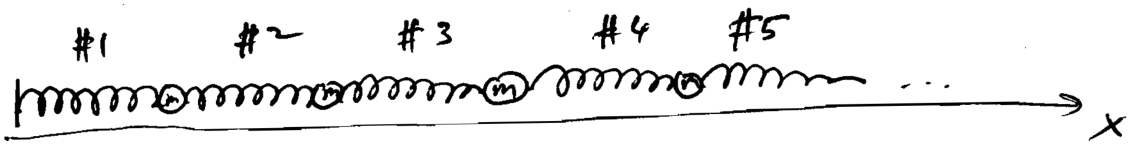
Ch.14 Wave Motion :

Wave : propagation of a local disturbance (or oscillation), not of a matter.

local disturbance : oscillation of a spring # 1



A disturbance is introduced by pulling on spring # 1, nothing
→ happens to spring # 80 (propagation is not instantaneous).
Propagation is going at the "wave speed" = $v = \frac{\lambda}{T}$

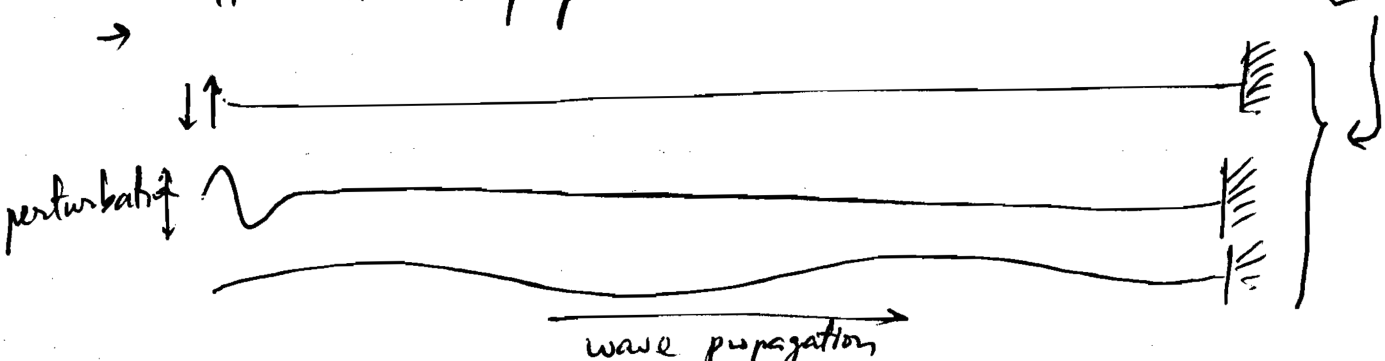
$\underline{\underline{\lambda}}$: lambda : wave length ; $\underline{\underline{T}}$: period

→ There is a time variation
→ Also a space variation

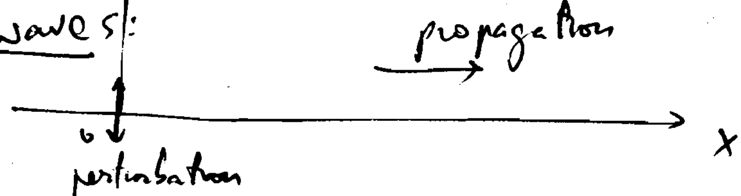
} periodic or harmonic : λ is the spatial separation between 2 consecutive peaks, T is the time separation b/w 2 consecutive peaks.

→ Wave travels in same direction as perturbation : longitudinal wave

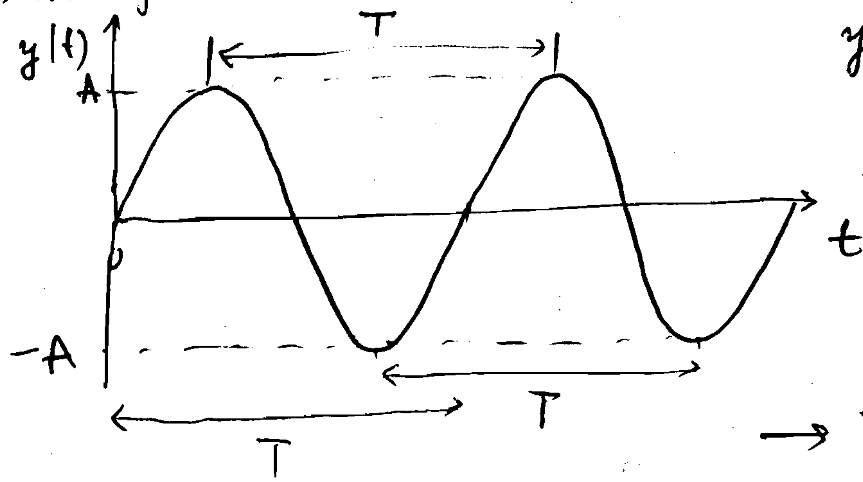
→ " " " perpendicular " " " : <u>transverse wave</u>



Transverse waves:



Perturbation at left end $(x=0)$



$y(t) = A\sin(\omega t)$

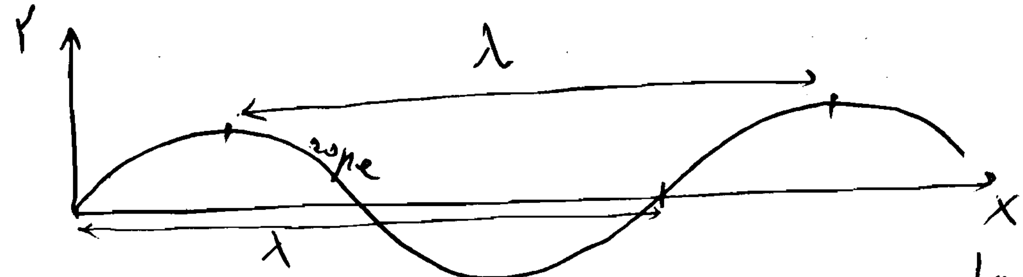
omega: angular freq.

$\omega = 2\pi f$

$\rightarrow T$: period $= \frac{1}{f}$

$\left(\omega = \frac{2\pi}{T}\right)$

Snapshot of rope at some later time t

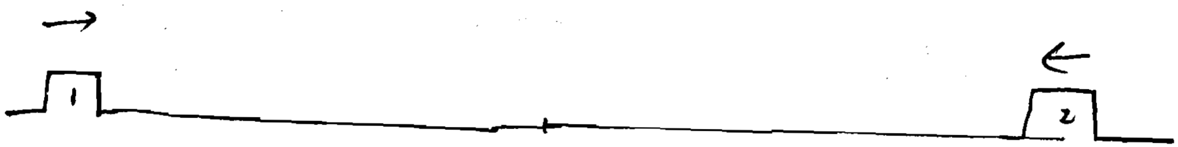


$y(x,t) = A\sin(kx - \omega t)$ → dimension: none.

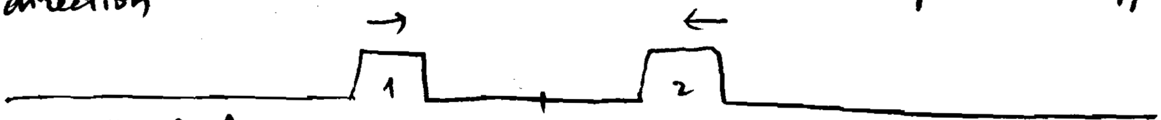
k: wave number $= \frac{2\pi}{\lambda}$; $\omega = \frac{2\pi}{T}$

wave speed: $v = \frac{\lambda}{T} = \frac{\omega}{k}$

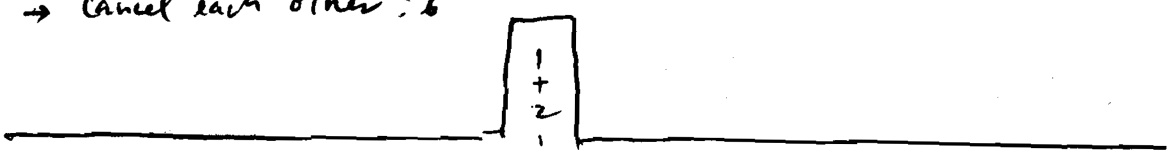
Wave superposition :

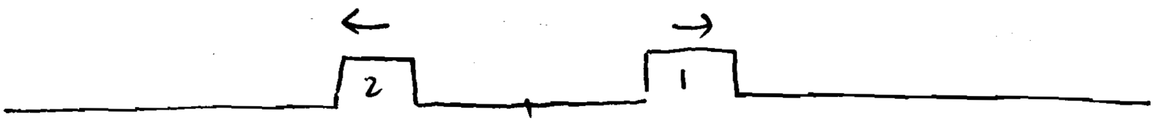


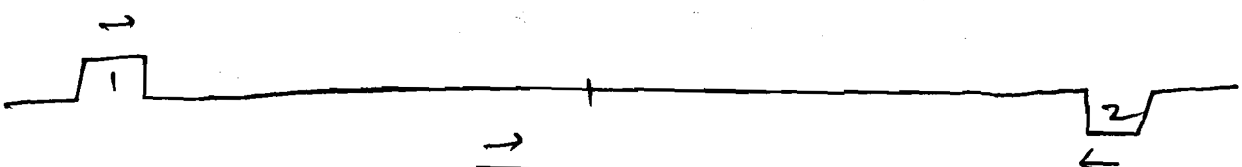
rectangular waves ~~……~~ propagating at same speed on opposite direction

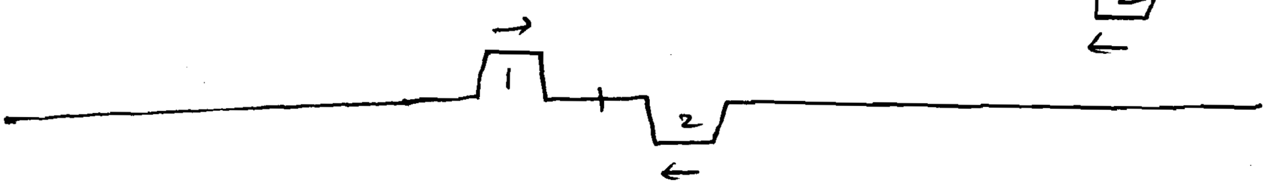


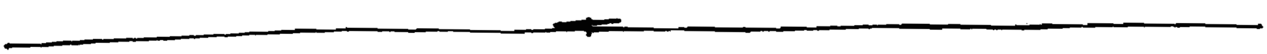
→ will reflect : 2
→ crossing, move on : 10
↓
superposition
→ cancel each other : 6

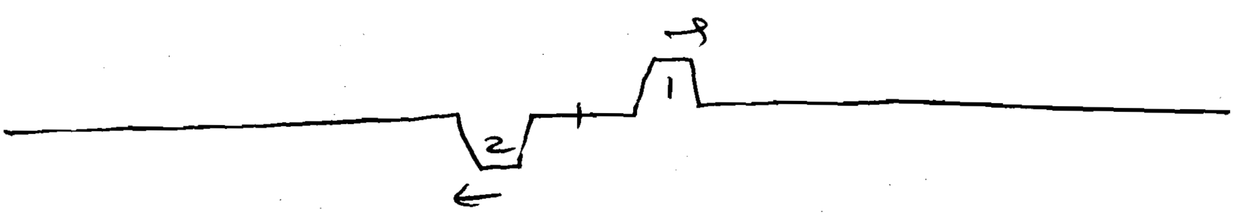












Quantitative expressions for wave superposition or interference

$$y(x,t) = A\cos(kx - \omega t) \quad \leftarrow \text{wave}$$

at $x=0$: $\quad y(0,t) = A\cos(-\omega t) = A\cos\omega t$

Two waves at $x=0$: $\quad \left\{ \begin{array}{l} y_1(0,t) = A\cos\omega_1 t \\ y_2(0,t) = A\cos\omega_2 t \end{array} \right\}$ local perturbations

$$y_1 + y_2 = A(\cos\omega_1 t + \cos\omega_2 t) = 2A\cos\left(\frac{\omega_1-\omega_2}{2}t\right)\cdot\cos\left(\frac{\omega_1+\omega_2}{2}t\right)$$

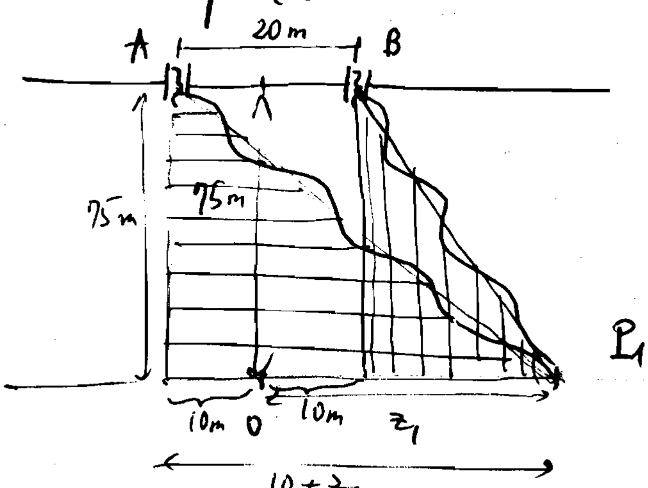
$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

→ Beat phenomenon: bimotor airplane

$w_1 = 7000\ s^{-1}$
$w_2 = 7010\ s^{-1}$ $\Big\rangle$ beats at $5\ s^{-1}$
→ we can hear.

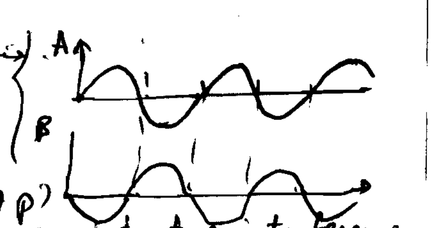
another application
(guitar tuning with a whistle)

Wave superposition in a pool:



$AP > BP$ → wave A will arrive at P at a different phase than wave B

at P :



what is the combined wave at P? → destructive interference

Destructive interference: (calm spots)

$$\begin{cases} AP_1 - BP_1 = \frac{\lambda}{2} \quad \text{(half wave length)} \\ AP_2 - BP_2' = \frac{3\lambda}{2} = \lambda + \frac{\lambda}{2} \\ AP_3 - BP_3 = \frac{5\lambda}{2} = 2\lambda + \frac{\lambda}{2} \\ \vdots \end{cases}$$

Constructive interference: (rough spots)

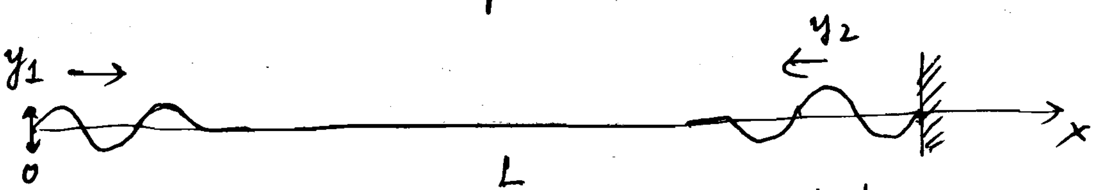
$$\begin{cases} AP_1^M - BP_1^M = \lambda \\ AP_2^M - BP_2^M = 2\lambda \\ AP_3^M - BP_3^M = 3\lambda \\ \vdots \end{cases}$$

$$AP_1 - BP_1 = \sqrt{75^2 + (z_1+10)^2} - \sqrt{75^2 + (z_1-10)^2} = \frac{\lambda}{2}$$

$\rightarrow$ if $\lambda = 16m$ $\rightarrow$ $z_1 = 33m$

Standing waves:

Transverse wave in a rope:



When wave reaches the fixed end:
1) disappears
2) reflects same phase
✓ 3) reflects ~~opposite~~ phase inverted

$$y(x,t) = \underbrace{y_1(x,t)}_{\text{incoming}} + \underbrace{y_2(x,t)}_{\text{reflected}}$$

$$= A\cos(kx - \omega t) - A\cos(kx + \omega t) \rightarrow$$

$$\boxed{\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$$\boxed{y(x,t) = -2A\sin kx \,\sin\left(-\frac{2\omega t}{2}\right) = \underbrace{2A\sin kx}\,\sin\omega t}$$

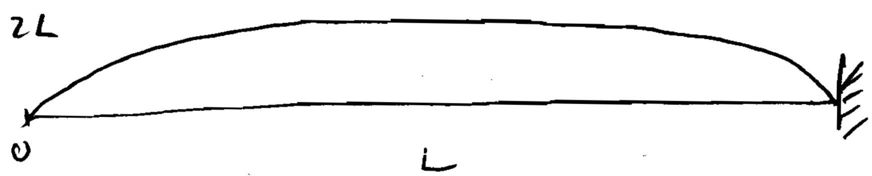
This can be zero along the rope: (nodes along rope)

At the fixed point $\left\{ \sin kL = 0 \quad \forall t : \left(y(L,t) = 2A\sin kL \sin\omega t \right) \right.$

$\hookrightarrow \underset{\substack{\downarrow \\ \text{wave number}}}{kL} = n\pi \quad \Big\} \quad \frac{2\cancel{\pi}}{\lambda} L = n\cancel{\pi} \rightarrow \boxed{\lambda_n = \frac{2L}{n}}$

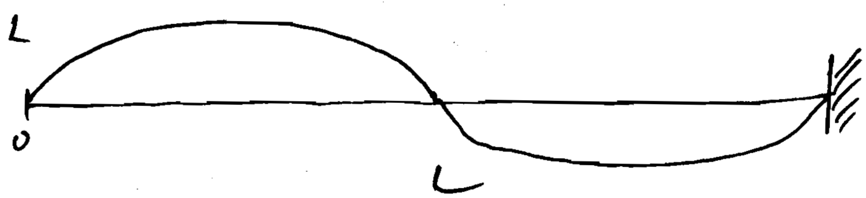
Only certain wavelengths λ_n can stand in a rope of length L with one end fixed: $\underbrace{\lambda_1 = 2L}$; $\lambda_2 = L$; $\lambda_3 = \frac{2L}{3}$, etc.

$\lambda_1 = 2L$



Fundamental mode (standing wave) (Lowest frequency)

$\lambda_2 = L$



Next mode

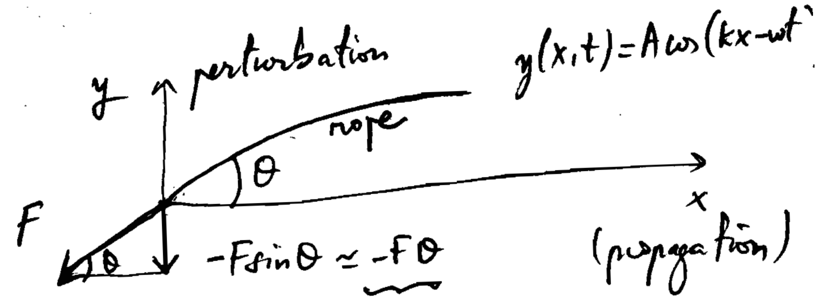
$\lambda_3 = \frac{2L}{3}$

⋮

Wave Power:

Recall: $P = F.v$ $\qquad \left(P = \frac{\text{Work}}{\text{time}} = \frac{F.d}{\text{time}} \right)$

Transverse wave in a rope:



$P_{wave} = -F\theta v$

$v \to$ perturbation speed. (or u)

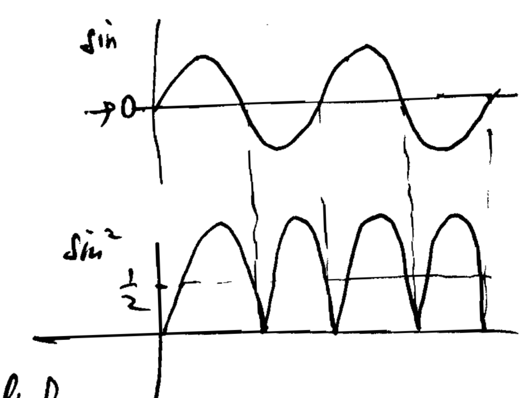
$\frac{dy}{dt} = -A\sin(kx-\omega t)(-\omega) = A\omega \sin(kx-\omega t)$

$\tan\theta \simeq \theta = \frac{dy}{dx} = -kA\sin(kx-\omega t)$

$P_{wave} = +FkA\sin(kx-\omega t)\, A\omega \sin(kx-\omega t)$
$= Fk\omega A^2 \sin^2(kx-\omega t)$

$\overline{P_{wave}} = Fk\omega A^2 \underbrace{\overline{\sin^2(kx-\omega t)}}_{\frac{1}{2}}$

$= \frac{1}{2} F\omega k A^2$



Rope: wave speed $v = \sqrt{\frac{F}{\mu}}$ — $F \to$ force applied (tension); $\mu \to$ linear mass density.

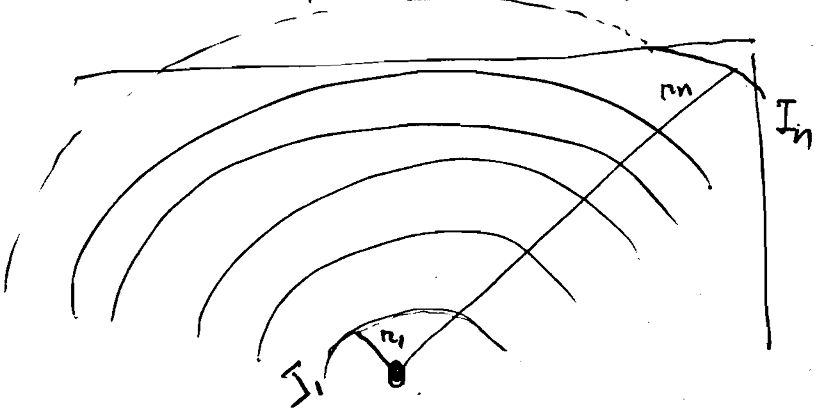
↳ $\frac{A}{T} = \frac{\omega}{k}$

↳ along direction of propagation (≠ perturbation speed $\frac{dy}{dt}$ along transverse direction)

$\boxed{\overline{P} = \frac{1}{2}\mu\omega^2 A^2 v}$

Wave intensity:

$$I = \frac{P}{\text{Area}}$$



$$I_1 = \frac{P}{4\pi r_1^2}$$

$$I_n = \frac{P}{4\pi r_n^2}$$

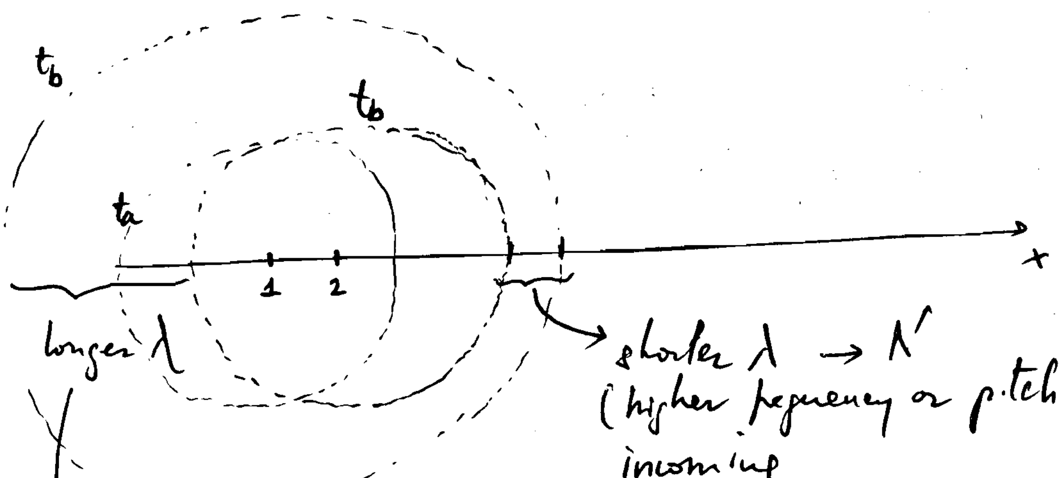
e.g. $r_n = 6r_1 \rightarrow \dfrac{I_n}{I_1} = \dfrac{r_1^2}{r_n^2} = \dfrac{1}{36}$

spherical waves

surface of a sphere is: $4\pi r^2$

Sound waves: Doppler Effect.

Consequence of a moving source



longer λ

(lower freq or pitch)

leaving

shorter $\lambda \rightarrow \lambda'$

(higher frequency or pitch)

incoming

$$\lambda' = \lambda - uT \qquad \left(T = \frac{\lambda}{v}\right)$$

u: source speed; since the wave travelled λ' in one period T, but now it is shortened by the distance travelled by the source at speed u in the same time period T

$\lambda' = \lambda\left(1 - \frac{u}{v}\right)$ (approaching source)

$\lambda' = \lambda\left(1 + \frac{u}{v}\right)$ (leaving source)

$$f' = \frac{f}{1 \mp \frac{u}{v}}$$

[redacted]

If observer is moving instead of the source

$$f' = f\left(1 \pm \frac{u}{v}\right)$$

$+$: approaching observer

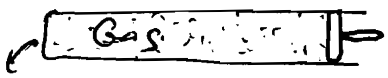
$-$: leaving observer.

<u>Sound:</u> $v = 340\ m/s$

Ch 15: Fluid Motion

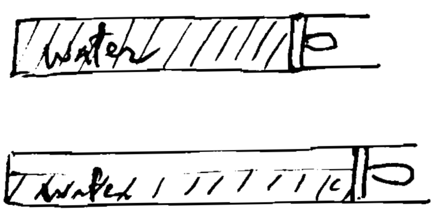
→ Gas : ρ can be variable (gas is compressible)
Liquid : ρ is constant (liquid is incompressible)

Density : ρ "rho" $= \dfrac{\text{mass}}{\text{volume}} = \dfrac{dm}{dV}$ $\left(\dfrac{kg}{m^3} \text{ in S.I.} \right)$



closed

m, V can be reduced under compression → ρ can be increased.
similarly V can be increased → ρ can be decreased



} can't change the density of a liquid.

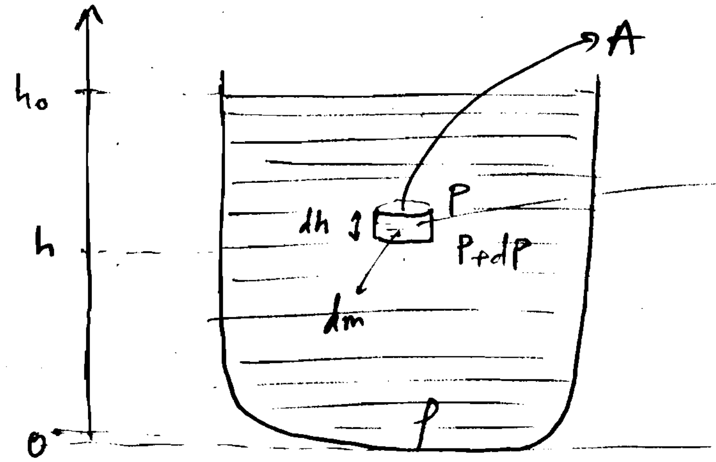
Pressure P : normal force per unit area :

$$P = \frac{F}{A} \quad \text{or} \quad \frac{dF}{dA}$$: (direction is not relevant, P is not a vector)

SI : $\dfrac{N}{m^2}$ or Pa (Pascal)

Atm (Atmosphere) $= 1.013 \times 10^5$ Pa

Hydrostatic equilibrium:



→ a piece of water, in equilibrium: $\vec{F}_{net} = 0$

(pressure is higher as we go down into the water: the difference in pressure at top and bottom of piece of water cancels its weight).

$$\rho = \frac{dm}{dV} \rightarrow dm = \rho\, dV = \rho A\, dh$$

dh ↕ A } $dV = A\,dh$

$$(P + dP)A - PA = g\, dm$$

$$\cancel{A}\, dP = g\rho \cancel{A}\, dh$$

$$\boxed{\frac{dP}{dh} = g\rho}$$

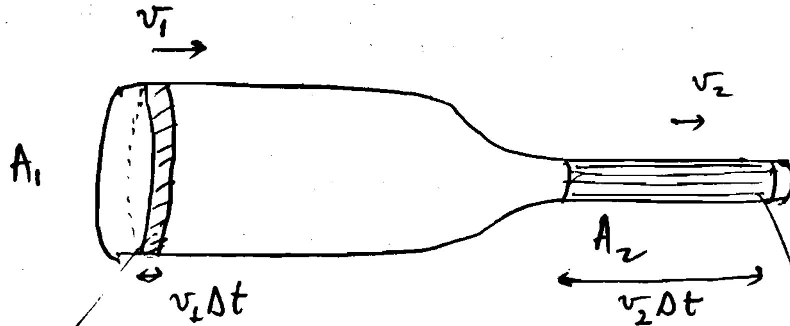
This equation describe how P changes with h

$$P(h) = \int_{h_0}^{h} g\rho\, dh \underset{\rho \text{ const.}}{=} g\rho\,(h - h_0)$$

For the ref. in the sketch → $\boxed{\frac{dP}{dh} = -g\rho}$ → $P(h) = g\rho(h_0 - h)$

$$P = g\rho h \rightarrow F = P.A = g\rho \underbrace{hA}_{Vol.} = F_{bouyant}.$$

Conservation of mass: (no leak or loss of fluid molecules)



a disc of fluid of mass m_1

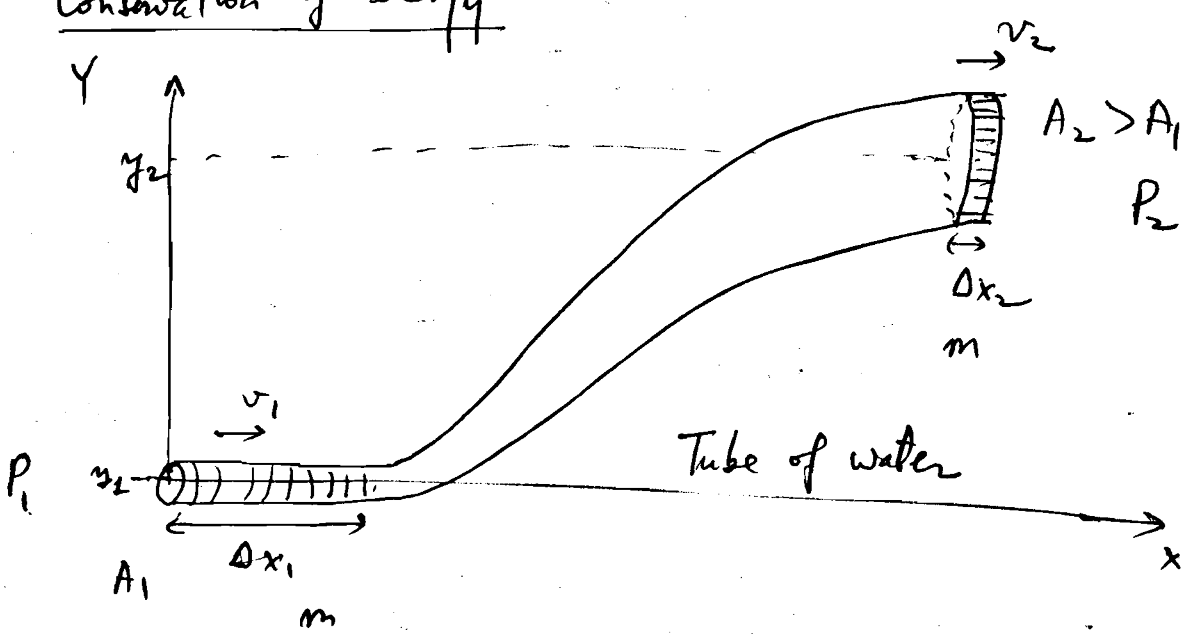
same disc of fluid of mass $m_2 = m_1$

$$\rho V_1 = \rho V_2$$

$$\boxed{A_1 v_1 \cancel{\Delta t} = A_2 v_2 \cancel{\Delta t}}$$

vA is constant. (conservation of mass)

Conservation of energy



Work done by the difference in pressure is:

$$\Delta W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

Conservation of energy:

$$\Delta W = \Delta KE + \Delta P.E$$

$$P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + m g y_2 - m g y_1$$

$$\rightarrow \frac{1}{2} m v_2^2 + m g y_2 + P_2 A_2 \Delta x_2 = \frac{1}{2} m v_1^2 + m g y_1 + P_1 A_1 \Delta x_1$$

$$\rightarrow \boxed{\frac{1}{2} m v^2 + m g y + P A \Delta x = \text{const.}}$$

Rearranging: dividing both sides by volume $V = A \Delta x$

$$\frac{1}{2} \frac{m}{V} v^2 + \frac{m}{V} g y + P = \text{const}$$

$$\boxed{\frac{1}{2} \rho v^2 + \rho g y + P = \text{const.}}$$

Bernoulli's equation.

15.60 → Spherical rubber balloon: $\begin{cases} m = 0.85 g \\ d = 0.3 m \end{cases}$

Filled with helium → $\rho = 0.18 \, kg/m^3$

How many paper clips of 1g can we hang before it loses its bouyancy?

$F_{bouyant} = g \rho_{air} V \rightarrow$ vol. of balloon (or vol. of fluid displaced by object)

Critical situation (hovering balloon + clips):

$$g \rho_{air} V_{balloon} = (m_{balloon} + m_{He} + m_{clips}) g$$

$$= m_{balloon} + \underline{\rho_{He}} V_{balloon} + N \times 10^{-3}$$

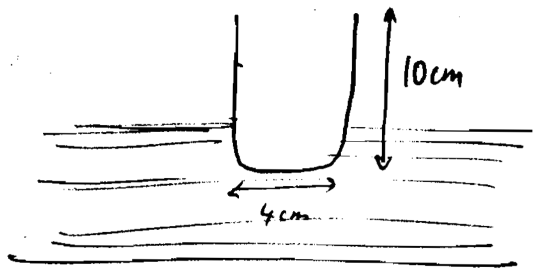
$$(\rho_{air} - \rho_{He}) V_{balloon} = m_{balloon} + N 10^{-3}$$

$\rho_{air} = 1 kg/m^3$

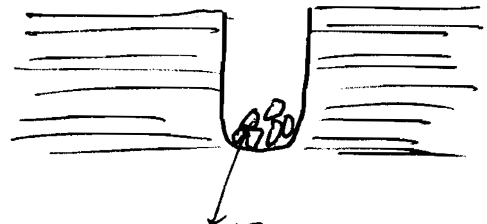
$$N = \frac{(1 - 0.18)\,\frac{4}{3}\pi(0.15)^3 - 0.85\times10^{-3}}{10^{-3}} = 10.74$$

$\rightarrow N = 10$ clips before it loses bouyancy

15-50



$d = 4cm$



empty: floats with $\frac{10 cm}{3}$ submerged

$\downarrow$

15g. How many rocks before it sinks?

$\frac{1}{3}$ submerged $\quad F_{bouyant} = g\rho_{water}\underbrace{V}_{\frac{V_{beaker}}{3}} = \underbrace{m}_{m_{beaker}}g$

Full submersion $F'_{bouyant} \Rightarrow$ Full $\rightarrow$ $\Delta F_{bouyant} = \frac{2}{3}F'_{bouyant}$

$$= 2F_{bouyant}$$

$$= g\rho_{water}\frac{2V_{beaker}}{3}$$

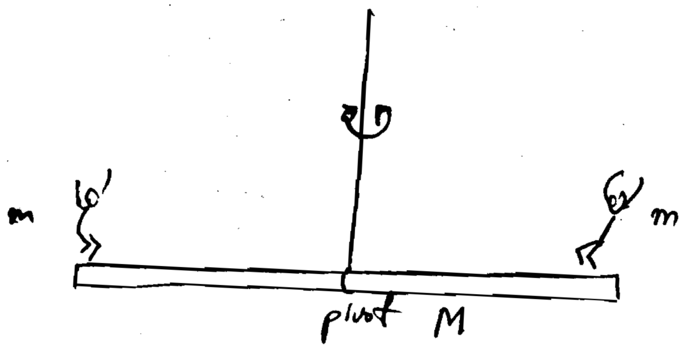
$\rightarrow$ This $\Delta F_{bouyant}$ will hold the rocks:

$\rho_{water} = 1000 kg/m^3$ $\qquad \cancel{g}\rho_{water}\frac{2}{3}V_{beaker} = N\,0.015\times\cancel{g}$

$$N = \frac{1000\,\frac{2}{3}\,0.1\times\pi\times0.02^2}{0.015} = 5.58 \text{ rocks}:$$

$N = 5$ rocks before it sinks.

13.50



Thin beam $L = 8\,m$

M?

$m = 75\,kg$; $\underset{\text{with people}}{w} = 0.8\, \underset{\text{without people}}{w_0}$. (due to the additional rotational inertia wrt pivot $2m\left(\frac{L}{2}\right)^2$

$$\omega = \sqrt{\frac{K}{I}} \quad K \rightarrow \text{"kappa"} \qquad ; \qquad \omega_0 = \sqrt{\frac{K}{I_0}}$$

$K \rightarrow$ same since torsions are due to the twisting of the same cable!

$$0.8 = \frac{\omega}{\omega_0} = \sqrt{\frac{I_0}{I}} = \sqrt{\frac{\frac{1}{12}M\cancel{L^2}}{\frac{1}{12}M\cancel{L^2} + 2m\frac{\cancel{L^2}}{4}}} = \sqrt{\frac{\frac{M}{12}}{\frac{M}{12} + \frac{m}{2}}}$$

$$0.8^2 = \frac{M}{M + 6m} \longrightarrow 0.8^2\,(M + 450) = M$$

$$M = \frac{450 \times 0.8^2}{1 - 0.8^2} = 800\,kg$$

(13.39) Front suspension $f_0 = 0.45\ \text{Hz}$

Shock absorbers worn out

At certain speed the car shakes violently. What is that speed?

Resonance at 0.45 bumps per sec or $\frac{1}{0.45}$ s b/w bumps:

$$v = \frac{40\text{m}}{\frac{1}{0.45}\text{s}} = 40 \times 0.45 = 18\ \text{m/s} = 64.8\ \text{km/h}$$

(14.45) Doppler effect.

Red light by H atoms at rest $\lambda_0 = 656$ nm.

From distant galaxy $\longrightarrow$ $\lambda = 708$ nm ($\lambda > \lambda_0$ $\rightarrow$ leaving source)

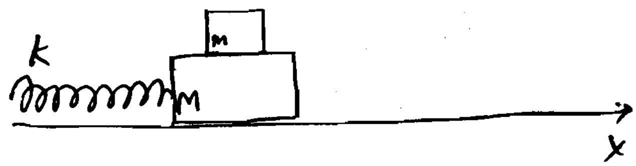
$$\lambda = \lambda_0 \left(1 + \frac{u}{v}\right)$$

u $\rightarrow$ speed of leaving galaxy

v $\rightarrow$ speed of light $= 3 \times 10^8$ m/s

$$\rightarrow \left(\frac{\lambda}{\lambda_0} - 1\right) v = u \rightarrow u = \left(\frac{708}{656} - 1\right) 3 \times 10^8 = 2.38 \times 10^7\ \text{m/s}$$

13.70



$M = 0.5\,kg$

$k = 8.7\,\frac{N}{m}$

SHM: $T = 1.8\,s$

When $A = 0.35\,m$ upper block m begins to slip. What is μ b/w the blocks.

Motion along x:

Focus on m: friction is holding it against M, up to certain acceleration: write this in term of A_{max} then compare ma_{max} with $\mu N = \mu mg$

$$\cancel{m} a_{max} = \mu \cancel{m} g$$

$$\downarrow$$

SHM: $\quad x(t) = A\cos(\omega t)\,; \quad v = \frac{dx}{dt} = -\omega A \sin(\omega t)$

$$\rightarrow a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t)$$

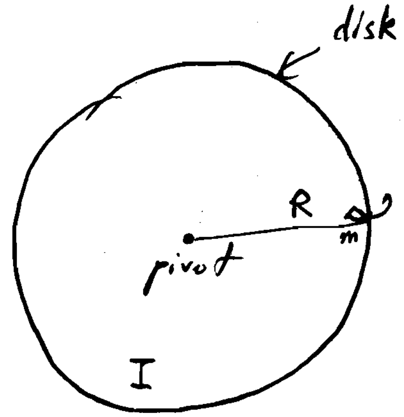
$$\boxed{a_{max} = \omega^2 A}$$

$$\mu = \frac{a_{max}}{g} = \frac{\omega^2 A}{g} = \frac{\left(\frac{2\pi}{1.8}\right)^2 \times 0.35}{9.81} = 0.44$$

$$\omega = \sqrt{\frac{k}{M_T}} = \frac{2\pi}{T} = \frac{2\pi}{1.8}$$ (can get m from here if would like but not asked for)

11.40

From above



$R = 0.25\,m$

$I_d = 0.0154\ kg\,m^2$

$\omega = 22\ rpm$

$m = 0.0195\ kg$

Mouse goes to center : a) New ω' b) W_{done} by mouse.

a) $\underset{\text{(mouse at edge)}}{I_{before}} = I_d + mR^2$; $\underset{\text{(mouse at center)}}{I_{after}} = I_d + m.0^2$

$\vec{\tau}_{net} = 0$

<u>Linear motion:</u>

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$

$\downarrow$

$\vec{F}_{net} = 0 \rightarrow \vec{p}$ is conserved.

<u>Angular or rotational motion:</u>

$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

$\vec{L}$ = angular momentum $= \vec{r} \times \vec{p}$

$\vec{\tau}_{net} = 0 \rightarrow \vec{L}$ is conserved.

$\rightarrow L = I\omega$ is conserved $\rightarrow I_{before}\,\omega = I_{after}\,\omega'$

$$\omega' = \frac{I_{before}}{I_{after}}\,\omega = \frac{0.0154 + 0.0195 \times 0.25^2}{0.0154}\,22\,rpm$$

$= 23\ rpm$

b) Work done by the mouse?

$$\Delta W = \Delta KE = \frac{1}{2} I_{after} w'^2 - \frac{1}{2} I_{before} w^2$$

$$= \frac{1}{2} I_{after} w'^2 \left(1 - \underbrace{\frac{I_{before}}{I_{after}}}_{\frac{w'}{w}} \frac{w^2}{w'^2}\right)$$

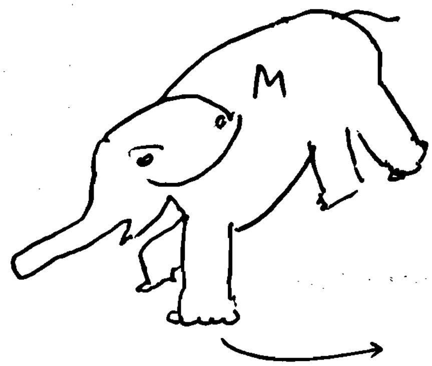
$$\boxed{= \frac{1}{2} I_{after} w'^2 \left(1 - \left(\frac{w}{w'}\right)\right)}$$

$$W = 22\,rpm = \frac{22 \times 2\pi}{60\,s} = \frac{44\pi}{60} s^{-1} = 2.3 s^{-1}$$

$$w' = 23.7\,rpm = \frac{23.7 \times 2\pi}{60\,s} = 2.41 s^{-1}$$

$$\Delta W = \frac{1}{2}\, 0.0154 \times 2.41^2 \left(1 - \frac{2.3}{2.41}\right) J = 3.49 mJ$$

15.22



$M = 4300\,kg$

0.3 m

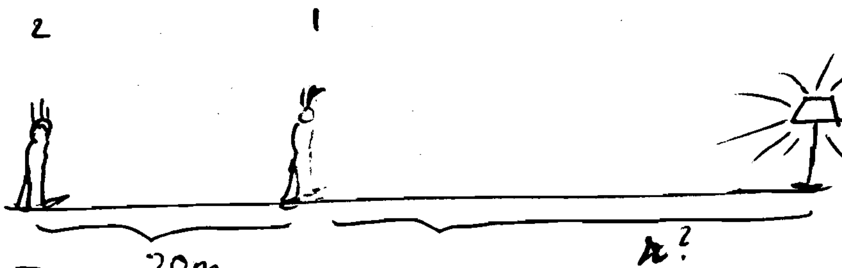
$\rightarrow A = \pi\left(\frac{0.3}{2}\right)^2$

$$P = \frac{F}{A} = \frac{Mg}{A} = \frac{4300 \times 9.81}{\pi\ 0.15^2} = 596\,000\ \text{Pa}$$

$$\downarrow$$

$$\frac{N}{m^2}$$

14.58



spherical light source

I_2 $\quad I_1 = 1.5\, I_2$

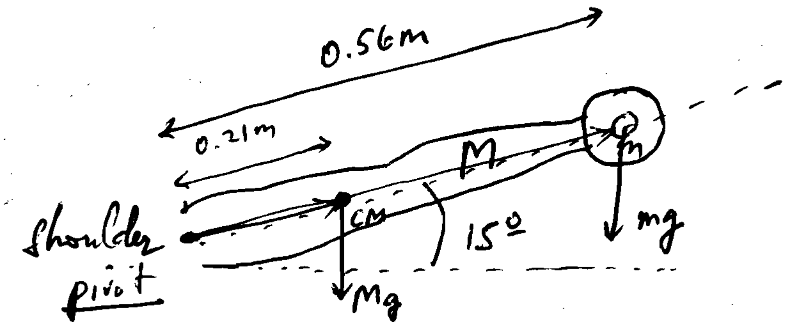
Light is an EM $\underline{\underline{\text{wave}}}$ $= \quad I = \frac{P}{4\pi r^2}$ (spherical wave)

$$1.5 = \frac{I_1}{I_2} = \frac{\frac{\cancel{P}}{\cancel{4\pi} r_1^2}}{\frac{\cancel{P}}{\cancel{4\pi} r_2^2}} = \frac{r_2^2}{r_1^2} = \frac{(r+20)^2}{r^2}$$

$$\rightarrow 1.5r^2 = r^2 + 40r + 400 \rightarrow 0.5r^2 - 40r - 400 = 0$$

$$r = \frac{40 \pm \sqrt{1600 + 800}}{1} = 40 \pm \sqrt{2400} \begin{cases} 88.9\,m \checkmark \\ -8.9\,m \end{cases}$$

(12-28)

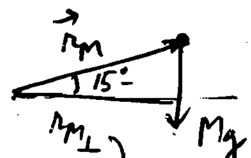


$M = 4.2\,kg$
$m = 6\,kg$

a) $\vec{\tau}$ about shoulder (pivot) due Mg & mg :

$$\vec{\tau} = \vec{r}_M \times \vec{F}_M + \vec{r}_m \times \vec{F}_m = \underbrace{r_{M\perp}}_{\text{component of } r_M \perp \text{ to } Mg} Mg + \underbrace{r_{m\perp}}_{\text{component } r_m \perp \text{ to } mg} mg \quad \text{(into the page)}$$

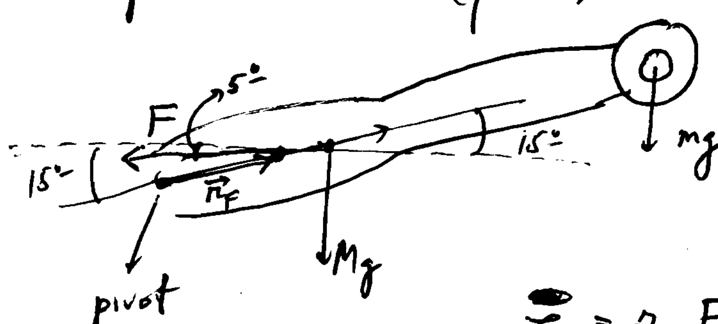
$\vec{r}_M$, $\vec{r}_m$: pivot to application point



$r_{M\perp} = 0.21 \cos 15°$

$$\tau = 0.21 \cos 15° \times 4.2 \times 9.81 + 0.56 \cos 15° \times 6 \times 9.81$$
$$= 40.2 \ N.m$$

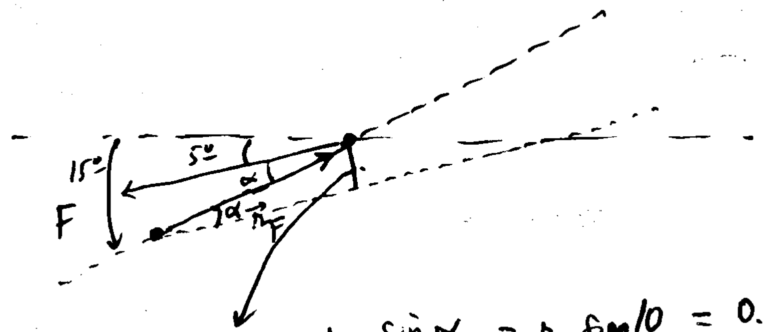
b) This torque is balance by that exerted by the deltoid muscle : force pointing 5° below horizontal at 0.18m from shoulder (pivot).



$\vec{F}$ gives a torque pointing out of the page → can balance

$$\tau_F = r_{F\perp} F = 40.2 \ N.m$$

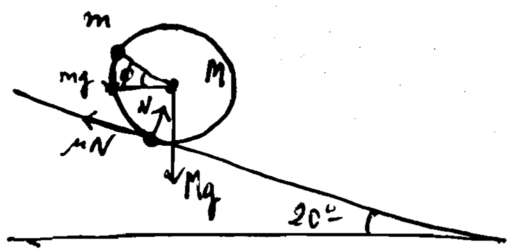
$$F = \frac{40.2}{r_{F\perp}} = \frac{40.2}{0.18 \sin 10°} N$$



$$r_{F\perp} = r_F \sin\alpha = r_F \sin 10 = 0.18 \sin 10°$$

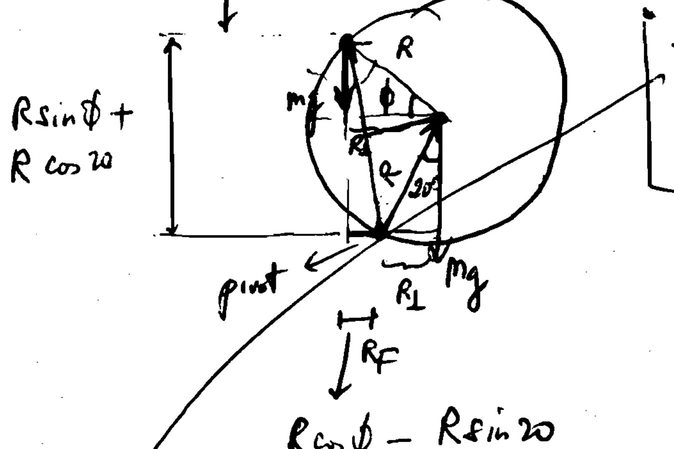
$$\rightarrow F = \frac{40.2}{0.18 \sin 10°} N = 1286 N$$

12-61



no slipping (friction)
uniform disk $M = 1.5 kg$
$m = 0.95 kg$
ϕ? so to prevent wheel from rolling down.

$\vec{\tau}_{net} = 0$ → pivot: contact with incline (no torque contribution from μN & N since $\vec{r} = 0$ for both)



$$\boxed{\tau_{net} = -\tau_{Mg} + \tau_{mg} = -R \sin 20° Mg + \cancel{R} \cos\phi \, m\cancel{g} = 0}$$

$-\sin 20°$

~~$\cos\phi = \frac{M}{m} \sin 20° = \frac{1.5}{0.95} \sin 20°$~~

~~$\phi = \cos^{-1}\left(\frac{1.5}{0.95} \sin 20°\right)$~~

$$\tau_{net} = -\cancel{R} \sin 20° M\cancel{g} + \cancel{R}(\cos\phi - \sin 20°)\, m\cancel{g} = 0$$

$$\rightarrow \cos\phi = \frac{M \sin 20° + m \sin 20°}{m} = \frac{M+m}{m} \sin 20° =$$

$$\phi = \cos^{-1}\left(\frac{2.45}{0.95}\sin 20^{\circ}\right) = 28.1^{\circ}$$