Ch 1 Doing Physics

Dimensional Analysis

→ Dimension of speed: $[v] = \dfrac{[\Delta s]}{[\Delta t]} = \dfrac{L}{T}$ (length) / (time)

"Δ" : delta, "change of" or "increment of"

→ Dimension of acceleration: $[a] = \dfrac{[\Delta v]}{[\Delta t]} = \dfrac{\frac{L}{T}}{T} = \dfrac{L}{T^2}$

→ Dimension of energy $[E] = [\frac{1}{2}mv^2] = [m][v^2]$

Numeric constants have dimension of 1

$$= M\left(\frac{L}{T}\right)^2 = \frac{ML^2}{T^2}$$

Application: check a formula:

$$v = \frac{1}{2}gh^2 \ ? \Rightarrow [\tfrac{1}{2}gh^2] = [g][h]^2 = \frac{L}{T^2}\cdot L^2 = \frac{L^3}{T^2}$$

$$v = \sqrt{gh} \ ? \Rightarrow [\sqrt{gh}\,] = \sqrt{[g][h]} = \sqrt{\frac{L}{T^2}L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} \ ✓$$

g: acceleration of gravity

h: height (length)

Units:

S.I. (system of international units)

L : m (meter)

T : s (second)

M : kg (kilogram ⇒ kilo : $1000 = 10^3$)

Area : m^2

Volume : m^3

Energy : $\frac{kg\, m^2}{s^2} = J$ (Joules)

L,

Unit conversion:

	nano	micro	centi						
	$1\,nm$	$1\,\mu m$	$1\,cm$	$1\,mm$	$1\,km$	1 light-year	1 mi	1 ft	1 in.
in m	$10^{-9}\,m$	$10^{-6}\,m$	$10^{-2}\,m$	$10^{-3}\,m$	$10^{3}\,m$	$9.46\times10^{15}\,m$	$1609\,m$	$0.3048\,m$	$2.54\,cm$

0.3048m

$1\,lb = 0.454\,kg$

$1\,h = 3600\,s$; $1\,day = 86\,400\,s$; etc.

$1\,km^2 = 10^6\,m^2$; $1\,cm^2 = 10^{-4}\,m^2$; $1\,mm^2 = 10^{-6}\,m^2$

$1\,km^3 = 10^9\,m^3$; $1\,cm^3 = 10^{-6}\,m^3$; $1\,mm^3 = 10^{-9}\,m^3$

Accuracy & Significant Figures:

- Scientific notation: $\Delta s = 3\,105\,000\ m = \underbrace{3.105}_{\substack{\text{number}\\ \text{less than}\\ 10}} \times \underbrace{10^6}_{\text{power of } 10}\ m$ (scientific notation)
  - ↳ easy for multiplication & division

in calculator: 3.105E6

$\Delta t = 3000 s = 3 \times 10^3 s$

$\rightarrow$ speed: $v = \dfrac{3.105 \times 10^6}{3 \times 10^3}\ m/s = \dfrac{3.105}{3} \cdot 10^{6-3}\ m/s$

$= 1.035 \times 10^3\ m/s$

Accuracy: $\pi = 3.1416$ is more accurate than 3.14

↳ # of decimal digits

Addition & subtraction: $3.1416 - 1.14 = 2.0016$

$= 2.00$

keep same accuracy as least accurate term in LHS

Significant figures: $\underbrace{6\,370\,000}_{3}$ (zeros at end) $\underbrace{6\,370\,001}_{7}$ (zeros in middle)

Multiplication & division: keep smallest number of significant figures (except for numeric constants)

Earth's circumference: $2\pi R_E = 2 \times 3.1416 \times \underbrace{6.37 \times 10^6}_{3\ s.f.'s}\ m$

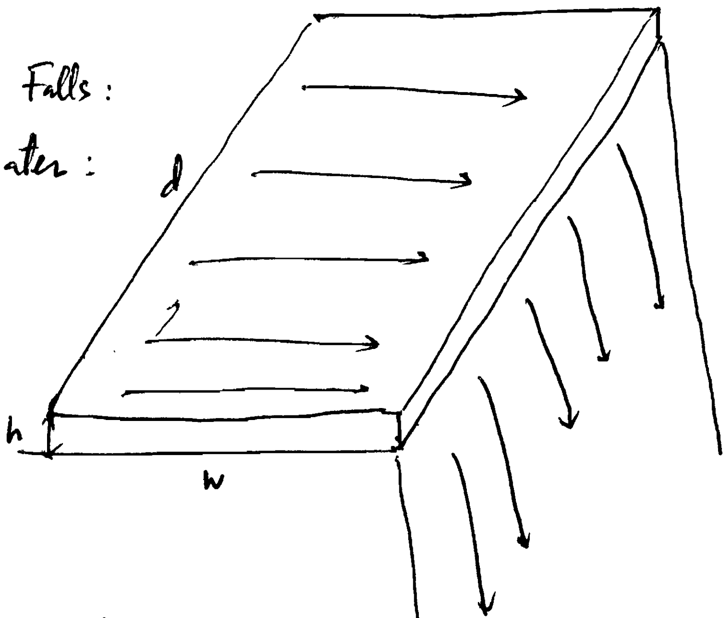
$= 4.002398 \times 10^7 = 4.00 \times 10$

1.40 a) Estimate volume of water flowing over Niagara Falls in 1s

$10^6 \frac{m^3}{s}$; $10^9 \frac{m^3}{s}$;

Visualize Falls:

slab of water:



$\hookrightarrow$ Vol = hwd

Flow rate (volume of water per second) $= \frac{Vol}{time} = \frac{hwd}{t}$

water speed in vertical direction

1) $\frac{h}{t} wd$

2) $h \frac{w}{t} d$

3) $hw \frac{d}{t}$ water speed along depth direction.

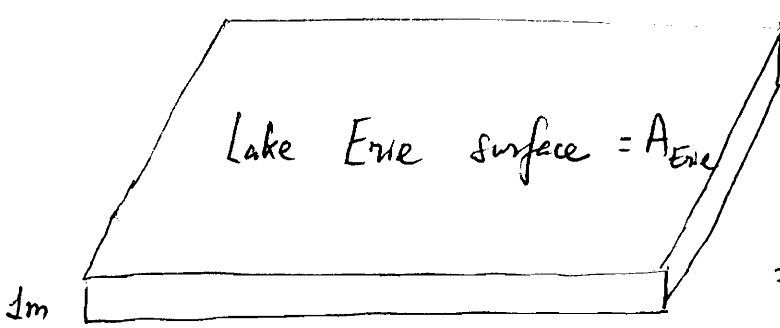
$\hookrightarrow hd \frac{w}{t} = 5000 \frac{m^3}{s}$ (educated guess).

water speed over falls

1m 1000m 5 m/s

$\frac{100\,km}{hr} \cdot \frac{1h}{3600\,s} \cdot \frac{1000\,m}{1\,km} = \frac{100}{3.6} \simeq 28\,m/s$

b)



How long ? $= \frac{V_{Erie}}{5000 \frac{m^3}{s}}$

$= \frac{1m\, A_{Erie}}{5000 \frac{m^3}{s}}$

(4b)

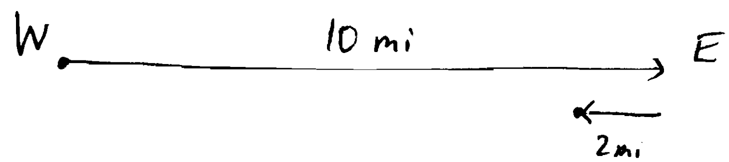
$$A_{Erie} = 75\,km \times 375\,km = 75 \times 375\ km^2$$

$$\text{How long} = \frac{75 \times 375 \times 10^6 \times 1\ \cancel{m^3}}{5000\ \frac{\cancel{m^3}}{s}} = 9 \times 10^6\ \cancel{s} \cdot \frac{1\,day}{84600\ \cancel{s}} = 100\ days.$$

Ch 2 Motion in a Straight Line

Average motion:

↳ $\text{speed} = \dfrac{\text{distance}}{\text{time}}$ $\qquad$ $\text{velocity} = \dfrac{\text{displacement}}{\text{time}}$



} Total time: 1hr.

$\text{speed} = \dfrac{12\text{ mi}}{\text{hr}}$ $\qquad$ $\text{velocity} = 8\ \dfrac{\text{mi}}{\text{hr}}$

(takes into consideration the direction of motion)

Average velocity: $\bar{v} = \dfrac{\Delta x}{\Delta t}$; "Δ" delta = displacement in or change of

Instaneous velocity: $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ ("derivative of x wrt. time")

x = position

→ example: $x = at^3 \rightarrow v = \dfrac{dx}{dt} = \dfrac{d(at^3)}{dt} = 3at^2$

$$\boxed{\frac{dt^n}{dt} = n\,t^{n-1}}$$

$x = at^n \rightarrow v = na\,t^{n-1}$

→ units: $\frac{m}{s}$; $\frac{km}{h}$; $\frac{mi}{h}$; etc...

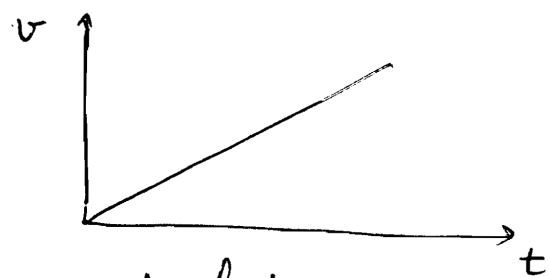
Acceleration change of velocity over time

Average acceleration: $\bar{a} = \frac{\Delta v}{\Delta t}$ (change of velocity over time)

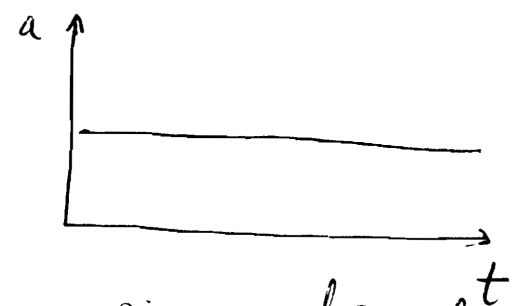
Instantaneous acceleration $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

↳ $x = at^3 \longrightarrow v = \frac{dx}{dt} = 3at^2 \rightarrow a = \frac{dv}{dt} = 3a\frac{d(t^2)}{dt}$
$= 6at$

↳ units: $[a] = \frac{L}{T^2} \rightarrow \frac{m}{s^2}; \frac{km}{s^2}; etc..$



~~Speed~~ Velocity increasing linearly with time



Since: $\frac{dv}{dt}$ is also the slope of the curve $v(t)$

Motion along a straight line (1D motion)

↳ **Constant acceleration:** $\boxed{a = \bar{a}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$

$\rightarrow v - v_0 = at \rightarrow \boxed{v = v_0 + at}$ (1)

1st Kinematic equation

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \quad \rightarrow \quad x = x_0 + \bar{v}t \qquad (A)$$

Average velocity $\bar{v}$ can also be calculated:

$$\bar{v} = \frac{1}{t-0}\int_0^t v\,dt$$

$$\overset{(1)}{=} \frac{1}{t}\int_0^t (v_0 + at)\,dt$$

$$= \frac{1}{t}\left[v_0 t + a\frac{t^2}{2}\right]_0^t = \frac{1}{t}\left[v_0 t + \frac{1}{2}at^2\right]$$

$$= v_0 + \frac{1}{2}at$$

$$= \frac{1}{2}v_0 + \frac{1}{2}a.0 + \frac{1}{2}v_0 + \frac{1}{2}a.t$$

$$= \frac{1}{2}\left[(v_0 + a.0) + (v_0 + a.t)\right]$$

$$\overset{(1)}{=} \frac{1}{2}\left[v_0 + v\right]$$

$$\Rightarrow \bar{v} = \frac{1}{2}(v_0 + v) \qquad (B)$$

(B) into (A) $\quad x = x_0 + \frac{1}{2}(v_0 + v)t \overset{(1)}{=} x_0 + \frac{1}{2}(v_0 + v_0 + at)t$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \quad (2)$$

Second kinematic equation.

Constant acceleration motion in 1D:

$$v = v_0 + at \quad (1)$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (2)$$

can derive from (1) & (2): $\dfrac{v^2 - v_0^2}{x - x_0} = 2a \quad (3)$

(no time)

x_0: initial position ; x = final position

v_0: initial velocity ; v = final velocity

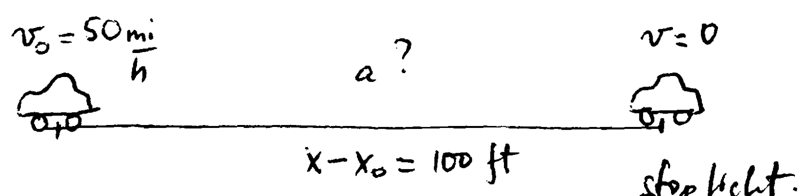
Applications:

2.35

car with $v_0 = 50\ mi/h$ constant deceleration (negative acceleration)

$x - x_0 = 100\ ft \rightarrow a$?

Sketch



No time information $\rightarrow$ try eq. (3) : $2a = \dfrac{0 - 22.35^2}{30.48} \Rightarrow \boxed{a = -8.192 \frac{m}{s^2}}$

deceleration, sign is provided by the equation!

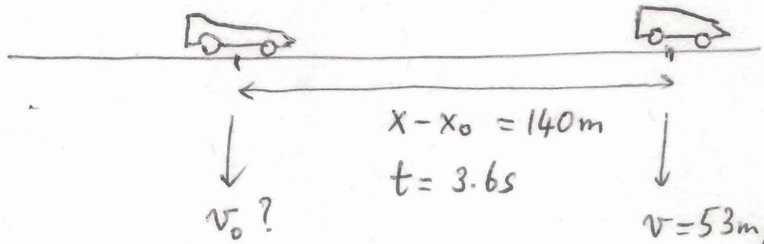
- Make sure units are in S.I.

$$v_0 = 50 \frac{\cancel{mi}}{\cancel{h}} \cdot \frac{1609 m}{1 \cancel{mi}} \cdot \frac{1 \cancel{h}}{3600 s} = 22.35 \frac{m}{s}$$

$$x - x_0 = 100 \cancel{ft} \cdot \frac{0.3048 m}{1 \cancel{ft}} = 30.48 m$$

2.61

a)



(1) $v_0 = v - at$ (now need a → bring in eq (2))

(2) $x - x_0 = v_0 t + \frac{1}{2} a t^2$

<u>One alternative:</u> Find a : eliminate v_0 use (1) in (2)

$$x - x_0 = (v - at)\,t + \frac{1}{2} a t^2 = vt - \frac{1}{2} a t^2$$

$$\rightarrow \quad a = \frac{-2(x - x_0 - vt)}{t^2} = -2\,\frac{140 - 53 \times 3.6}{3.6^2} = 7.83\,\frac{m}{s^2}$$

Back in (1): $v_0 = v - at = 53 - 7.83 \times 3.6 = 24.8\,\frac{m}{s}$

<u>Second alternative:</u> Elimate a :

(1) $a = \dfrac{v - v_0}{t}$

(2) $v_0 t = x - x_0 - \frac{1}{2} a t^2 = x - x_0 - \frac{1}{2}(v - v_0) t$

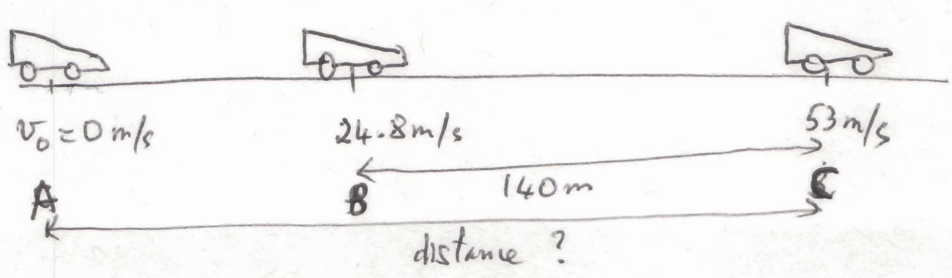
$$= x - x_0 - \frac{1}{2} v t + \frac{1}{2} v_0 t$$

$$\frac{1}{2} v_0 t = x - x_0 - \frac{1}{2} v t$$

$$v_0 = 2\,\frac{(x - x_0)}{t} - v = 2\,\frac{140}{3.6} - 53$$

$$= 24.8\,m/s.$$

b)



Alternatives: 1) Find AB , then $AC = AB + \underbrace{BC}_{140m}$

2) Look at motion b/w A & C : constant acceleration $a = 7.83\ \frac{m}{s^2}$

$v_0 = 0$ (at A), $v = 53\,m/s$ (at C)

↳ Eq(3): $(x - x_0)_{AC} = \dfrac{v^2 - v_0^2}{2a} = \dfrac{53^2 - 0}{2 \times 7.83}$

$= 179.37m$

(AB → 39.37m)

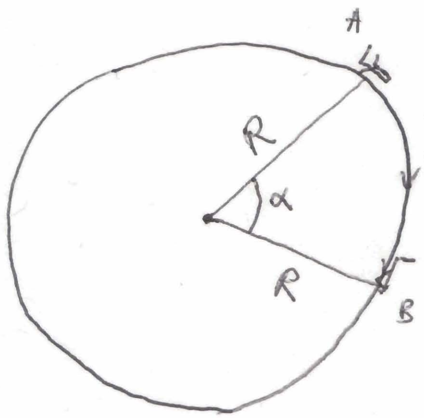
2.73

1.31 $\quad (6.4\times10^{19})^{\frac{1}{3}} = \sqrt[3]{6.4\times10^{19}}$

$= (64\times10^{18})^{\frac{1}{3}}$

$= (4^3\times10^{18})^{\frac{1}{3}} = 4\times10^6$

1.17



$R = 3.4\,\text{km}$ (radius)

$s = 2.1\,\text{km}$ (arc)

$\alpha = \dfrac{\text{arc}}{\text{radius}} = \dfrac{2.1\,\text{km}}{3.4\,\text{km}} = 0.62\,\text{rad}$

↓ "unit" for angles.

1.39 → Consumption 1.5 kW (energy rate = energy use per unit time)

→ Collection (solar): 300 W per m^2 : " energy rate density

→ Total area with solar cells to get needed energy rate :

$$\text{energy rate} = \text{energy rate density} \times \text{area}$$

$$\text{area} = \frac{\text{energy rate}}{\text{energy rate density}} = \frac{1.5\times10^3\,\cancel{W}}{300\,\frac{\cancel{W}}{m^2}}$$

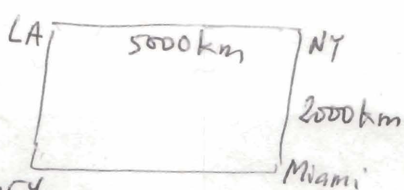
solar cell efficiency = 20%

$$= 5\ m^2 \quad \text{(per person)}$$

Total area w/ solar cells. $= 5m^2 \times 250\times10^6 = 1.25\times10^9\ m^2$

$= 1250\ km^2$.

Total surface of US =



$\rightarrow 10^7\ km^2$

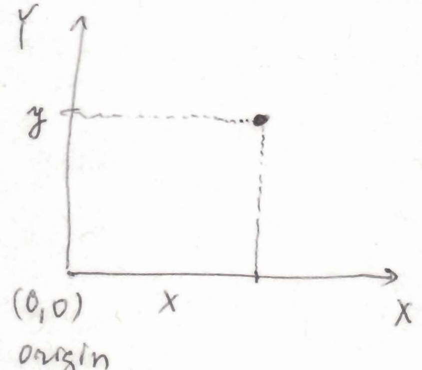
Fraction: 1.25×10^{-4} = $\boxed{0.0125\,\%}$ $\xrightarrow{\times 5}$ 0.0625%

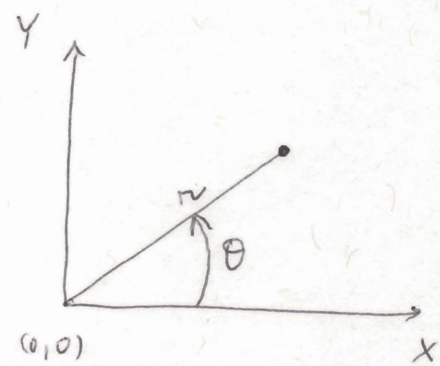
Ch 3: Motion in 2 & 3 Dimensions:

2D: → Position is determined by 2 quantities:

$$\begin{cases} (x, y) \longrightarrow \text{Cartesian coordinates} \\ (r, \theta) \longrightarrow \text{Polar coordinates} \end{cases}$$

r: radius; θ: "theta", the angle

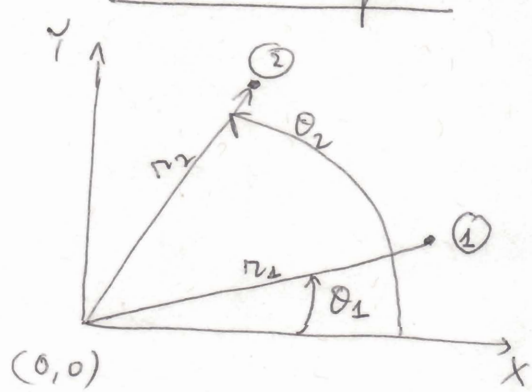




θ: measured from the x-axis

Position $\boxed{\text{vector}}$: $\vec{r} = (x, y) = (r, \theta)$

Direction is important: ⇒ use a vector to describe position.

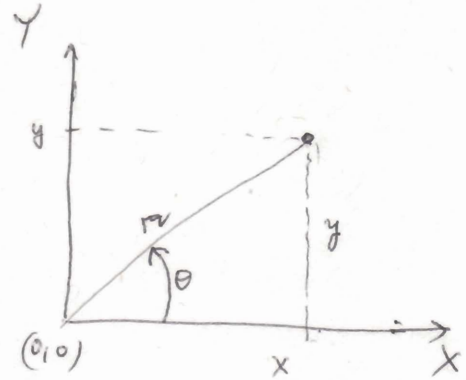


even if $r_1 = r_2$ we still have different positions because $\theta_1 \neq \theta_2$

$$2D \begin{cases} \text{Position vector } \vec{r} = (x, y) = (r, \theta) \\ \text{Velocity vector } \vec{v} = (v_x, v_y) = (v, \theta_v) \\ \text{Acceleration vector } \vec{a} = (a_x, a_y) = (a, \theta_a) \end{cases}$$

Changing from Cartesian coords to Polar coords:

$$\vec{r} = (x, y) \longrightarrow \left(r = \sqrt{x^2+y^2} \ , \ \theta = \tan^{-1}\frac{y}{x} \right)$$



Reversing $\vec{r} = (r, \theta) \longrightarrow \left(x = r\cos\theta \ , \ y = r\sin\theta \right)$

Unit vectors : vectors with magnitude 1
↳ radius or length of vector

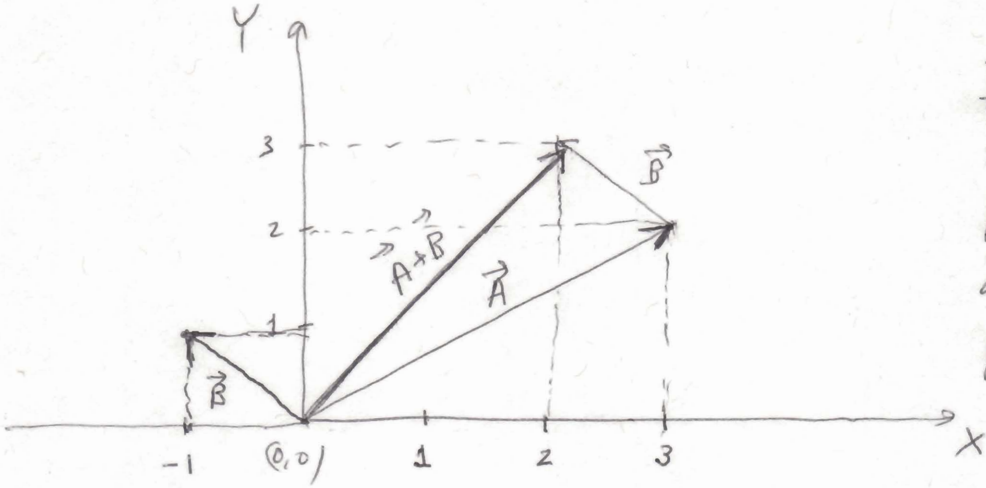
↳ along x axis: $\hat{i} = (1, 0) = (1, 0°)$ (length is 1)
along y axis: $\hat{j} = (0, 1) = (1, 90°)$ (length is 1!)

↳ $\vec{r} = (x, y) = x\hat{i} + y\hat{j} = x(1,0) + y(0,1)$
$= (x, 0) + (0, y) = (x, y)$

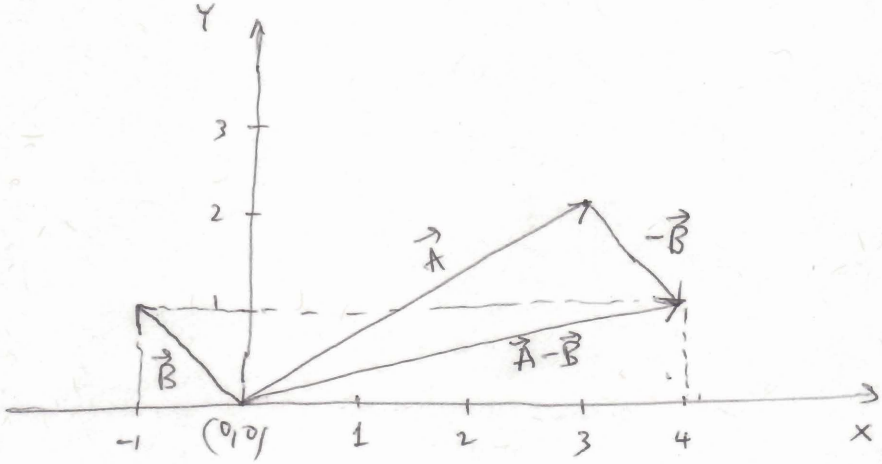
$\vec{v} = (v_x, v_y) = v_x\hat{i} + v_y\hat{j}$

Preparing for 2D (and 3D) equations of motion (or Kinematic equations) → Look at addition & substraction of vectors:

Addition & subtraction of vectors:



$\vec{A}+\vec{B}$: $\boxed{\text{graphically}}$: draw a copy of $\vec{B}$ from tip of $\vec{A}$: addition vector $\vec{A}+\vec{B}$ from origin of $\vec{A}$ to tip of copy of $\vec{B}$



$\vec{A}-\vec{B}$: repeat the same but reversing direction of copy of $\vec{B}$

$\boxed{\text{Mathematically}}$ using Cartesian coordinates (best for addition & substraction, polar coord's would be best for multiplication & division)

$$\vec{A} = A_x \hat{i} + A_y \hat{j} \qquad (A_x = 3;\ A_y = 2 \text{ in the example above})$$

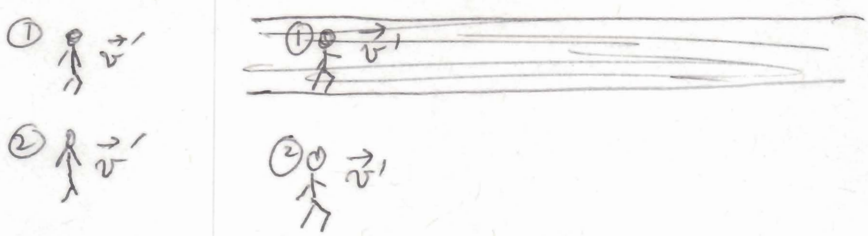
$$\vec{B} = B_x \hat{i} + B_y \hat{j} \qquad (B_x = -1;\ B_y = 1 \ \text{''} \quad)$$

$$\vec{A}+\vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$
$$= (3-1)\hat{i} + (2+1)\hat{j} = 2\hat{i} + 3\hat{j}$$

$$\vec{A}-\vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$
$$= 4\hat{i} + 1\hat{j}$$

Relative Motion:

1D:



automatic walkway at Boston Logan moving at constant velocity $\vec{V}$ (upper case)

Both ① & ② walk at constant velocity $\vec{v}'$, then ① chooses to use walkway while ② stay out. ① gets to terminal faster. Why?

Relative motion: person ① $\begin{cases} \text{wrt. walkway}: & \vec{v}' \\ \text{wrt. floor}: & \vec{v}' + \vec{V} \end{cases}$

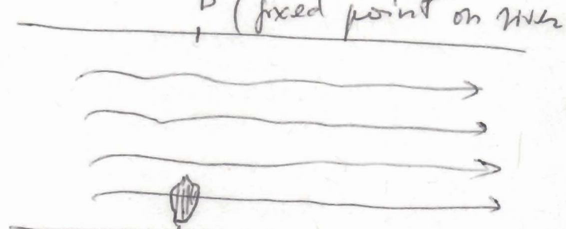
person ② $\{ \text{wrt. floor} = \vec{v}'$

Since $\vec{V}$ is in the same direction as $\vec{v}' \rightarrow \vec{v}' + \vec{V} > \vec{v}'$

→ wrt floor $\begin{cases} \vec{v}_1 = \vec{v}' + \vec{V} \\ \vec{v}_2 = \vec{v}' \end{cases}$

2D:

B (fixed point on river bank)



River, water flows at velocity $\vec{V}$ (upper case)

A (fixed point on river bank)

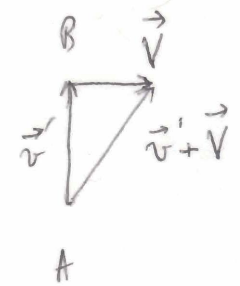
Rowing boat going at velocity $\vec{v}'$ (wrt water). To get to B where should $\vec{v}'$ point to:

1) To B 2) Left of B 3) Right B

Velocity of boat wrt water = $\vec{v}'$

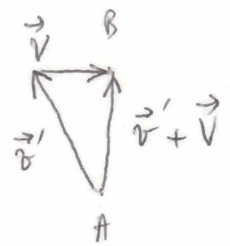
" " " wrt river bank (ground) = $\vec{v}' + \vec{V}$

1) $\vec{v}'$ pointing A to B:



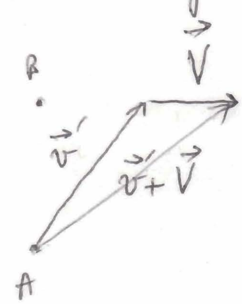
since $\vec{v}' + \vec{V}$ points to right of B → boat will get to the right of B

2) $\vec{v}'$ pointing A to left of B



Boat can get from A to B

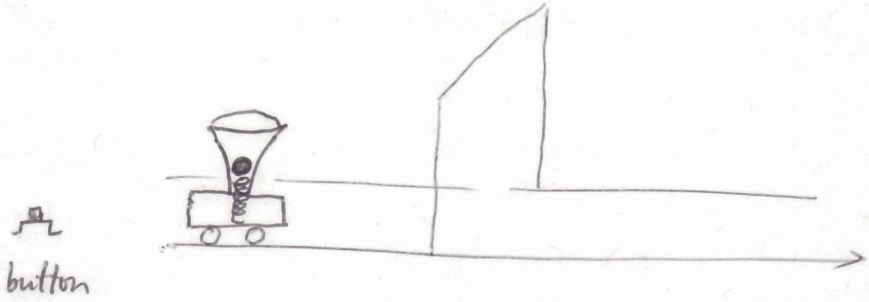
3) $\vec{v}'$ points A to right of B



Boat goes further away from B than situation 1).

Equations of motion in 2D :

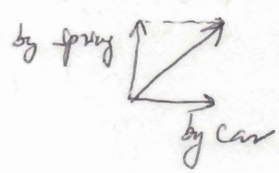
→ Motions along perpendicular directions are <u>independent</u>



Give car a push, release the spring by pressing button, ball is lanched vertically upward, car keeps going forward.

1) Ball falls back down behind car
2) Ball falls back in front of car
3) " " " into car (funnel) ✓

→ car motion is along x (1D)
ball motion is composed of
→ vertical lanch by spring
→ horizontal motion of car



→ Motions along perpendicular directions combine, however they are independent of each other.

With this observation, what are the kinematic equations in 2D? (constant acceleration)

$$1D: \begin{cases} v = v_0 + at & (1) \\ x = x_0 + v_0 t + \frac{1}{2} a t^2 & (2) \end{cases} \rightarrow 2D \begin{cases} \left.\begin{array}{l} v_x = v_{0x} + a_x t \\ v_y = v_{0y} + a_y t \end{array}\right\} (1) \\ \left.\begin{array}{l} x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \end{array}\right\} (2) \end{cases}$$

using vector notation:

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

$$2D \begin{cases} \vec{v} = \vec{v}_0 + \vec{a} t & (1) \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t & (2) \end{cases}$$

In 3D, add a third component z, kinematic equations in vector notation look exactly the same

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

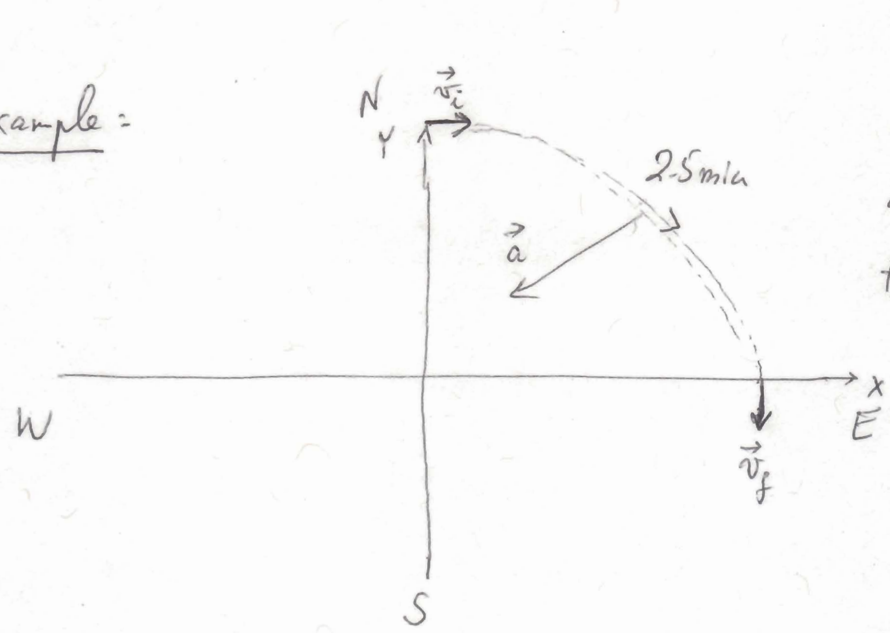
$\hookrightarrow$ unit vector along z-direction

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$3D \begin{cases} \vec{v} = \vec{v}_0 + \vec{a} t & (1) \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t & (2) \end{cases}$$

Example:



airplane flies eastward at $\vec{v}_i = 2100 \frac{km}{h} \hat{i}$

turns southward to $\vec{v}_f = -1800 \frac{km}{h} \hat{j}$

Average acceleration? $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t} = \dfrac{-500\hat{j} - 583.3\hat{i}}{150s} \frac{m}{s}$

$|\vec{v}_i| = 2100 \frac{\cancel{km}}{\cancel{h}} \cdot \frac{1000m}{1\cancel{km}} \cdot \frac{1\cancel{h}}{3600s} = 583.3 \frac{m}{s} \rightarrow \vec{v}_i = 583.3 \frac{m}{s} \hat{i}$

$|\vec{v}_f| = 1800 \frac{km}{h} = 500 \frac{m}{s} \rightarrow \vec{v}_f = 500 \frac{m}{s} (-\hat{j})$

$\vec{a} = -3.3\hat{j} - 3.9\hat{i} \quad \frac{m}{s^2}$ (cartesian) $\rightarrow$

$a = \sqrt{(-3.9)^2 + (3.3)^2}$

$= 5.1 m/s^2$

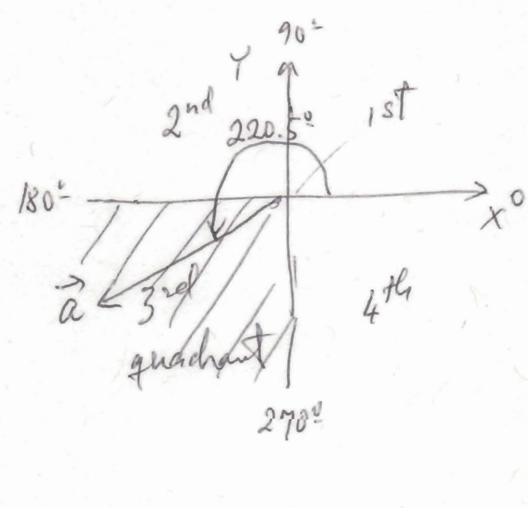
$\theta_a = \tan^{-1}\left(\frac{-3.3}{-3.9}\right)$

$= 40.5°$

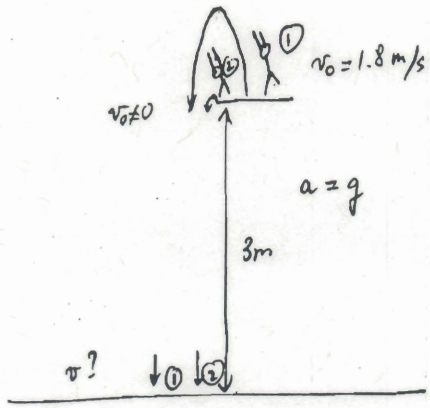
Not correct!

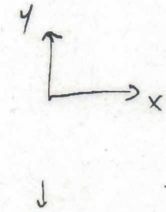
$\theta_a = 40.5° + 180°$

$= 220.5°$ ✓.



2.69

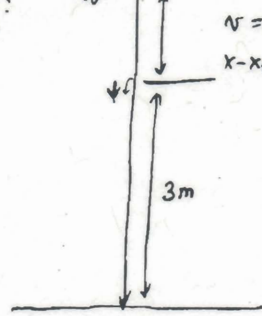




Diver # ② : $(3) \rightarrow \dfrac{v^2 - v_0^2}{x - x_0} = 2a \rightarrow v^2 = 2g(x - x_0)$

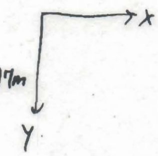
$$v_{②} = \sqrt{2 \times 9.81 \times 3} = 7.67 \text{ m/s}$$

Diver # ① :



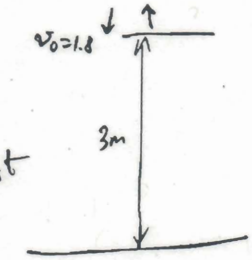
$v = v_0 - gt = 0 \rightarrow t = \dfrac{v_0}{g}$

$x - x_0 = v_0 t - \frac{1}{2} g t^2 = \frac{1}{2} \dfrac{v_0^2}{g} = \frac{1}{2} \dfrac{1.8^2}{9.81} = 0.17\text{m}$



$$v_1 = \sqrt{2 \times 9.81 \times (3 + 0.17)} = 7.88 \text{ m/s}$$

or



$$v_1^2 = 2g(x - x_0) + v_0^2$$

$$v_1 = \sqrt{2 \times 9.81 \times 3 + 1.8^2} = 7.88 \text{ m/s}$$

→ Diver #1 will hit water first

$t_2 = \dfrac{v_2 - v_0}{g} = \dfrac{7.67 - 0}{9.81} = 0.782\text{s}$; $t_1 = \dfrac{v_1 - v_0}{g} = \dfrac{7.88 - 1.8}{9.81} = 0.62\text{s}$