Ch 3 (cont.)  Projectile Motion

Kinematic equations

1) \[ \vec{v} = \vec{v}_0 + \vec{a}t \]

(constant acceleration)

2) \[ \vec{x} = \vec{x}_0 + \vec{v}_0t + \frac{1}{2}\vec{a}t^2 \]

Assumption: motions along perpendicular directions are independent of each other.

Example: marble going along a horizontal track at a constant velocity, then it gets launched up vertically, the combined motion is a projectile motion.

\[ \begin{align*}
\text{Horizontal motion:} & \quad \text{uniform constant velocity} \\
\text{Vertical motion:} & \quad \text{constant acceleration of gravity (up: deceleration; down: acceleration)}
\end{align*} \]

Graphical description:

1) \[ \begin{align*}
\dot{v}_x &= v_{0x} = v_0 \cos \theta \\
\dot{v}_y &= v_{0y} - gt = v_0 \sin \theta - gt
\end{align*} \]

2) \[ \begin{align*}
x &= x_0 + v_{0x}t \\
y &= y_0 + v_{0y}t - \frac{1}{2}gt^2
\end{align*} \]

Separation of different positions along x: 1) increasing 2) decreasing 3) same

Separation of different positions along y: 1) increasing 2) decreasing 3) same (up)
In our sketch \((x_0, y_0) = (0, 0)\).

Eliminate time \(t\) to arrive at the "trajectory equation":

2) \[x = v_0 \cos \theta \cdot t \quad \Rightarrow \quad t = \frac{x}{v_0 \cos \theta}\]

\[y = v_0 \sin \theta \cdot \frac{x}{v_0 \cos \theta} - \frac{1}{2} g \cdot \left(\frac{x^2}{v_0^2 \cos^2 \theta}\right)\]

\[y = x \tan \theta - \frac{g}{2v_0^2 \cos^2 \theta} \cdot x^2\]

Projectile motion is described with a parable in the XY plane.

**Maximum altitude point:** \((x_{\text{max}}, y_{\text{max}}) = \left(\frac{v_0^2 \sin 2\theta}{2g}, \frac{v_0^2 \sin^2 \theta}{2g}\right)\)

1) \[v_y = v_0 \sin \theta - gt\]

At \((x_{\text{max}}, y_{\text{max}})\) \(v_y = 0\) \(\Rightarrow t_{\text{max}} = \frac{v_0 \sin \theta}{g}\)

2) \[x_{\text{max}} = v_0 \cos \theta \cdot t_{\text{max}} = v_0 \cos \theta \cdot \frac{v_0 \sin \theta}{g} = \frac{v_0^2 \cos \theta \sin \theta}{g}\]

\[y_{\text{max}} = v_0 \sin \theta \cdot \frac{v_0 \sin \theta}{g} - \frac{1}{2} g \cdot \left(\frac{v_0^2 \sin^2 \theta}{2g}\right) = \frac{v_0^2 \sin^2 \theta}{2g}\]

**Range point:** \((x_{\text{max}}, 0) = \left(\frac{v_0^2 \sin 2\theta}{g}, 0\right)\)
Uniform Circular Motion:

- constant speed along a circular trajectory (not constant velocity)

\[ \begin{align*}
\theta & \in [0^\circ, 180^\circ] \\
\theta & \in [90^\circ, 180^\circ]
\end{align*} \]

Assume it takes time \( t \) to go from A to B: 

\[ \Theta = \frac{\text{arc AB}}{R} = \frac{vt}{R} \]

(\( \Theta \) is increasing with \( t \))

\[ \vec{z} = x \hat{i} + y \hat{j} = R \cos \Theta \hat{i} + R \sin \Theta \hat{j} = R \left[ \cos \left( \frac{vt}{R} \right) \hat{i} + \sin \left( \frac{vt}{R} \right) \hat{j} \right] \]

(position vector \( \vec{z} \) has fixed length \( R \), but varying orientation)

\[ \vec{v} = \frac{d \vec{z}}{dt} = R \left[ - \frac{v}{R} \sin \left( \frac{vt}{R} \right) \hat{i} + \frac{v}{R} \cos \left( \frac{vt}{R} \right) \hat{j} \right] \]

\[ - \vec{v} = -v \left[ - \sin \left( \frac{vt}{R} \right) \hat{i} + \cos \left( \frac{vt}{R} \right) \hat{j} \right] \]

(velocity vector \( \vec{v} \) has fixed length \( v \), but varying direction)

\[ |\vec{v}| = \sqrt{v^2 \left( - \sin \left( \frac{vt}{R} \right) \right)^2 + v^2 \left( \cos \left( \frac{vt}{R} \right) \right)^2} = v \sqrt{\sin^2 \left( \frac{vt}{R} \right) + \cos^2 \left( \frac{vt}{R} \right)} = v \]

\[ \rightarrow \text{uniform motion} \]

\[ \vec{a} = \frac{d^2 \vec{z}}{dt^2} = v \left[ - \frac{v}{R} \cos \left( \frac{vt}{R} \right) \hat{i} + \frac{v}{R} \sin \left( \frac{vt}{R} \right) \hat{j} \right] \]

\[ = -v^2 \left[ \cos \left( \frac{vt}{R} \right) \hat{i} + \sin \left( \frac{vt}{R} \right) \hat{j} \right] \]

Since \( \vec{v} \) changes with time.
(acceleration vector has fixed length \( \frac{v^2}{R} \), direction radially inward)

\[ \begin{aligned}
\vec{v}_0 &= 11 \hat{i} + 14 \hat{j} \text{ m/s} ; \\
\vec{a} &= -1.2 \hat{i} + 0.26 \hat{j} \text{ m/s}^2
\end{aligned} 
\]

(const. acceleration)

\[3.60\]

a) When does particle cross \( y \)-axis?

\[ \begin{aligned}
\text{Will it cross the } y \text{-axis?} \Rightarrow \text{Yes}
\end{aligned} \]

2D \[
\begin{align*}
1) \quad \vec{v} &= \vec{v}_0 + \vec{a} t \\
2) \quad \vec{r} &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2
\end{align*}
\]

1) \[
\begin{aligned}
\vec{v}_x &= v_{0x} + a_x t \\
\vec{v}_y &= v_{0y} + a_y t
\end{aligned}
\]

2) \[
\begin{align*}
x &= 0 + v_{0x} t + \frac{1}{2} a_x t^2 \\
y &= 0 + v_{0y} t + \frac{1}{2} a_y t^2
\end{align*}
\]

When particle crosses \( y \)-axis: \( x = 0 \Rightarrow 0 = 11 t - \frac{1}{2} \cdot 1.2 \cdot t^2 \)

\[ t = \frac{11}{0.6} = 18.3 \text{ s} \]

b) What is \( y \) at the crossing?

\[ y = 14 \cdot (18.3) + \frac{0.26}{2} (18.3)^2 = 300 \text{ m} \]

c) How fast & in what direction it goes at crossing with \( y \)-axis?

\[\begin{aligned}
\vec{v}_x &= 11 - 1.2 \cdot (18.3) = -10.96 \text{ m/s} \Rightarrow \vec{v} = \sqrt{10.96^2 + 18.3^2} = 21.7 \text{ m/s} \\
\vec{v}_y &= 14 + 0.26 \cdot (18.3) = 18.8 \text{ m/s} \Rightarrow \theta_y = \tan^{-1} \frac{18.8}{10.96} = -60^\circ \\
\end{aligned}\]
Quadrent check:

\[ v = \Theta_{v, \text{calc}} + 180^\circ = 60^\circ + 180^\circ = 240^\circ \]

\[ \vec{v} = \begin{pmatrix} 21.7 \text{ m/s} \\ 120^\circ \end{pmatrix} \]

What was \( v_{0x} \)? \( (v_{0y} = 0) \)

\[
\begin{align*}
2D \quad (1) \quad \vec{v} &= v_0 + a \cdot t \\
(2) \quad \vec{v} &= v_{0x} + \frac{1}{2} a \cdot t^2 \\
\end{align*}
\]

\[
\begin{align*}
\vec{x} &= x_0 + v_0 \cdot t + \frac{1}{2} a \cdot t^2 \\
\vec{y} &= y_0 + v_{0y} \cdot t + \frac{1}{2} a \cdot t^2 \\
\end{align*}
\]

After water leaves gun:

free fall along \( y \) &
uniform motion along \( x \) =
projectile motion from
\((x_{max}, y_{max})\) to \((x_{proj}, y_{proj})\)

\[
\begin{align*}
0.93 - 1.6 &= -\frac{1}{2} g \cdot t^2 \\
0.93 - 1.6 &= -9.81 \cdot t^2 \\
t &= \sqrt{\frac{2 \cdot 0.67}{9.81}} \\
t &= 0.37 \text{ s} \\
\end{align*}
\]
\[ T = \frac{2\pi \sqrt{\frac{26380000}{0.058 \times 9.81}}}{3600} = 11.88 \text{ hrs} \approx 12 \text{ hours.} \]

3.46

\[ \vec{v}_o = (v_{ox}, v_{oy}) \]

\[ \begin{align*}
& a_x = 0 \\
& a_y = -g 
\end{align*} \]

\text{Projectile motion}

\[ \begin{align*}
& \vec{v} = \vec{v}_o + \vec{a} t \\
& \vec{v}_x = v_{ox} \\
& \vec{v}_y = 0 = v_{oy} - gt 
\end{align*} \rightarrow \text{Need } t \text{ to continue.} \]

We also have

\[ \begin{align*}
& x-x_0 = 8.6 \cos 39^\circ \\
& y-y_0 = 8.6 \sin 39^\circ 
\end{align*} \]

(2) \[ \vec{r} = \vec{r}_0 + \vec{v}_o t + \frac{1}{2} \vec{a} t^2 \]

\[ \begin{align*}
& x-x_0 = v_{ox} t \\
& y-y_0 = v_{oy} t - \frac{1}{2} g t^2 
\end{align*} \]

\[ \begin{align*}
& 8.6 \cos 39^\circ = v_{ox} t \\
& 8.6 \sin 39^\circ = v_{oy} t - \frac{9.81}{2} t^2 
\end{align*} \]

(c)

\[ t = \frac{v_{oy}}{9.81} \]

(c) in (1) \[ 8.6 \sin 39^\circ = \frac{v_{oy}^2}{9.81} - \frac{9.81}{2} \left( \frac{v_{oy}}{9.81} \right)^2 = \frac{1}{2} \frac{v_{oy}^2}{9.81} \]

\[ v_{oy} = \sqrt{2 \times 9.81 \times 8.6 \times \sin 39^\circ} = 10.3 \text{ m/s} \]

(a) \[ v_{ox} = \frac{8.6 \cos 39^\circ}{t} = \frac{9.81 \times 8.6 \times \cos 39^\circ}{10.3} = 6.36 \text{ m/s} \]
\[ v_0 = v_{ox} \hat{i} + v_{oy} \hat{j} = 6.76 \hat{i} + 10.3 \hat{j} \text{ m/s} \]

1st quadrant

Alternative solution: observation: here \( x-x_0, y-y_0; a_x, a_y \);

Don't have time information \( \Rightarrow \) third equation:

\[ \frac{v_x^2 - v_{ox}^2}{a_x} = 2a_x (i) \text{ Not useful} \]

\[ \frac{v_y^2 - v_{oy}^2}{a_y} = 2a_y \] (ii)

\( \implies \) \( v_{oy} = \sqrt{v_y^2 - 2a_y(y-y_0)} \)

\[ v_{ox} = \frac{x-x_0}{t} = \frac{8.6 \cos 39^\circ}{10.3/9.81} = 6.36 \text{ m/s} \]

\[ v_y = v_{oy} - gt \]

\[ 0 = v_{oy} - gt \Rightarrow t = \frac{v_{oy}}{g} \]

\[ \Rightarrow v_0 = 6.36 \hat{i} + 10.3 \hat{j} \text{ m/s} \]

\[ \sqrt{6.36^2 + 10.3^2} = 12.1 \text{ m/s} \]

\[ \theta_0 = \tan^{-1} \left( \frac{10.3}{6.36} \right) = 58.3^\circ \]
Ch. 4 Force & Motion

Very icy: 1 likely happens. Why? a **force** (friction) that would change direction of motion under normal conditions is absent. Conclusion: a force is needed to change a motion → change a velocity, \( \vec{v} = (v, \theta) \)

Even if \( v \) stays constant, changing its direction requires a force. Direction is important for force → **Force is a vector**.

Sometimes there could be more than one force involved; in that case, the **net force** is the one that will change the motion.

1st Newton's Law: a body in uniform motion will stay in uniform motion, a body at rest will stay at rest, unless there is a net force acting on the body. (law of inertia)

2nd Newton's Law: \( \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \) \( \vec{p} = m\vec{v} \)

\[
\vec{F}_{\text{net}} = \frac{d}{dt} \left( m \vec{v} \right) = \frac{dm}{dt} \vec{v} + m \frac{d\vec{v}}{dt}
\]
If the body's mass is not changing with time: \( \frac{dm}{dt} = 0 \)

\[ \vec{F}_{net} = m \vec{a} \]

If the body's mass changes significantly with time, as in a rocket ship where lots of fuel is burned. \( \rightarrow \) 1st term should be included.

Dimension of the force:

\[ [F] = \frac{[p]}{[t]} = \frac{[m][v]}{[t]} = \frac{M}{T} \frac{L}{T} = \frac{ML}{T^2} \]

Unit in SI: \( \text{kg} \frac{m}{s^2} = \text{N} \) (Newton)

3rd Newton's Law: If A exerts a force on B, B exerts an equal and opposite force on A. (Law of action and reaction)

\[ F \]
\[ \begin{array}{c}
\text{No friction} \\
\vec{F}_1 = m_1 \vec{a} \\
F \text{ is applied on box } 0; \text{ the two move; what force is } \\
\text{changing motion of } 0? \\
\vec{F} = \vec{F}_1 \\
\text{Net force on } 0 \big/ F - F_1
\end{array} \]
a) What is the direction of $\vec{v}'$? (velocity vector of boat w.r.t. water)

We want $\vec{v} = \vec{v}' + \vec{V}$, which is the velocity vector of boat w.r.t. ground, to point along AB (straight across river)

So $\vec{v}'$ needs to point to left of AB, forming an angle $\theta$:

$$\theta = \sin^{-1}\left(\frac{\vec{V}}{|\vec{V}|}\right) = \sin^{-1}\left(\frac{0.57}{1.3}\right) = 26^\circ$$

b) How long will it take you to cross the river?

$$t = \frac{63}{|\vec{v}'|} = \frac{63}{\sqrt{V_{x}^2 + V_{y}^2}} = \frac{63}{\sqrt{1.3^2 - 0.57^2}} = \frac{63}{1.17} = 53.9s$$
Also \( \vec{B}_2 \) would satisfy \( \vec{A} + \vec{B}_2 \parallel y \)-axis.

\[
\begin{align*}
\vec{B}_1 &= -0.82 \hat{x} + 1.6 \hat{y} \quad (\theta_B = 117^\circ \text{ CCW from } x\text{-axis}) \\
\vec{B}_2 &= 0.82 \hat{x} - 1.6 \hat{y} \quad (\theta_B = 243^\circ \text{ CCW from } x\text{-axis})
\end{align*}
\]

3.7.3

**Projectile motion**

\( v_0 \)?

**Trajectory equation**: 

\[
y = x \tan \theta - \frac{g}{2v_0^2 \cos^2 \theta} x^2
\]

(initial point at origin of coordinates 
\( x_0 = y_0 = 0 \) & \( \theta \) the angle of the initial velocity)

Can I find \( v_0 \) from this equation? 

\[
\begin{align*}
x &= 390 \text{ m} \\
y &= 270 \text{ m} \\
g &= 9.81 \text{ m/s}^2
\end{align*}
\]

\[
\frac{1}{v_0^2} = \left( x \tan \theta - y \right) \frac{2 \cos^2 \theta}{g x^2} \Rightarrow v_0 = \frac{x}{\cos \theta} \sqrt{\frac{g}{2 \left( x \tan \theta - y \right)}}
\]

\[
v_0 = \frac{390}{\cos 70^\circ} \sqrt{\frac{9.81}{2} \left( 390 \tan 70^\circ - 270 \right)} = 89.2 \text{ m/s}
\]
Alternative Solution:

1) \[ v_x = v_{0x} \quad \text{\(a_x = 0\)} \] Projectile motion,
   \[ v_y = v_{0y} - gt \quad \text{\(a_y = -g\)} \]

2) \[ x - x_0 = v_{0x}t \quad \rightarrow \quad 390 = v_{0x}t \quad \text{(a)} \]
   \[ y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \quad \rightarrow \quad 270 = v_{0y}t - \frac{1}{2}gt^2 \quad \text{(b)} \]
   \[ \tan 70^\circ = \frac{v_{0y}}{v_{0x}} \quad \rightarrow \quad v_{0y} = v_{0x}\tan 70^\circ \]

\( \begin{align*}
\text{(a)} & \quad 390 = v_{0x}t \quad \rightarrow \quad t = \frac{390}{v_{0x}} \\
\text{(b)} & \quad 270 = v_{0x}\tan 70^\circ t - \frac{1}{2}gt^2 \\
& \quad 270 = 390\tan 70^\circ - \frac{1}{2} \times 9.81 \times \frac{390^2}{v_{0x}^2} \\
& \quad \rightarrow v_{ix} = \sqrt{\frac{9.81 \times 390^2}{2 \times (390\tan 70^\circ - 270)}} \approx 30.5 \text{ m/s} \\
& \quad \rightarrow v_{0y} = v_{0x}\tan 70^\circ = 83.5 \text{ m/s}.
\end{align*} \]

\[ \rightarrow v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{30.5^2 + 83.5^2} \approx 89.2 \text{ m/s} \]
\[ m = 60 \text{ kg} \]

Passenger with seat belt
\[ v_0 = 110 \text{ km/h} \]

\[ 0 = v_0 - at \Rightarrow a = \frac{v_0}{t} \]

\[ v_0 = 110 \text{ km/h} \cdot \frac{1 \text{ m}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \]

\[ = 30.56 \text{ m/s} \]

\[ v = 0 \text{ km/h} \]

Seat belt brings passenger from \( v_0 \) to 0

\[ \Rightarrow \text{exerting a stopping force} \]

\[ F = m \cdot a \]

\[ = 60 \text{ kg} \cdot \frac{30.56 \text{ m/s}}{0.14 \text{ s}} \]

\[ = 13095 \text{ N} \]

(like under ~20 people)
\[ a_1 = a_2 \text{ same as both objects are connected together.} \]

\[ F_{net1} = T = m_1 a \]

\[ m_2 g - N = 0 \]

\[ \begin{aligned}
   & m_2 g - m_1 a = m_2 a \\
   & m_2 g = (m_1 + m_2) a \\
   & a = \frac{m_2}{m_1 + m_2} g \\
\end{aligned} \]

**#2)** Double \( m_2 \) \[ a' = \frac{2m_2}{m_1 + 2m_2} g \]

**#3)** Double \( m_2 \) & \( m_1 \) \[ a'' = \frac{2m_1}{2m_1 + 2m_2} g = a \]
4.38

\[ k = 340 \text{ N/m} \]

\[ m = 6.7 \text{ kg} \]

\[
\text{Looking at } F_{\text{net}} \text{ on fish:}
\]

\[ F_{\text{net}} = 0 = \text{mg} - F = \text{mg} - k\Delta x \]

\[ \Delta x = \frac{\text{mg}}{k} = \frac{6.7 \times 9.81}{340} \approx 0.193 \text{ m} \]

4.53

\[ m = 4300 \text{ kg} \]

\[ F_{\text{th}} \]

\[ \text{Hovering: } a = 0 \]

\[ \text{F}_{\text{th}}: \text{ thrust force by air on blades; } -F_{\text{th}} \text{ is force exerted by blades on air (Law of action & reaction or Newton's third law)} \]

\[ F_{\text{net}} = ma = 0 = \text{mg} - F_{\text{th}} \rightarrow -F_{\text{th}} = -mg = -4300 \times 9.81 \]

\[ F_{\text{th}} = 42 \times 10^3 \text{ N} \]

b) Helicopter dropping at 21 m/s with \( a = -3.2 \text{ m/s}^2 \) (upward acceleration of 3.2 m/s²)

\[ F_{\text{net}} = ma = \text{mg} - F_{\text{th}} \rightarrow -F_{\text{th}} = ma - mg = m(a - g) \]

\[ -F_{\text{th}} = -55 \times 10^3 \text{ N} \]
c) Helicopter rising at $17 \text{m/s}$, speed increasing at $a = 3.2 \text{m/s}^2$.

$$F_{\text{net \ upward}} = ma = F_{\text{th}} - mg$$

$$-F_{\text{th}} = -ma - mg = -m(a + g)$$

$$= -4300(3.2 + 9.81) = -55.9 \times 10^3 \text{ N}$$

d) Helicopter rising at steady $15 \text{m/s}$, $a = 0$.

$$F_{\text{net \ upward}} = ma = 0 = F_{\text{th}} - mg$$

$$-F_{\text{th}} = -mg = -6300 \times 9.81 = -62 \times 10^3 \text{ N}$$

e) Helicopter rising at $15 \text{m/s}$, speed decreasing at $3.2 \text{m/s}^2$.

Let $a = -3.2 \text{m/s}^2$ up.

$$F_{\text{net \ upward}} = ma = F_{\text{th}} - mg$$

$$-F_{\text{th}} = -ma - mg = -m(a + g)$$

$$= -m(-3.2 + 9.81) = -28.4 \times 10^3 \text{ N}$$
\[ m_1 = 2 \text{ kg} \quad m_2 = 3 \text{ kg} \]
\[ k = 140 \text{ N/m} \]

Both are going at some acceleration, since they are connected together.

What is \( \Delta x \) for spring? \[ \Delta x = \frac{T_{st}}{k} \]
\( T_{st} \): tension force of \( m_2 \) on spring

\( \rightarrow \) by Newton's third law: tension force of spring on \( m_2 \) = \( T_{st} = T_s \)

We only add forces applying on a same object!

Similar happen b/w spring & \( m_1 \)

\[ T_{s1} = T_{s2} \quad \rightarrow \quad T = m_2 a \]

Looking at:

\[ \begin{align*}
T_{s1} &= \text{ Net on } m_1: F_{net1} = T_{s1} = T_{s2} = \frac{T}{m_1} a \\
T_{s2} &= \text{ Net force on } m_2: F_{net2} = F - T_{s2} = [F - T = m_2 a]
\end{align*} \]

\[ \begin{align*}
F = m_1 a & \quad \rightarrow \quad a = \frac{T}{m_1} \quad J \\
F - T = m_2 a & \quad \rightarrow \quad F - T = \frac{m_2 a}{m_1} T \quad \rightarrow \quad T = \frac{F}{(1 + \frac{m_2}{m_1})}
\end{align*} \]

\[ T = \frac{15}{1 + \frac{3}{2}} \]

\[ T = \frac{15}{\frac{5}{2}} = 6 \text{ N} \]

\[ \Delta x = \frac{T}{k} = \frac{6 \text{ N}}{140 \text{ N/m}} = 0.0429 \text{ m} \]
The two boxes are moving together → same acceleration $a$.

No friction, b/w boxes & surface they're sliding on.

**Box #2**:
Net force is $F_{12}$: exerted by 1 on 2.
(The weight and normal forces are not relevant to horizontal motion along $x$)

2nd Newton's Law:
$$F_{12} = m_2a$$

**Box #1**: Net force is $F - F_{21} = m_1a$  (2nd Newton's Law)
$$F_{21} = F_{12}$$  (3rd Newton's Law)

$$F - m_2a = m_1a$$

$$F = (m_2 + m_1)a$$

$$a = \frac{F}{m_1 + m_2}$$

Measuring forces:

- **Spring balance**

  Hooke's law:
  $$F = -k\Delta x$$
  $k$: Spring constant
  $\Delta x$: Stretch in displacement
  $x$: Natural length
\[ F_{\text{on ball}} = -k \Delta x = 0 \]

Force is opposite to displacement of the ball.

\[ F_{\text{on ball}} = -k(-\Delta x) + k\Delta x \]

Force is opposite to displacement of the ball.

Net force on \( m \):

\[ mg - k \Delta y \]

If the mass is not moving, \( a = 0 \)

\[ mg - k \Delta y = ma = 0 \]

Free-body diagram:

- Force on mass: now described as a point.
  - \( F = -k \Delta y \)
  - \( mg \)
Frictional force: when a body is in contact with a surface

Normal force exerted by surface on body

Static friction: \[ F_s = \mu_s N \]
- \( \mu_s \) sub \( s \): coefficient of static friction
- Threshold force for some object to start moving

Kinetic friction: \[ F_k = \mu_k N \]
- while object is moving on a surface

Observation: when trying to push a heavy box, we apply increasing force, suddenly it starts moving (we reached the threshold to overcome static friction). As we continue without increasing the force applied the object acquires good acceleration = kinetic friction is lower than static friction.

Explanation: bottom of box develops bonding with surface
Static friction:

\[ F_s = \mu_s N \]

Car is parked on a slope of angle \( \alpha \)

Force on car:

\[ \begin{align*}
1. & \ mg \quad \text{(vertical)} \\
2. & \ N \\
3. & -mg\cos\alpha + N = 0 \\
\end{align*} \]

(no motion perpendicular to slope)

\[ mg \sin\alpha \]

If there is no friction, car would slide down.

\[ F_s = \mu_s N \]

Friction has no unique direction like weight (always downward, toward Earth's center); it always tends to oppose motion. In this case, car tends to slide down \( \rightarrow F_s \) points uphill.

With magnitude \( \mu_s N = mg \cos\alpha \).

A minimum of \( \mu_s \) is needed:

\[ \mu_s N \geq mg \sin\alpha \]

\[ \mu_s \cos\alpha \geq \sin\alpha \]

\[ \mu_s \geq \tan\alpha \]

(if \( \mu_s < \tan\alpha \rightarrow \text{car will slide down hill} \))

\( \mu_s \) depends on surface area in contact & \( \mu_s \) & roughness of materials.
3.45 \hspace{1cm} \text{Uniform Circular Motion} \hspace{1cm} \text{can rounding a turn of } R = 75 \text{ m} \text{ at constant speed} . \hspace{1cm} v = \sqrt{aR} \\
\hspace{1cm} v = \sqrt{gR} = \sqrt{9.81 \times 75} = 27.1 \text{ m/s} \\
\hspace{1cm} 3.46 \hspace{1cm} \text{Moon's Orbital period is 277 days} \hspace{1cm} \text{to cent of curvature} \\
R = 385 \times 10^6 \text{ m} \\
a = \frac{v^2}{R} = \frac{(\frac{2\pi R}{T})^2}{R} \\
\hspace{1cm} = \frac{4\pi^2 R}{T^2} \hspace{1cm} = \frac{4 \pi^2 \times 385 \times 10^6}{2332800^2} \hspace{1cm} = 0.00299 \text{ m/s}^2 \hspace{1cm} = 2.99 \times 10^{-3} \text{ m/s}^2
a) Bucket at rest: \( a = 0 \)
\[
\text{Free on surgeon} = ma = 0
\]
\[
mg - N = 0 \rightarrow N = mg = 74 \times 9.81 = 725 \text{N}
\]

b) Bucket moving upward at steady \( v = 2.4 \text{ m/s} \) \( \rightarrow a = 0 \)
\[
\text{Free on surgeon} = ma.
\]
\[
N - mg = 0 \rightarrow N = 725 \text{N}
\]

c) Bucket moving downward at steady \( v = 2.4 \text{ m/s} \) \( \rightarrow a = 0 \)
\[
\text{Free on surgeon} = ma.
\]
\[
mg - N = 0 \rightarrow N = 725 \text{N}
\]

d) Bucket accelerating upward at \( a = 1.7 \text{ m/s}^2 \)
\[
N - mg = ma \rightarrow N = m(g+a) = 74(9.81+1.7) = 851 \text{N}
\]

e) Bucket accelerating downward at \( a = 1.7 \text{ m/s}^2 \)
\[
mg - N = ma \rightarrow N = m(g-a) = 74(9.81-1.7) = 599 \text{N}
\]
A on the soccer ball trajectory (equation) such that
\[ \tan 15^\circ = \frac{y}{x} \]

\[ y = x \tan \theta - \frac{\theta}{2\nu_0 \cos \theta} x^2 \]

\[ x = 19.05 \text{ m} \]
Ch5. Using Newton's Laws

- Equilibrium
- Multiple objects
- Circular motion
- Friction

Common strategies:

1) Understand the problem, make a good sketch
2) Select a convenient coordinate system
   (most free involved pointing along either x or y axis)
3) Draw a free-body diagram of forces on each object
4) Draw components for forces, not lined up along x or y axis
5) Write 2nd Newton’s Law for each object using net forces, possibly for both x & y directions.
6) Solve these equations to obtain numeric solutions with correct units.

Equilibrium:

\[ F_{net} = 0 \]

Pack is in equilibrium:

\[ m = 26 \text{ kg} \]

Hose \( T_h \) ?

Tension \( T_v \) ?

Force on pack \( \{ T_h, T_v \} \)

\[ T_h \]

\[ T_v \]

\[ mg \]

1) Sketch
2) Bond XY (mg along -Y)
3) Free-body diagram
4) Component of \( T_h, T_v \)
5) Equations
Step 5)
1 object, pack:

\[
\begin{align*}
\text{Net}_x &\Rightarrow T_2 \cos 28^\circ - T_1 \cos 71^\circ = 0 \\
\text{Net}_y &\Rightarrow T_2 \sin 28^\circ + T_1 \sin 71^\circ - mg = 0
\end{align*}
\]

2nd Newton's Law

From free-body diagram.

Step 6)
2 equations w/ two unknowns: \(T_1 \& T_2\)

\[
\begin{align*}
T_2 \cos 28^\circ - T_1 \cos 71^\circ &= 0 \\
T_2 \sin 28^\circ + T_1 \sin 71^\circ - mg &= 0
\end{align*}
\]

\[
\begin{align*}
T_2 \sin 28^\circ + T_2 \cos 28^\circ \tan 71^\circ &= mg \\
T_2 &= \frac{mg}{\sin 28^\circ \cos 71^\circ + \cos 28^\circ \tan 71^\circ} = 84 \text{N}
\end{align*}
\]

\[
T_1 = \frac{\cos 28^\circ}{\cos 71^\circ} \times 84 \text{N} = 228 \text{N}
\]

\(T_1 > T_2\)

[Diagram]

5.49