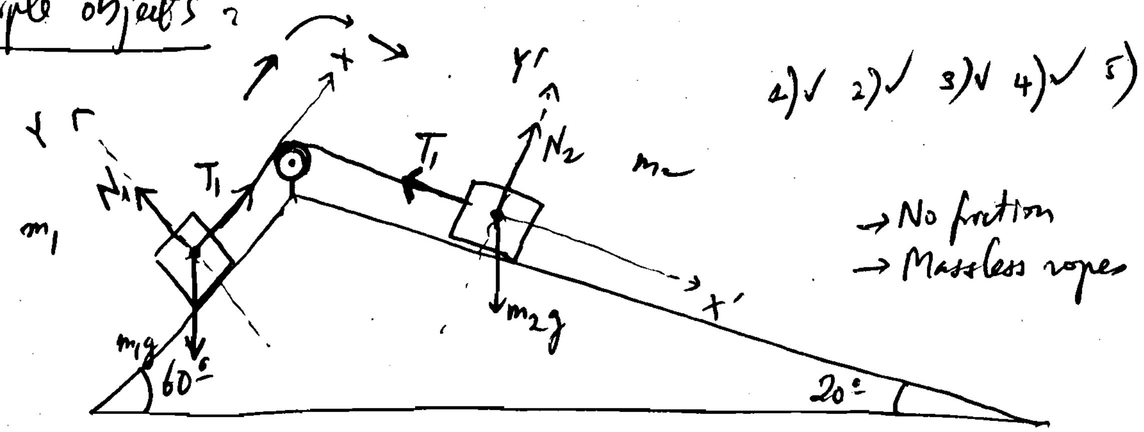
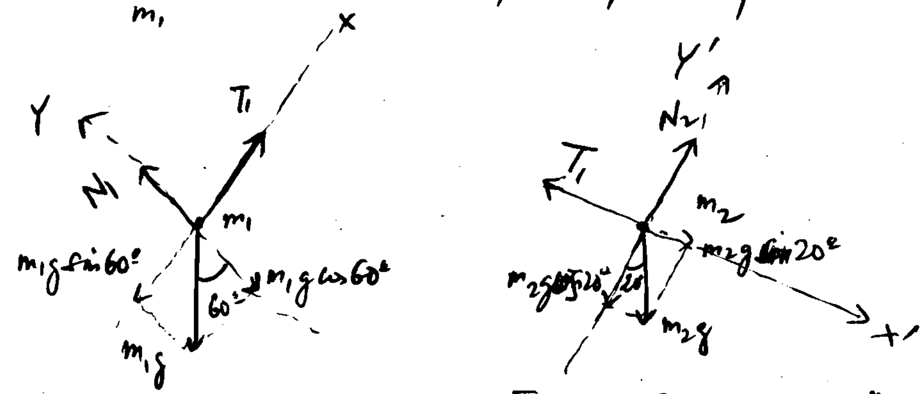
Multiple objects:

1)✓ 2)✓ 3)✓ 4)✓ 5)

→ No friction
→ Massless ropes



Find $\frac{m_2}{m_1}$ such that they stay in equilibrium (not moving)



Step 5)

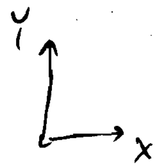
$$m_1 \begin{cases} x: & F_{net_x} = T_1 - m_1 g \sin 60° = m_1 \underbrace{a_{1x}}_{0} = 0 & (a) \\ y: & F_{net_y} = N_1 - m_1 g \cos 60° = 0 & (b) \end{cases}$$

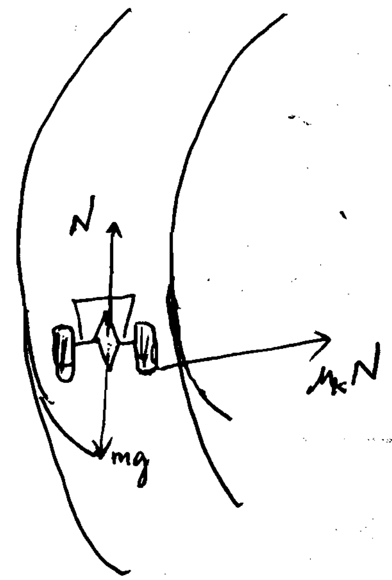
$$m_2 \begin{cases} x: & F_{net_{x'}} = m_2 g \sin 20° - T_1 = m_2 a_{2x'} = 0 & (c) \\ y: & F_{net_{y'}} = N_2 - m_2 g \cos 20° = 0 & (d) \end{cases}$$

(a) & (c) $\rightarrow \left.\begin{array}{l} T_1 = m_1 g \sin 60° \\ T_1 = m_2 g \sin 20° \end{array}\right\}$ $\frac{m_2 \sin 20°}{m_1 \sin 60°} = 1$

$$\boxed{\frac{m_2}{m_1} = \frac{\sin 60°}{\sin 20°} \approx 2.5}$$

• Circular motion:





$F_{net_y} = N - mg = 0$

$F_{net_x} = \mu_k N = m\dfrac{v^2}{R}$

Friction force changes direction of the car

UCM: $\dfrac{mv^2}{R} < \mu_k N = \mu_k mg$

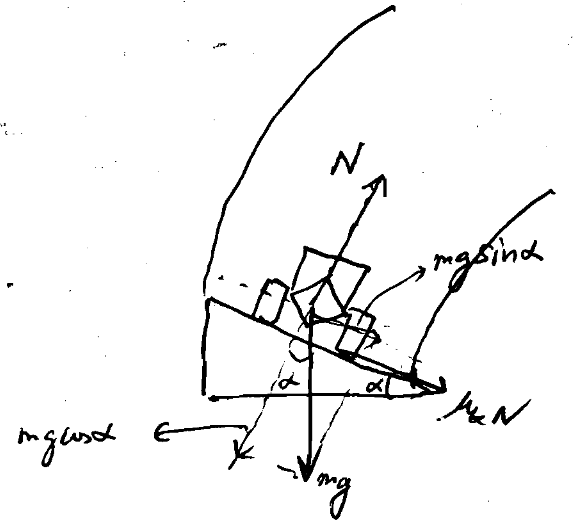
↓

v: car constant speed $\rightarrow v < \sqrt{\mu_k g R}$

R: radius of curvature

Race car: { wide tires

Track: { lower speed limit at close turn (smaller R)



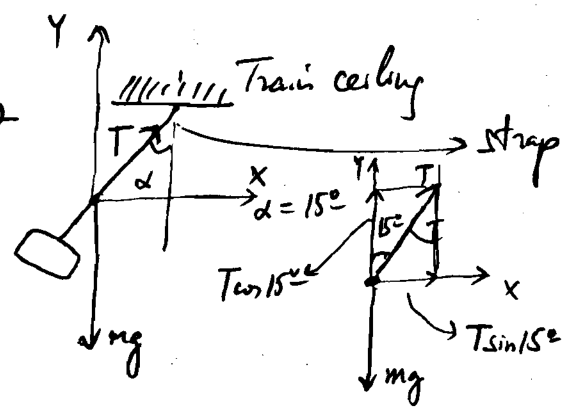
$F_{net_y} = N - mg\cos\alpha = 0$

$F_{net_x} = \mu_k N + mg\sin\alpha = m\dfrac{v^2}{R}$

↓

→ speed limit is higher

(5.25) Train taking a turn at constant $v_{train} = 67\,\frac{km}{h} = \frac{67}{3.6}\,m/s$

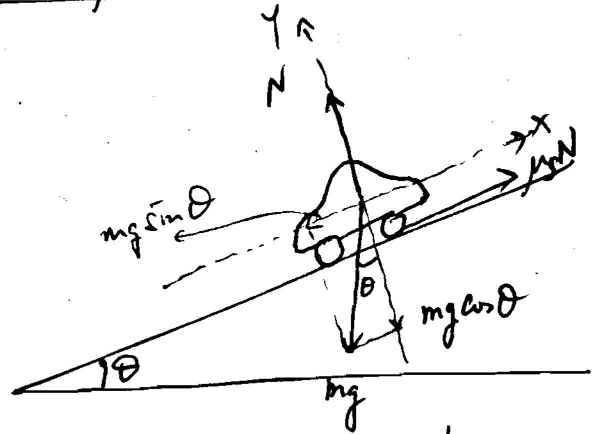


Force on strap: { mg, T

R?

1) Sketch ✓ 2) Coords. XY ✓ 3) Free-body 4) Components ✓ 5)

Friction & Equilibrium:



$\mu_s = 0.14$

What is max θ we can park safely?

Step 5) :

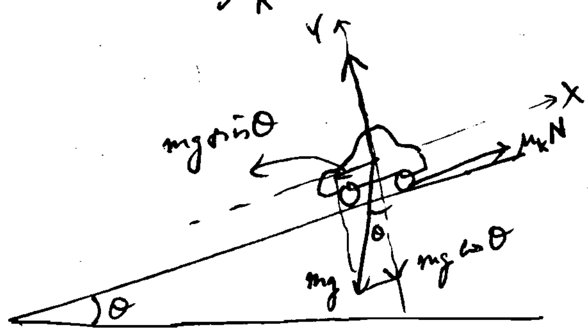
$$F_{net_x} = \mu_s N - mg\sin\theta = 0$$

$$F_{net_y} = N - mg\cos\theta = 0 \rightarrow N = mg\cos\theta$$

$$\mu_s \cancel{m}g\cos\theta - \cancel{m}g\sin\theta = 0$$

$$\tan\theta = \mu_s \rightarrow \theta = \tan^{-1}\mu_s$$
$$= 7.97^\circ$$

What if $\theta \geq 7.97^\circ$? At $\theta = 7.971^\circ$ Car starts to slip $\rightarrow$ what will be its acceleration. $\mu_k = 0.088$



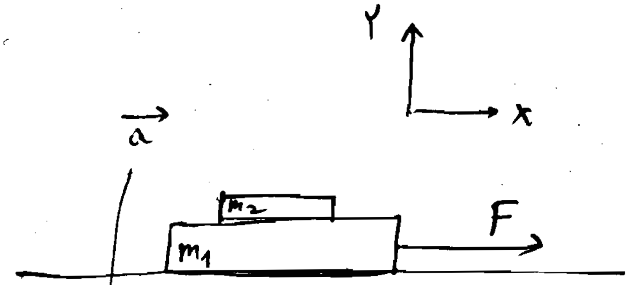
$$F_{net_x} = mg\sin\theta - \mu_k N = m a_x$$

$$F_{net_y} = N - mg\cos\theta = 0 \rightarrow N = mg\cos\theta$$

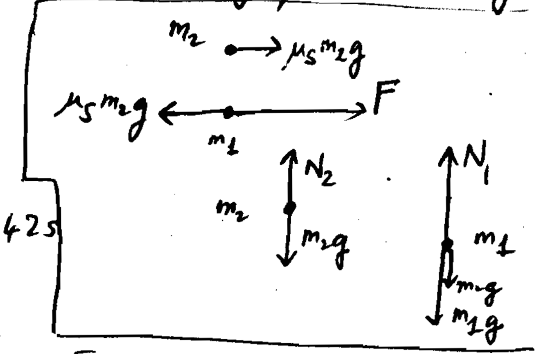
$$\cancel{m}a_x = \cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta \rightarrow a_x = g(\sin\theta - \mu_k\cos\theta)$$

$$a_x = 9.81(\sin 7.971^\circ - 0.088 \cdot \cos 7.971^\circ) = 0.5053 \frac{m}{s}$$

5.48



$m_2 = 0.31\,kg$; $m_1 = 1.2\,kg$



- Book - surface: no friction
- Book - book yes friction

$0\,m/s \longrightarrow 96\,cm/s$ in $0.42\,s$
$= 0.96\,m/s$

$$v_f = v_0 + at \rightarrow a = \frac{v_f - v_0}{t} = \frac{0.96 - 0}{0.42} = 2.28\,m/s^2$$

They accelerate together.

→ Static friction b/w books can provide $m_2 a$.

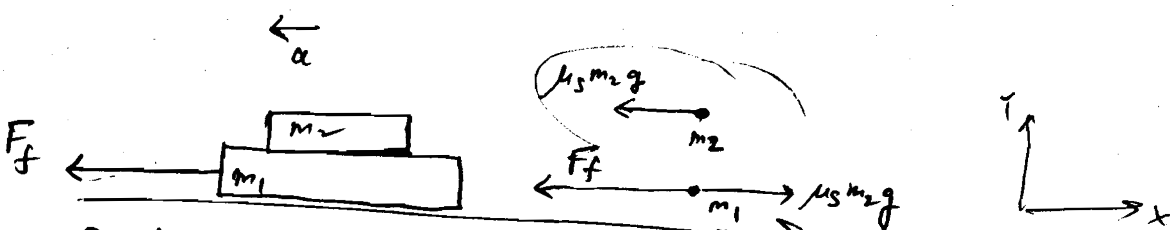
Focus on book # 2 : $F_{net\,x_2} = \mu_s \cancel{m_2} g = \cancel{m_2} a$

$$\mu_s \geq \frac{a}{g} = \frac{2.28}{9.81} = 0.23$$

→ $\underbrace{m_1 \text{ is brought to a stop}}_{\text{deceleration.}}$ in $0.33\,s$ → m_2 slipped off

$$v_{ff} = v_i + a_f t_f$$

$$0 = 0.96 + a_f\,(0.33) \rightarrow a_f = -\frac{0.96}{0.33} = -2.91\,m/s^2$$



Deceleration to the right = acceleration to the left.

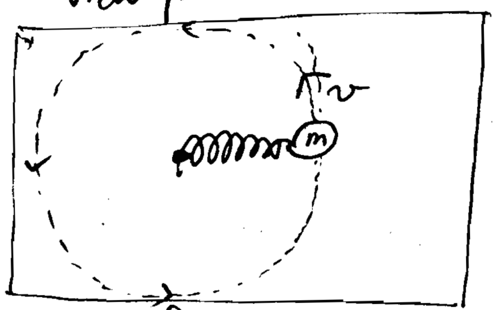
Focus on book # 2 : $F_{net\,x_2} = -\mu_s \cancel{m_2} g = \cancel{m_2} a_f$

→ Static friction cannot provide this deceleration: $\mu_s < \frac{a_f}{-g} = \frac{-2.91}{-9.81} = 0.3$

→ $\boxed{0.23 \leq \mu_s < 0.3}$

5.62

View from above



horizontal table

↳ forces like mg & N are balanced.

$m = 2.1\,kg$

$k = 150\,N/m$

$x_0 = 0.18\,m$

no friction (air table)

$v = 1.4\,m/s$

(uniform circular motion)

<u>View from a side:</u>

m

$\ominus k(x - x_0) \rightarrow$ magnitude is $k(x - x_0)$

$$F_{net} = ma$$

$$k(x - x_0) = m\frac{v^2}{x} \rightarrow kx^2 - kx_0x = mv^2$$

$$150x^2 - 27x - \underbrace{2.1 \times 1.4^2}_{4.12} = 0$$

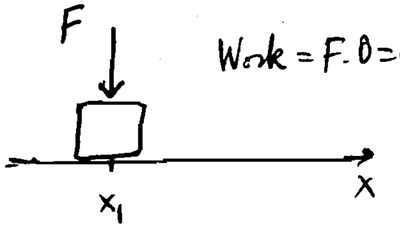
$$x = \frac{27 \pm \sqrt{27^2 + 600 \times 4.12}}{300}$$

$$x = \frac{27 \pm 56.58}{300} \begin{cases} 0.28\,m \checkmark \\ -0.097\,m \end{cases}$$

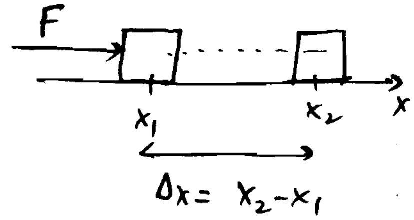
Ch6 : Work, Energy and Power :

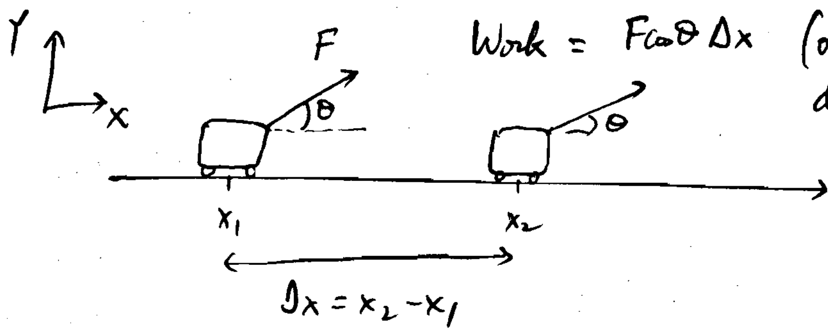
Work: $F.\Delta x$ (unit: SI. $Nm = J$ (Joule))

Work $= F.0 = 0$



Work $= F.\Delta x$





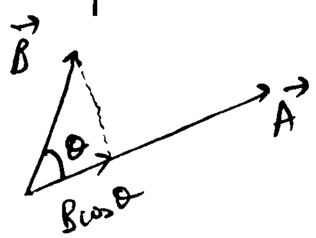
Work $= F\cos\theta\,\Delta x$ (only force applied in the direction of the displacement did work)

Along y-direction : Work $= F\sin\theta \underbrace{\Delta y}_{0} = 0$

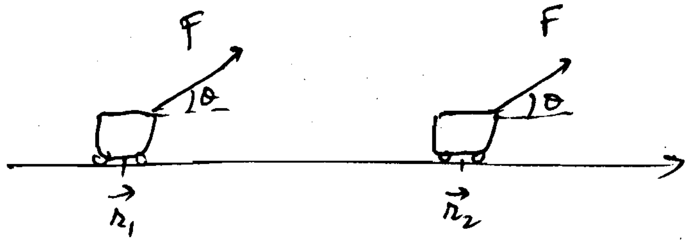
↳ Work done $= \vec{F} \odot \Delta\vec{r}$

- $\vec{F}$ → Force applied (vector)
- $\odot$ → scalar product (b/w 2 vectors produce a number)
- $\Delta\vec{r}$ → displacement

Scalar product: b/w two vectors $\vec{A}$ & $\vec{B}$

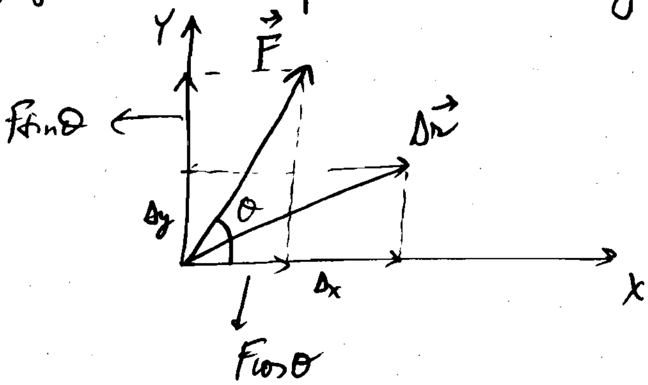


$$\vec{A}\bullet\vec{B} = A\,\underbrace{B\cos\theta}_{\text{projection of } \vec{B} \text{ onto the direction of } \vec{A}}$$



$$\vec{F}\cdot\Delta\vec{r} = F\,\Delta r\cos\theta$$
$$= \Delta r\,\underbrace{F\cos\theta}_{\text{projection of the force onto the direction of displacement}}$$

With the scalar product we can calculate the work done by forces and displacements along any direction:



$$\text{Work done} = \vec{F}\cdot\Delta\vec{r}$$
$$= (F\cos\theta\,\hat{i} + F\sin\theta\,\hat{j})\cdot(\Delta x\,\hat{i} + \Delta y\,\hat{j})$$
$$= F\cos\theta\,\Delta x + F\sin\theta\,\Delta y$$

$\hat{i}\cdot\hat{i} = 1$; $\hat{j}\cdot\hat{j} = 1$

$\hat{i}\cdot\hat{j} = 0$; $\hat{j}\cdot\hat{i} = 0$ ($\cos 90° = 0$)

If the force applied is changing with the position:

$$\text{Work done} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

↳ infinitesimal displacement vector

↳ Work done in stretching a spring:

Hooke's law: $F = -kx$ (force applied by a person is kx)

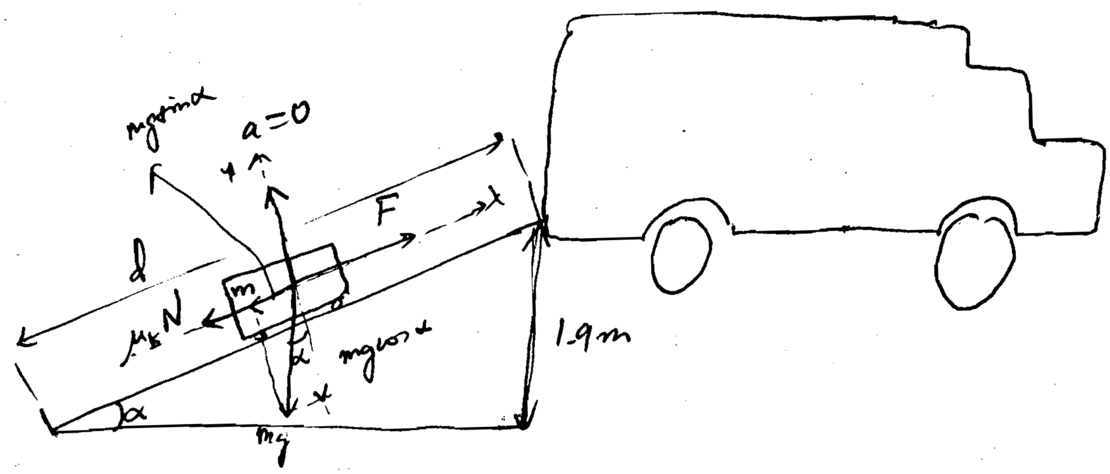
$$\int_0^x F\,dx = \int_0^x kx\,dx = k\frac{x^2}{2}\Big]_0^x = \frac{1}{2}kx^2$$

$$\left(\int_a^b x^n\,dx = \frac{x^{n+1}}{n+1}\Big]_a^b\right)$$

Example:

$m = 460\,kg$

$\mu_k = 0.62$

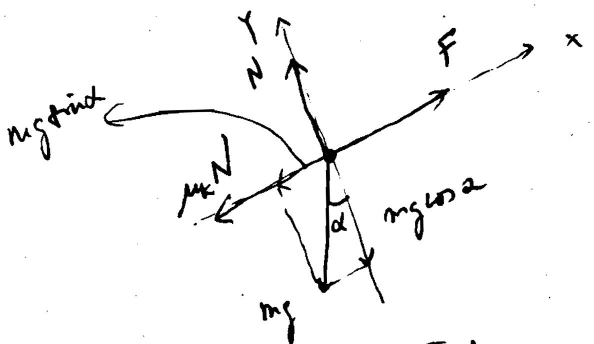


Work done in pushing the box from bottom to top of ramp of $\alpha = 15^\circ$ and if $\alpha = 30^\circ$?

→ Find needed F then calculate Work = F.d

$$\sin\alpha = \frac{1.9}{d}$$

$$\rightarrow \boxed{d = \frac{1.9}{\sin\alpha}}$$



steps):

$$\begin{cases} F_{net\,x} = F - mg\sin\alpha - \mu_k N = m.0 \\ F_{net\,y} = N - mg\cos\alpha = 0 \rightarrow N = mg\cos\alpha \end{cases}$$

$$\rightarrow F = mg\sin\alpha + \mu_k mg\cos\alpha = mg(\sin\alpha + \mu_k\cos\alpha)$$

$$F = \begin{cases} \alpha = 15^\circ = 460 \times 9.81(\sin 15^\circ + 0.62\cos 15^\circ) = 3.88\,kN \\ \alpha = 30^\circ = 4.68\,kN \end{cases}$$

$$\text{Work done} = F.d = F_x \cdot \frac{1.9}{\sin\alpha} \begin{cases} \alpha = 15^\circ = 3.88\,kN \cdot \frac{1.9}{\sin 15^\circ} = 28.5\,kJ \\ \alpha = 30^\circ = 4.68\,kN \cdot \frac{1.9}{\sin 30^\circ} = 17.8\,kJ \end{cases}$$

Kinetic Energy:

2nd Newton's Law: $\vec{F}_{net} = \frac{d\vec{p}}{dt} \overset{\text{m is constant}}{=} m\frac{d\vec{v}}{dt}$

$$\Delta K.E. = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net} \cdot d\vec{r} = m\int_{\vec{r}_1}^{\vec{r}_2} \frac{d\vec{v}}{dt} \cdot d\vec{r} = m\int_{\vec{r}_1}^{\vec{r}_2} d\vec{v} \cdot \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}}$$

$$= m\int_{\vec{r}_1}^{\vec{r}_2} v\,dv = \frac{1}{2}mv^2\Big]_{\vec{r}_1}^{\vec{r}_2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

linear motion: $\vec{v} \parallel d\vec{v} \rightarrow \vec{v} \cdot d\vec{v} = v\,dv\,\cos 0 = v\,dv$

Power: P work or energy per unit time

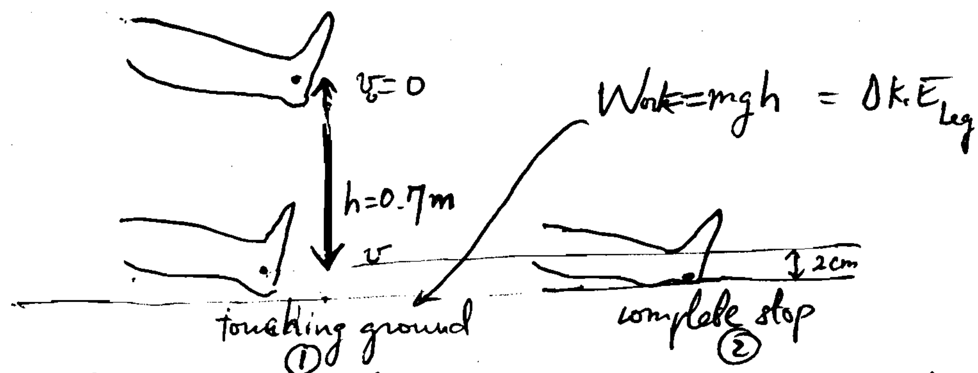
Average power: $\overline{P} = \frac{\Delta \text{Work}}{\Delta t}$

Instantaneous power: $P = \frac{d\,\text{Work}}{dt}$

Unit: $\frac{J}{s} = W$ (Watt)

Power & velocity: $P = \frac{d\text{Work}}{dt} = \frac{d}{dt}(\vec{F} \cdot \Delta\vec{r}) = \vec{F} \cdot \frac{d(\Delta\vec{r})}{dt} = \vec{F} \cdot \vec{v}$

6.81



① Leg + heel received a change in kinetic energy of mgh as they touch ground.
increase

② They have lost all of this K.E. : a stopping force exerted by the floor eliminated this KE in $\Delta y = 0.02\,m$

$$mgh = F_{stopping} \times \Delta y \rightarrow F_{stopping} = \frac{mgh}{\Delta y}$$

$$= \frac{8 \times 9.81 \times 0.7}{0.02} N$$

$$= 2744\,N$$

Alternative:

(i) Free fall: $\frac{v^2 - 0}{h} = 2g \rightarrow v = \sqrt{2gh}$

$$= \sqrt{2 \cdot 9.81 \times 0.7} =$$

$$= 3.7\,m/s$$

(2) $\frac{0 - 3.7^2}{0.02} = +2a \rightarrow a = \frac{-3.7^2}{0.04} = -342\,m/s^2$

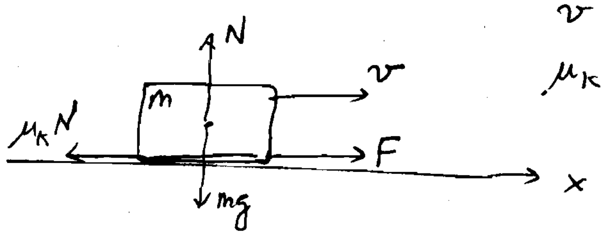
$$\rightarrow F_{stopp} = ma = 8 \times 342 = 2744\,N$$

6.67

$m = 95\,kg$

$v = 0.62\,m/s$

$\mu_k = 0.78$



a) P ? $= F.v = \mu_k\, mg\, v = 0.78 \times 95 \times 9.81 \times 0.62 = 450\,W$

$F_{net_x} = F - \mu_k N = ma_x = m.0 \rightarrow \boxed{F = \mu_k mg}$

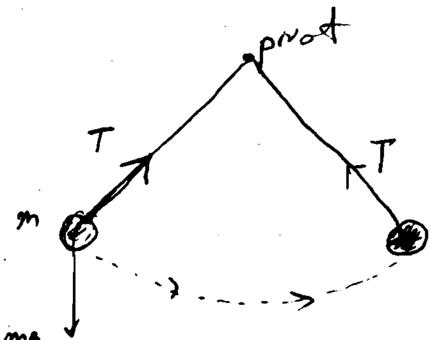
$F_{net_y} = N - mg = 0 \rightarrow N = mg$

b) How much work done in pushing the chest 11 m?

$Work = F.d = \mu_k\, mg\, d = 0.78 \times 95 \times 9.81 \times 11 = 8\,kJ$

or $Work = P.\Delta t = 450 \frac{J}{s} \cdot \frac{11\,m}{0.62\,\frac{m}{s}} = 8\,kJ$

6.8



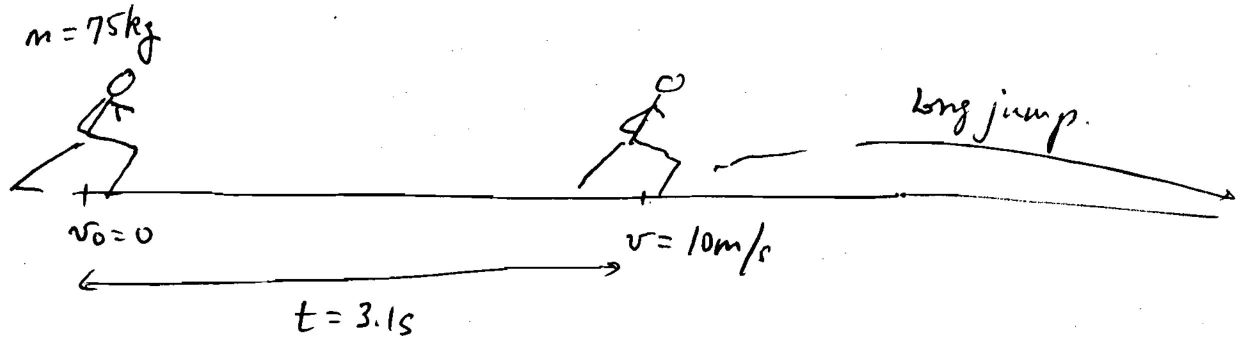
Does T do any work?

Mass is always at a same separation to the center of the circle (= pivot point)

Since T is acting along the radial direction and $\Delta r = 0$

$\rightarrow Work = T.\Delta r = 0$

6.38



or $P = \dfrac{Work}{\Delta t} = \dfrac{\Delta KE}{\Delta t} = \dfrac{\frac{1}{2}mv^2}{\Delta t} = \dfrac{1}{2}\dfrac{75 \times 10^2}{3.1} = 1.21\,kW \approx 1.6\,hp$

or $P = F.\bar{v} = ma\dfrac{v_0+v}{2} = 75\,\dfrac{10}{3.1}\,\dfrac{10}{2} = 1.21\,kW$

$v = \underset{0}{v_0} + at \rightarrow a = \dfrac{v}{t} = \dfrac{10}{3.1}$

6.80

1 food calorie = 4.186 kJ

How many lifts (45kg, h = 2.5m) to burn off 230 × 4.186kJ

Work in one lift = $mgh = 45 \times 9.81 \times 2.5 = 1.1\,kJ$

$\rightarrow \dfrac{230 \times 4.186\,kJ}{1.1\,kJ} = 873$ times

$\dfrac{12\ \text{times}}{\text{min}} \cdot \dfrac{60\ \text{min}}{h} \rightarrow 720$

6.20

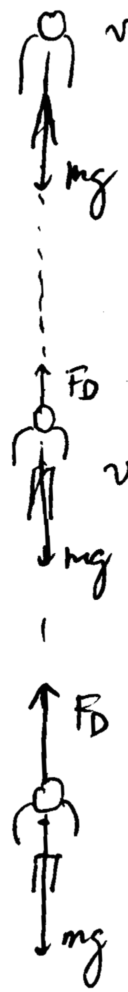
$\vec{F}.\vec{r} = \underset{N}{(1.8\hat{i} + 2.2\hat{j})}.\underset{m}{(56\hat{i} + 31\hat{j})} = 1.8 \times 56 + 2.2 \times 31$

$= 169\,J$

Drag forces: $F_D = \frac{1}{2} C \rho_{fluid} A_{body} v_{body}^2$

C : is a constant
ρ = fluid density (air or water)
A_{body} = surface area
v = speed.

Sky diving:



$v_0 = 0$ $F_D = 0$

$v = gt$ increasing with time $F_D \propto (gt)^2$ getting larger & larger.

At certain time it could be enough to cancel the acceleration of gravity:

Net force: $mg - F_D = 0$ → the diver achieves terminal speed. v_T
(the sooner you ~~arr~~ arrive at v_T is better)

$$\boxed{v_t = \sqrt{\frac{2mg}{C \rho_{fluid} A_{Body}}}}$$