{\text{net on surgeon}} = ma = 0$$

$$mg - N = 0 \longrightarrow N = mg = 74 \times 9.81 = 725\,N$$

b) Bucket moving upward at steady $v = 2.4\,m/s \rightarrow a = 0$

$$F_{\text{net on surgeon}} = ma$$

$$N - mg = 0 \rightarrow N = 725\,N$$

c) Bucket moving downward at steady $v = 2.4\,m/s \rightarrow a = 0$

$$F_{\text{net on surgeon}} = ma$$

$$mg - N = 0 \rightarrow N = 725\,N$$

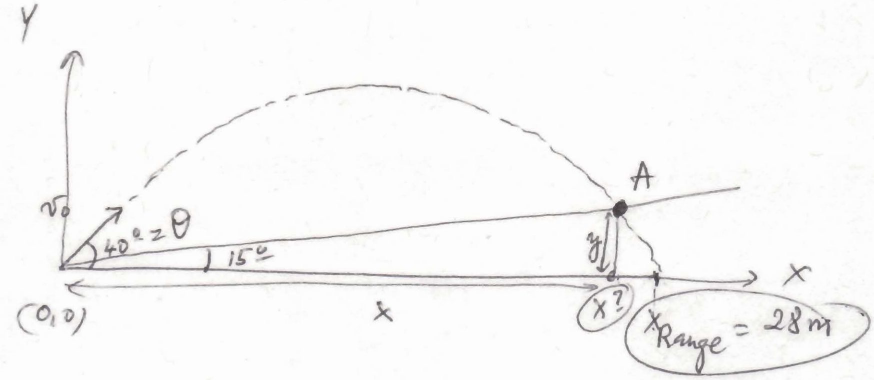
d) Bucket accelerating upward at $a = 1.7\,m/s^2$

$$N - mg = ma \rightarrow N = m(g+a) = 74(9.81 + 1.7) = 851\,N$$

e) Bucket accelerating downward at $a = +1.7\,m/s^2$

$$mg - N = ma \rightarrow N = m(g-a) = 74(9.81 - 1.7) = 599\,N$$

3.76



A on the soccer ball trajectory (equation) such that

$\tan 15° = \frac{y}{x}$

$$\boxed{y = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2}$$

x, y, v_0 $\rightarrow$

$$\begin{cases} y = x\tan 40° - \dfrac{9.81}{2v_0^2\cos^2 40}x^2 & (a) \\ \dfrac{y}{x} = \tan 15° \rightarrow y = x\tan 15° & (b) \\ \dfrac{v_0^2 \sin 80°}{9.81} = 28 \rightarrow v_0^2 = 9.81 \times 28 / \sin 80° & (c) \end{cases}$$

(b) & (c) in (a) : $\cancel{x}\tan 15° = \cancel{x}\tan 40° - \dfrac{\cancel{9.81}\ x^{\cancel{2}}}{2\times\cancel{9.81}\times 28\times[\sin 80]^{-1}\times\cos^2 40}$

$$x = (\tan 40° - \tan 15°)\times 2\times 28\times[\sin 80]^{-1}\times\cos^2 40$$

$$\boxed{x = 19.05\,m.}$$

Ch5. Using Newton's Laws

- Equilibrium
- Multiple objects
- Circular motion
- Friction

Common strategies:

1) Understand the problem, make a good sketch
2) Select a convenient coordinate system (most forces involved pointing along either x or y axis)
3) Draw a free-body diagram of forces on each object
4) Draw components for forces not lined up along x or y axis
5) Write 2nd Newton's Law for each object using net forces, possibly for both x & y directions.
6) Solve these equations to obtain numeric solutions with correct units.

Equilibrium:

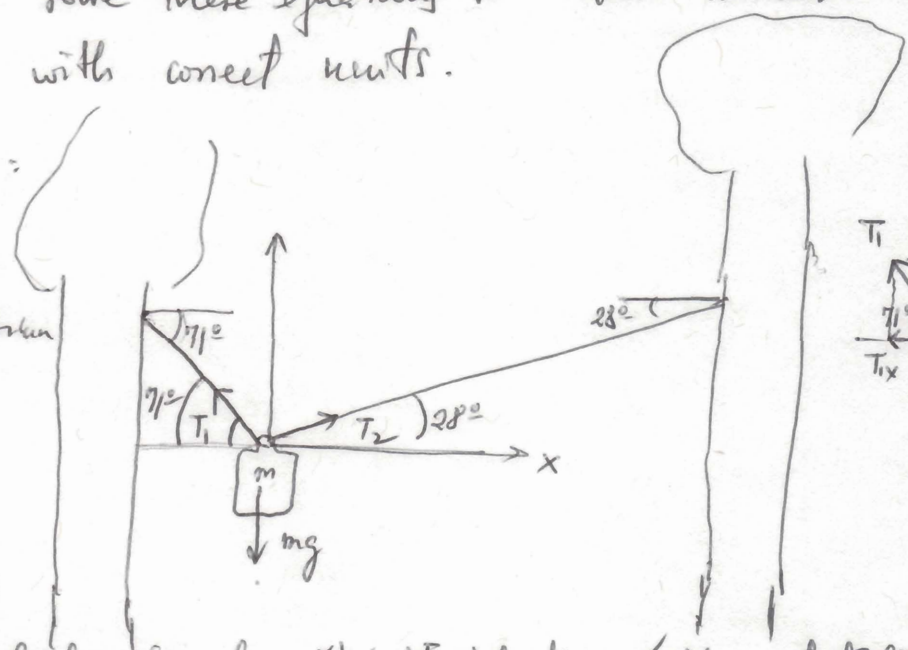
5-36

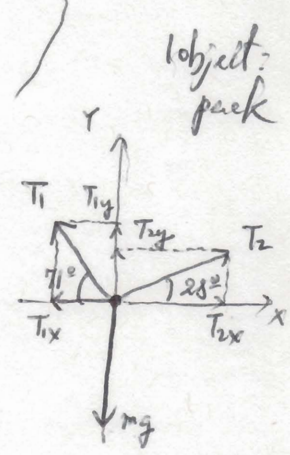
pack is in equilibrium

$m = 26\,kg$:

Rope tensions $\begin{cases} T_1\,? \\ T_2\,? \end{cases}$

Forces on pack $\{T_1;\ T_2;\ mg$





1) Sketch ✓ 2) Coord XY (mg along −Y) ✓ 3) Free-body diagram ✓ 4) components of T_1 & T_2 5) Equations

Step 5)

1 object : pack :

$$\begin{cases} x \begin{cases} F_{net_x} \overset{(**)}{=} T_{2x} - T_{1x} = \boxed{T_2 \cos 28^\circ - T_1 \cos 71^\circ} \\ \overset{(*)}{=} m a_x = \boxed{0} \end{cases} \\ y \begin{cases} F_{net_y} \overset{(**)}{=} T_{2y} + T_{1y} - mg = \boxed{T_2 \sin 28^\circ + T_1 \sin 71^\circ - mg} \\ \overset{(*)}{=} m a_y = \boxed{0} \end{cases} \end{cases}$$

(*) 2nd Newton's Law

(**) From free-body diagram.

Step 6) 2 equations w/ two unknows : T_1 & T_2

$$T_2 \cos 28^\circ - T_1 \cos 71^\circ = 0 \rightarrow T_1 = \frac{\cos 28^\circ}{\cos 71^\circ} T_2$$

$$T_2 \sin 28^\circ + T_1 \sin 71^\circ - mg = 0$$

$$\hookrightarrow T_2 \sin 28^\circ + T_2 \cos 28^\circ \tan 71^\circ = mg$$

$$T_2 = \frac{mg}{\sin 28^\circ + (\cos 28^\circ \cdot \tan 71^\circ)} = 84\,N$$

$$\rightarrow T_1 = \frac{\cos 28^\circ}{\cos 71^\circ} 84\,N = 228\,N$$

$T_1 > T_2$

$\downarrow \quad \downarrow$

$71^\circ \quad 28^\circ$

5.49