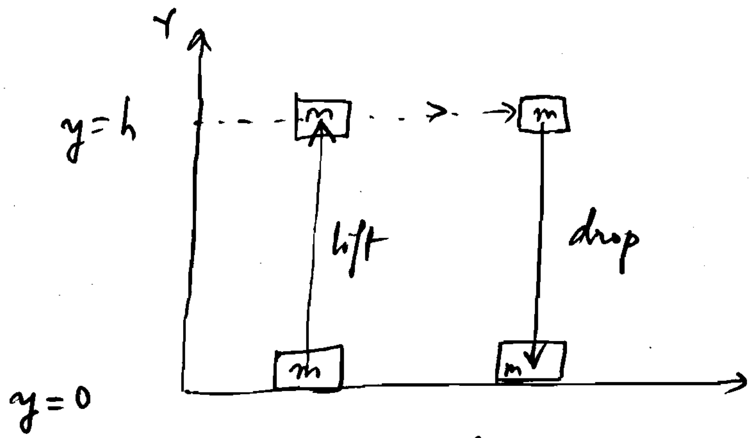
Ch. 7: Conservation of Energy



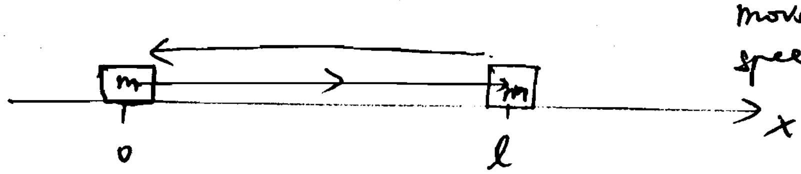
Lifting $\begin{cases} \text{work done by lifter} = +mgh \\ \text{Work done by gravity} = -mgh \text{ (force times displacement)} \end{cases}$

dropping $\begin{cases} \text{work done by dropper} : 0 \\ \text{work done by gravity} : +mgh \end{cases}$

(potential) energy is conserved
↓
gravitation is a conservative force

μ_K

move the box with constant speed. $\rightarrow F_{min} = \mu_K mg$
↓
Force applied



$0 \rightarrow l$ $\begin{cases} \text{work done by friction} : -\mu_K mgl \\ \text{work done by pusher} : \mu_K mgl \end{cases}$

$l \rightarrow 0$ $\begin{cases} \text{work done by friction} : -\mu_K mgl \\ \text{work done by pusher} : \mu_K mgl \end{cases}$

non-zero: energy is not conserved. Friction is not a conservative force

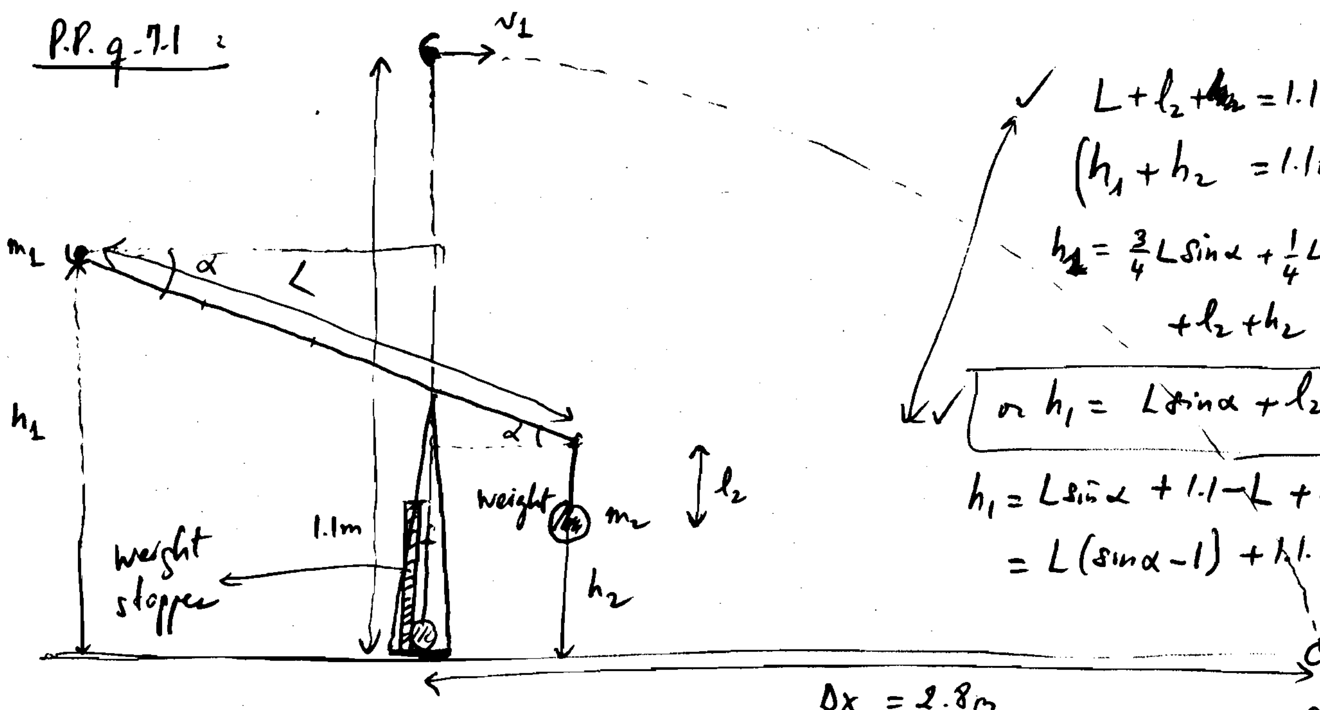
Conservation of Mechanical Energy:

$\underbrace{\text{Mechanical Energy}}$ → Sum of Kinetic energy and gravitational potential energy

$$\left(\frac{1}{2}mv^2 + mgh\right)_{\text{time }1} = \left(\frac{1}{2}mv^2 + mgh\right)_{\text{time }2}$$

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

P.P. 9.7.1:



✓ $L + l_2 + \cancel{h_2} = 1.1m$

$(h_1 + h_2 = 1.1m)$

$h_1 = \frac{3}{4}L\sin\alpha + \frac{1}{4}L\sin\alpha + l_2 + h_2$

✓ $\boxed{\text{or } h_1 = L\sin\alpha + l_2 + h_2}$

$h_1 = L\sin\alpha + 1.1 - L + h_2$

$= L(\sin\alpha - 1) + 1.1 + h_2$

Conservation of mechanical energy: $\begin{cases} \text{initial: weight at } h_2\text{; ball at } h_1 \\ \text{final: weight at } 0\text{; ball at } 1.1m \end{cases}$

This way we can calculate v_1 and relating it to $2.8m = \Delta x$.

$$M.E._{\text{initial}} = M.E_{\text{final}}$$

$$KE_i + P.E_i = \underbrace{KE_f} + \underbrace{PE_f}$$

$$\underset{0}{\downarrow} \quad m_1gh_1 + m_2gh_2 = \frac{1}{2}m_1v_1^2 + \cancel{\frac{1}{2}m_2 0^2} + m_1g1.1 + \cancel{m_2g0}$$

Projectile motion

Once the ball left the spoon: it followed a projectile motion:

$$\begin{cases} \text{horizontal speed} = \text{constant} : & \Delta x = v_1 t \rightarrow v_1 = \dfrac{\Delta x}{t} = \dfrac{2.8}{\sqrt{\frac{2.2}{9.81}}} = 5.9\,m/s \\ \text{vertical motion} = & \Delta y = \frac{1}{2} g t^2 \rightarrow 1.1 = \frac{1}{2} 9.81\, t^2 \rightarrow t = \sqrt{\dfrac{2.2}{9.81}} \end{cases}$$

In order to reach the target at 2.8m away, v_1 has to be 5.9 m/s

→ Plug v_1 into conserv. of M.E. to find h_2:

$$m_1 g h_1 + m_2 g \overset{\text{unknown.}}{h_2} = \frac{1}{2} m_1 \underset{5.9\,m/s}{v_1^2} + m_1 g\, 1.1$$

$$m_2 g h_2 = \underbrace{\frac{1}{2} 0.1 \times 5.9^2}_{1.8\,J} + \underbrace{m_1 g}_{0.1 \times 9.81} \underbrace{(1.1 - h_1)}_{\underbrace{[L(1-\sin\alpha) - h_2]}_{\alpha \simeq 30° \;\to\; \left(\frac{L}{2} - h_2\right)}}$$

Do good estimate

Ch. 8 : Gravity :

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$ Universal Gravitation

$G \rightarrow$ Universal Grav. constant $= 6.67\times10^{-11}\ \frac{Nm^2}{kg^2}$

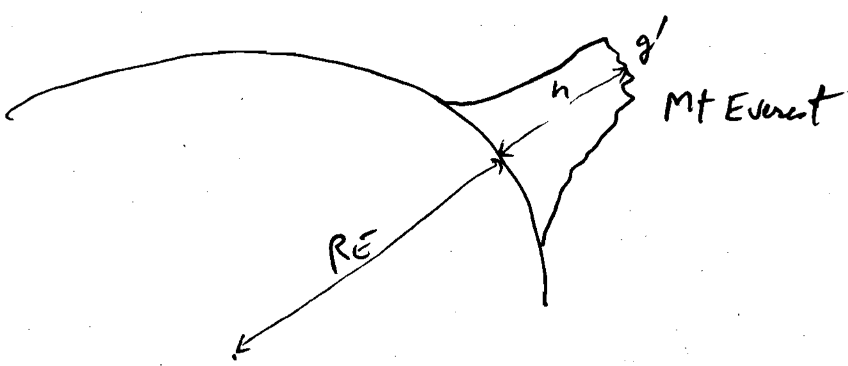
$F \rightarrow$ Force of attraction b/w objects of masses m_1 and m_2, separated by r.

Why $g = 9.81\,m/s^2$. Gravitation by our planet : $\begin{cases} M_E = 5.97\times10^{24}\,kg \\ R_E = 6.37\times10^{6}\,m \end{cases}$

For an object of mass m on the surface : $r = R_E$

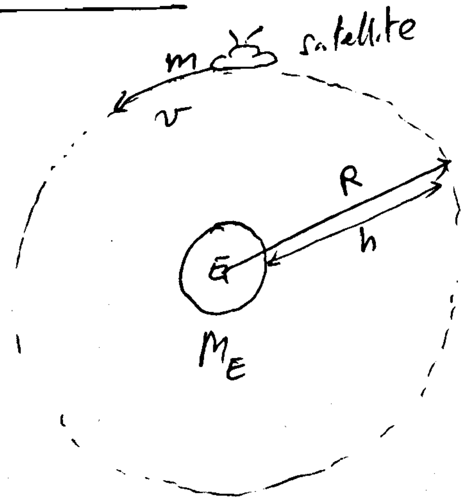
$$F = G\frac{M_E \cdot m}{R_E^2} = mg \rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6.37\times10^6)^2}$$

$$= 9.81\,m/s^2$$



$$g' = \frac{GM_E}{(R_E+h)^2} < g$$

Orbital Motion:



→ Period for orbital motion T (time to complete one turn around the Earth)

Uniform circular motion:

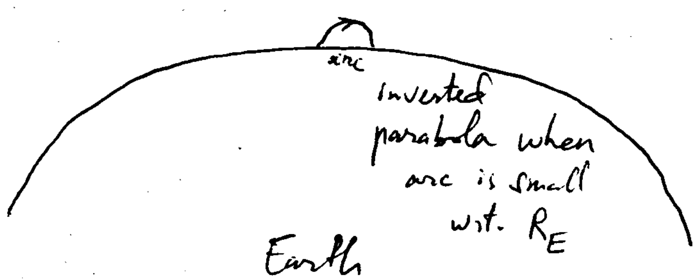
$$\frac{G M_E \cancel{m}}{R^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{R}}$$

$$\boxed{v = \sqrt{\frac{G M_E}{R}}}$$

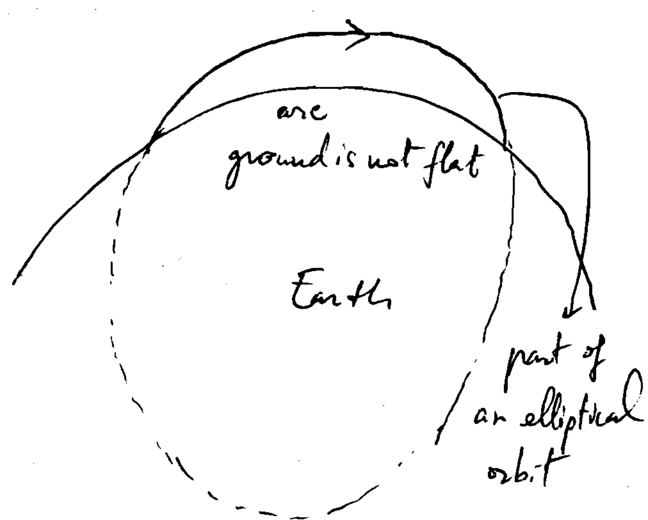
$$\boxed{T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{G M_E}{R}}} = \frac{2\pi}{\sqrt{G M_E}} R^{3/2}}$$

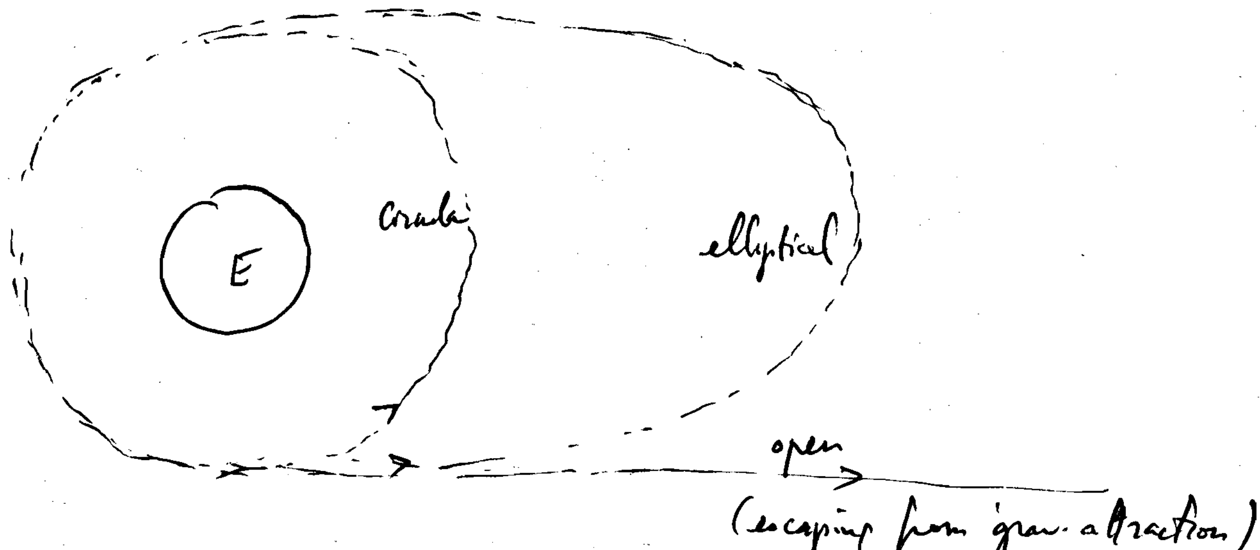
A satellite at $h = 250$ km → $T = \frac{2\pi}{\sqrt{G . M_E}}(R_E + h)^{3/2} = 5400s$
$= 1.5h$

Projectile motion:



More generally:





Escape speed:

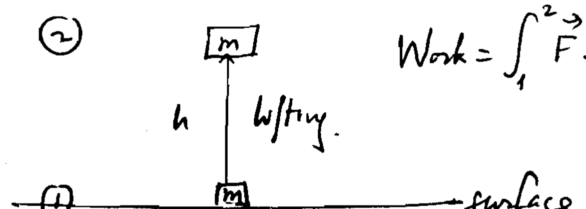
$$KE + PE = 0 \quad \text{(Total energy of object under gravity)}$$

$$\frac{1}{2} m v_{esc}^2 - \frac{G M_E m}{r} = 0 \rightarrow v_{esc} = \sqrt{\frac{2 G M_E}{r}}$$

$$r = R_E \longrightarrow v_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.37 \times 10^{6}}} = 11.2 \frac{km}{s}$$

$$= 40320 \frac{km}{h}$$

Gravitational Potential Energy:



$$\text{Work} = \int_1^2 \vec{F} \cdot d\vec{r}$$

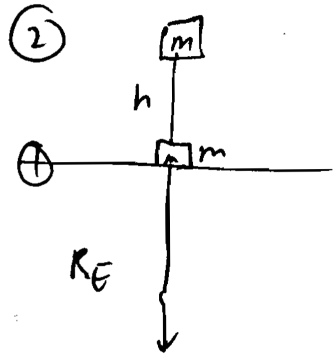
Grav. pot. energy at (2) is higher/lower than at (1)

$$\underset{(U_2 - U_1)}{\Delta U} = \int_1^2 \frac{G M_E m}{r^2} dr = G M_E m \left(-\frac{1}{r}\right)_1^2 = G M_E m \left(-\frac{1}{R_E + h} + \frac{1}{R_E}\right)$$

$$= G M_E m \frac{h}{R_E (R_E + h)}$$

When $h \ll R_E = 6370 km \longrightarrow \Delta U \approx G M_E m \frac{h}{R_E^2} = mgh$

$$\Delta U = U_2 - U_1 = GM_E m\left(\underbrace{-\frac{1}{R_E+h}}_{2} - \underbrace{\left(-\frac{1}{R_E}\right)}_{1}\right)$$

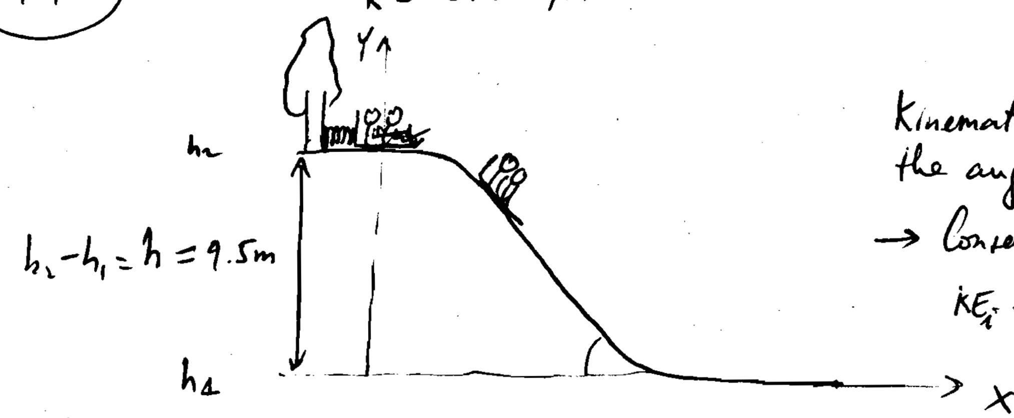


$$U_2 = -GM_E m\frac{1}{R_E+h}\ ;\ U_1 = -GM_E m\frac{1}{R_E}$$

$$\boxed{U(r) = -GM_E m\,\frac{1}{r}}$$

7.43

$k = 890\ N/m$ $\qquad$ $\Delta x = 2.6\ m$



Kinematic equations require the angle of slope.

→ Conserv. of M.E.

$$KE_i + PE_i + EPE_i = KE_f + PE_f + EPE_f$$

Elastic potential energy: $\frac{1}{2}kx^2$

initial: as shown, top of slope

final: bottom of slope.

a) $0 + mgh + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2 + 0 + 0$

$$v = \sqrt{\frac{2mgh + k(\Delta x)^2}{m}} = \sqrt{2\times 9.81\times 9.5 + \frac{890}{80}2.6^2}$$

$$v = 16.1\ m/s$$

b) Fraction of final KE stored in the spring?

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}k(\Delta x)^2} = \frac{80\times 16.1^2}{890\times 2.6^2} = 3.44$$ (gravity also give speed)

→ percentage from spring: $\frac{1}{3.44} = 0.29$

$\approx 29\%$

11.1 Impulse and Collisions

In a collision of two objects, the velocity of one or both objects is suddenly changed. The strong but short-duration force associated with the collision is called an <u>impulsive force</u>.

Based on Newton's law

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} \cdot dt = \int_{t_1}^{t_2} d\vec{p} = \Delta\vec{p}$$

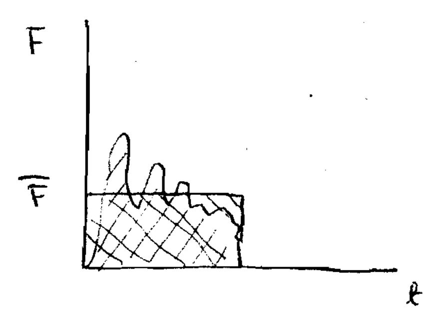
$\Delta\vec{p}$: change in momentum occurs during the collision.

$\vec{I} = \int_{t_1}^{t_2} \vec{F} \cdot dt$: impulse.

Although the impulsive force in a collision may be complicated and varies during the collision, we can define

$$\overline{\vec{F}} = \frac{\vec{I}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

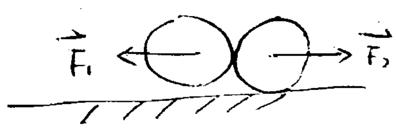
— average impulsive force.



11.2 Collisions and the Conservation Laws

The conservation laws allow us to relate the interacting objects' motions before and after collision. It provides a "short-cut" without having to deal with details of the collision.

Consider a system consisting of both colliding objects, then impulsive forces are internal to the system. Therefore the total momentum of the system should be conserved.



$\vec{F}_1$ and $\vec{F}_2$: internal

$\vec{F}_1 = -\vec{F}_2$.

What about external forces?

Since collision occurs in a very short time period, they do not have time to alter the system momentum dramatically during the collision. To a good approximation, the total momentum is conserved.

$$F_{col} >> F_{ext}.$$

$$\Rightarrow I_{col} >> I_{ext} \quad \text{in } \Delta t \text{ (short)}.$$

What about energy?

Depending on actual collisions, it may or may not be conserved during a collision.

If kinetic energy is conserved, the collision is elastic.

If not, it is inelastic.

In reality, elastic collision is impossible to realize just like the frictionless surface. But in some situation it gets close.

When the colliding objects stick together after collision to form one composite object — totally inelastic collision.

In general, we can only determine the motion after collision for the case of completely elastic or totally inelastic collisions. For intermediate cases, we have to know just how much energy is lost.

11.3 Inelastic Collisions

Consider two objects with masses m_1 and m_2 and initial velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ that undergo a totally inelastic collision.

Initially, $m_1 : \vec{v}_{1i}$ Finally $m_1+m_2 : \vec{v}_f$
$m_2 : \vec{v}_{2i}$

Conservation of momentum

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1+m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1+m_2}$$

Kinetic energy loss: (one dimension collision)

$$|\Delta K| = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 - \frac{1}{2}(m_1+m_2)v_f^2$$

$$= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}(m_1+m_2)\left(\frac{m_1v_1+m_2v_2}{m_1+m_2}\right)^2$$

$$= \frac{m_1m_2}{2(m_1+m_2)}(v_1-v_2)^2$$

If $v_1 = v_2$, no collision, $|\Delta K| = 0$ no loss.

Otherwise, $|\Delta K| > 0$. always energy loss for the total inelastic collisions.

11.4 Elastic Collisions

In elastic collisions, not only the momentum is conserved, but also the kinetic energy.

Initially: m_1, $\vec{v}_{1i}$; m_2, $\vec{v}_{2i}$

Finally: m_1, $\vec{v}_{1f}$; m_2, $\vec{v}_{2f}$

Conservations.

momentum: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

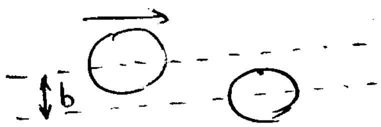
Energy: $\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$

In general, given the initial velocities, we would like to predict the outcome of a collision.

For one dimension collisions, the above two equations are enough to solve for v_{1f} and v_{2f}.

For two dimension collisions, including the two components of the momentum conservation equation, there are totally only three equations, but there are four unknowns v_{1fx}, v_{1fy} and v_{2fx}, v_{2fy}. We need additional information to help us solve them.

The additional information can be obtained with the so-called <u>impact parameter</u> which is a measure of how much the collision differs from being head-on



impact parameter : b.

However, we will not study this in any detail due to the time constraint.

Elastic Collisions in One Dimension

When two objects collide head-on, the internal forces act along the same line as the incident motion, and the objects' subsequent motion must therefore be along that same line.

From previous equations for momentum and energy conservations:

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \quad (1)$$

and

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2) \quad (2)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \quad (3)$$

$$\frac{(3)}{(1)} \Rightarrow \quad v_{1i} + v_{1f} = v_{2f} + v_{2i} \qquad (4)$$

or

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \qquad (5)$$

This equation tells us that the relative speed remains unchanged after the collision, although the direction reverses. If one object is approaching another, then after collision it will recede at the same relative speed.

$$\begin{cases} v_{1i} + v_{1f} = v_{2f} + v_{2i} \\ m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \end{cases}$$

$$\Rightarrow \begin{cases} v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i} + \dfrac{2m_2}{m_1 + m_2} v_{2i} \\[2ex] v_{2f} = \dfrac{2m_1}{m_1 + m_2} v_{1i} + \dfrac{m_2 - m_1}{m_1 + m_2} v_{2i} \end{cases}$$

Final velocities are given in terms of initials.

To examine the relationship, we choose a frame of reference moving with m_2 at its initial velocity.

Then, $v_{2i} = 0$.

Now let's consider several special cases.

Case 1 $m_1 << m_2$

Example: ① a light ball striking a much heavier ball at rest (Bouncing a basketball on earth surface).

② Any object colliding elastically with a perfectly rigid wall.

$$\begin{cases} v_{1f} = -v_{1i} \\ v_{2f} = 0 \end{cases}$$

The lighter object rebounds back with the same speed, while the much heavier object remains at rest. Obviously the kinetic energy is conserved.

What about the momentum?

Light object: momentum change of $2 m v_{1i}$.

Heavy object: momentum is 0 ? Wrong!

The heavy object can absorb huge amount of momentum without acquiring significant speed. If we "back off" from the extreme case that m_1 can be neglected ($m_1 << m_2$), then we would find that the lighter object rebounds with slightly reduced speed and that the heavier object begins moving very slowly.

$\Rightarrow$ Momentum is conserved.

Case 2 : $m_1 = m_2$

$$\begin{cases} v_{1f} = 0 \\ v_{2f} = v_{1i} \end{cases}$$

So the first object stops, transfering all its energy and momentum to the second object initially at rest.

Example In pool shooting, if the cue ball strikes a ball head-on, the cue ball will stop abruptly the kinetic energy and momentum will be carried over by the ball being hit.

Case 3 : $m_1 >> m_2$

$$\begin{cases} v_{1f} = v_{1i} \\ v_{2f} = 2v_{1i} \end{cases}$$

The more massive object moves on with no change in motion, while the lighter one heads off in the same direction with thrice the speed. It appears that both kinetic energy and momentum are not conserved.