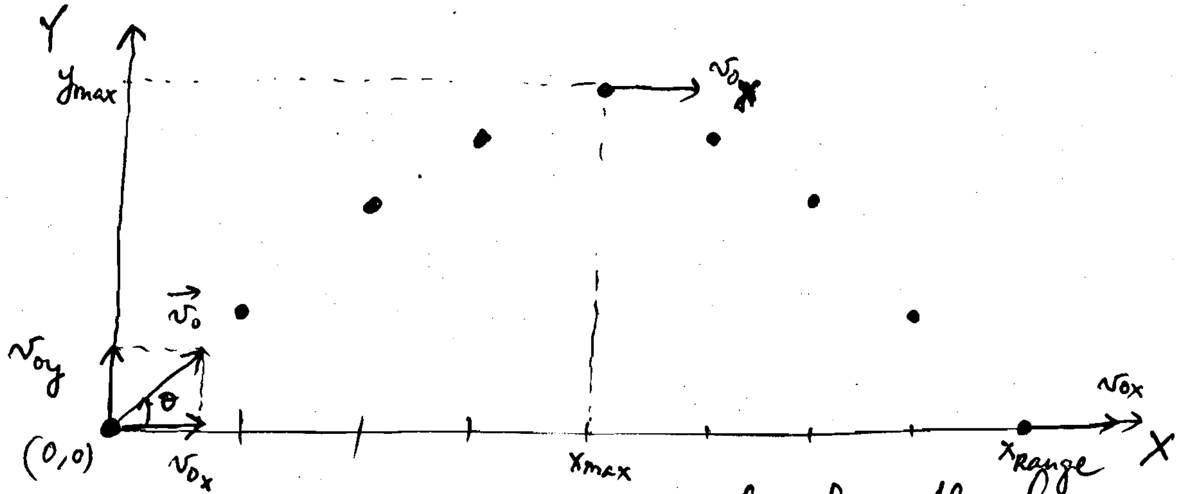
Projectile Motion

Kinematic equations (constant acceleration)
$$\begin{cases} (1)\ \vec{v} = \vec{v}_0 + \vec{a}\,t \\ (2)\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \end{cases}$$

Example: marble going along a horizontal track at constant velocity, then it gets tossed up vertically. The combine motion is a projectile motion.



Snapshots at regular time intervals:
- → Sep. of marble along x: 1) Decreasing 2) Increasing 3) Same ✓
- → Sep. of marble along y: 1) Decreasing ✓ 2) Increasing 3) Same

deceleration of gravity

$$1)\begin{cases} v_x = v_{0x} = v_0\cos\theta \\ v_y = v_{oy} - gt = v_0\sin\theta - gt \end{cases}$$

$$2)\begin{cases} x = x_0 + v_0\cos\theta\, t \\ y = y_0 + v_0\sin\theta\, t - \frac{1}{2}gt^2 \end{cases}$$

deceleration of gravity on the way up to the max. altitude point. (x_{max}, y_{max})

For simplicity $(x_0, y_0) = (0,0)$ (Can always put back if required)

Eliminate time to get "trajectory equation":

2) $x = v_0 \cos\theta \, t \rightarrow t = \dfrac{x}{v_0 \cos\theta}$

$$\rightarrow y = \cancel{v_0} \sin\theta \frac{x}{\cancel{v_0} \cos\theta} - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta}$$

$$\boxed{y = x \tan\theta - \frac{g}{2 v_0^2 \cos^2\theta} x^2} \rightarrow$$ Projectile motion is a parabola (quadratic) (inverted)

$$(x_{max}, y_{max}) = ? = \left(\frac{v_0^2 \sin 2\theta}{2g}, \frac{v_0^2 \sin^2\theta}{2g} \right)$$

1) $v_y = v_0 \sin\theta - gt \rightarrow$ at $(x_{max}, y_{max}) \Rightarrow v_y = 0$

$$\Rightarrow \boxed{t_{max} = \frac{v_0 \sin\theta}{g}}$$

$$\rightarrow \boxed{x_{max} = v_{0x} t_{max} = v_0 \cos\theta \frac{v_0 \sin\theta}{g} = \frac{v_0^2}{g} \frac{\sin 2\theta}{2}}$$

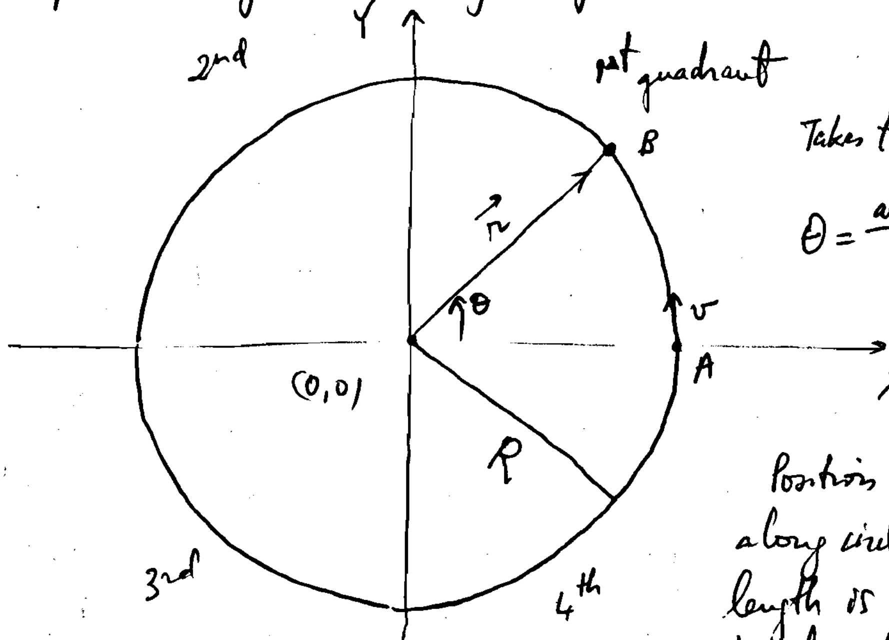
$$2 \cos\theta \sin\theta = \sin 2\theta$$

$$\rightarrow \boxed{y_{max} = v_0 \sin\theta \frac{v_0 \sin\theta}{g} - \frac{1}{2} \cancel{g} \frac{v_0^2 \sin^2\theta}{g^{\cancel{2}}} = \frac{v_0^2 \sin^2\theta}{2g}}$$

$$(x_{Range}, y_{Range}) = \left(\frac{v_0^2 \sin 2\theta}{g}, 0 \right)$$

Uniform Circular Motion:

↳ constant speed along circular trajectory (not constant velocity!)



Takes t to go from A to B

$$\theta = \frac{\text{arc } AB}{R} = \frac{vt}{R}$$

Position vector $\vec{r}$ changes along circle: although its length is fixed (R), its angle changes from $0°$, to 2π, to 4π, etc.

$$\vec{r} = x\hat{i} + y\hat{j} = R\cos\theta\ \hat{i} + R\sin\theta\ \hat{j} = R\left[\cos\left(\frac{vt}{R}\right)\hat{i} + \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \cancel{R}\left[-\frac{v}{\cancel{R}}\sin\left(\frac{vt}{R}\right)\ \hat{i} + \frac{v}{\cancel{R}}\cos\left(\frac{vt}{R}\right)\ \hat{j}\right]$$

$$= v\left[-\sin\left(\frac{vt}{R}\right)\hat{i} + \cos\left(\frac{vt}{R}\right)\hat{j}\right]$$

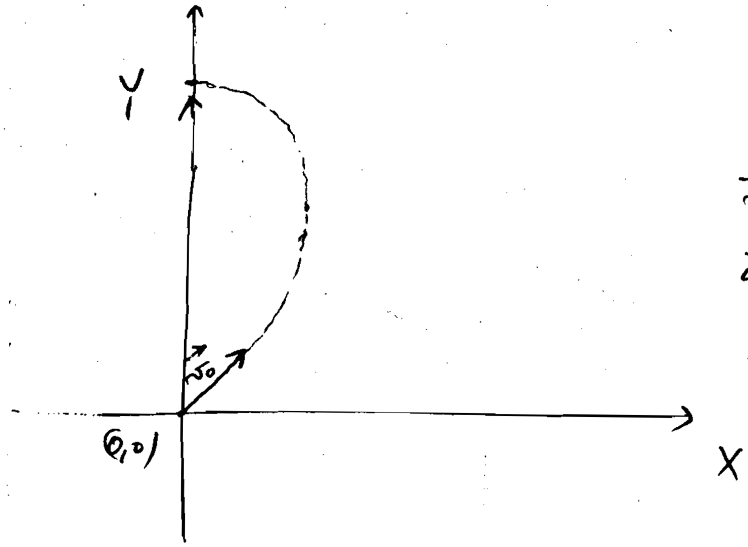
$|\vec{v}| = v$ ↓ const speed along circle

$$\vec{a} = \frac{d\vec{v}}{dt} = v\left[-\frac{v}{R}\cos\left(\frac{vt}{R}\right)\ \hat{i} - \frac{v}{R}\sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$= -\frac{v^2}{R}\left[\cos\left(\frac{vt}{R}\right)\hat{i} + \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$a = |\vec{a}| = \frac{v^2}{R} \quad (\text{direction is radially inward}).$$

3.60



$\vec{v}_0 = 11\hat{i} + 14\hat{j}$ m/s

$\vec{a} = -1.2\hat{i} + 0.26\hat{j}$ m/s^2

a) When does the particle cross the y-axis?
(makes sense since acceleration is predominantly along $-x$ $\Rightarrow$ trajectory will meet the y-axis as shown above)
Constant acceleration equations of motion will apply:

(2) $\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ $\begin{cases} x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \\ 0 = 11t + \frac{1}{2}(-1.2)t^2 \end{cases}$

when particle crosses y-axis

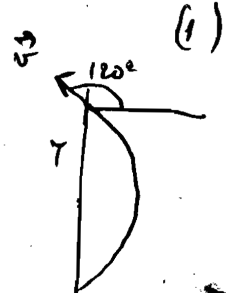
$t = \frac{22}{1.2}$ s $= 18.3$ s

b) What is y_{Range}?

(2) $\begin{cases} y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \\ y = 14 \times 18.3 + \frac{1}{2}(0.26) \times 18.3^2 = 300\text{m} \end{cases}$

c) How fast & in what direction at that time?

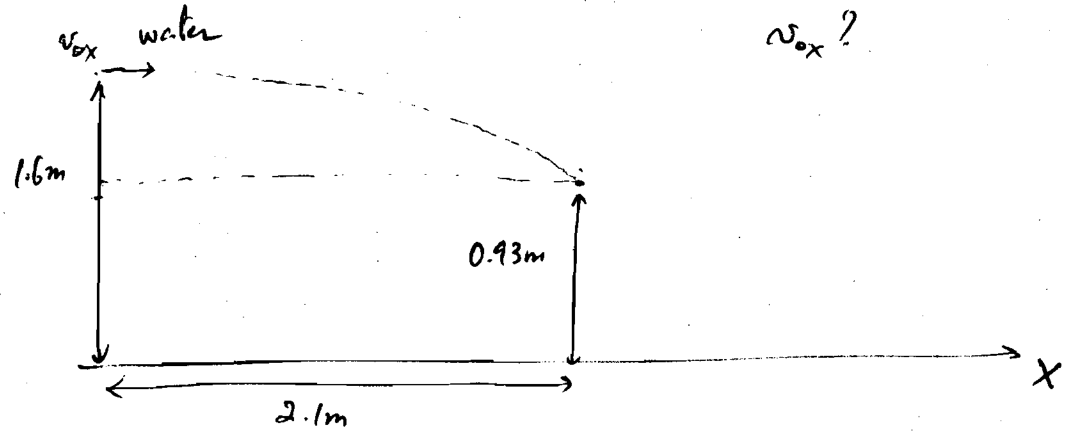
(1) $\vec{v} = \vec{v}_0 + \vec{a}t = (v_{0x} + a_x 18.3)\hat{i} + (v_{0y} + a_y 18.3)\hat{j}$
$= -10.96\hat{i} + 18.8\hat{j}$ m/s (2nd quadrant)



$\vec{v} = \begin{cases} v = \sqrt{10.96^2 + 18.8^2} = 21.7 \text{ m/s} \\ \theta_v = \tan^{-1}\frac{18.8}{-10.96} = -59.8° = -60° \xrightarrow{+180°} 120° \checkmark \end{cases}$

3.61

v_{ox} ?

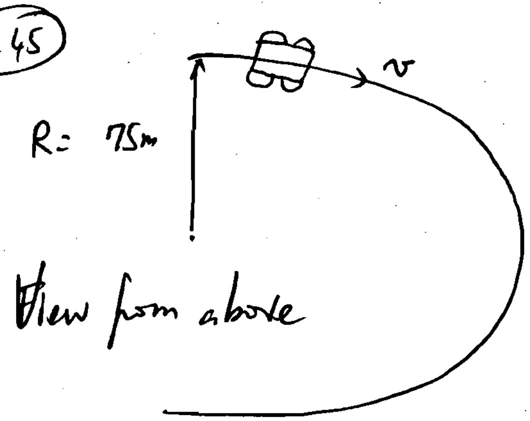


(2) $$y - y_0 = v_{oy} t + \frac{1}{2} g t^2$$

$$1.6 - 0.93 = 0 + \frac{1}{2} 9.81 t^2 \rightarrow t = \sqrt{\frac{2 \times 0.67}{9.81}} \text{ s} = 0.37 \text{ s}$$

(2) $$x - x_0 = v_{ox} t + 0 \rightarrow v_{ox} = \frac{2.1}{0.37} = 5.68 \text{ m/s}$$

3.45



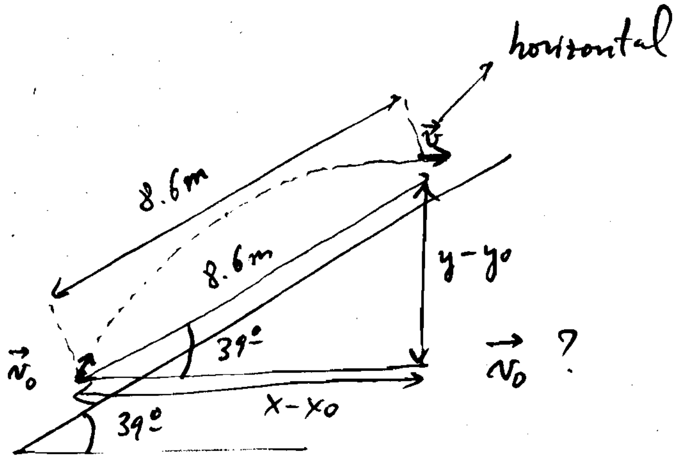
$$a = \frac{v^2}{R} = 9.81 \rightarrow v = \sqrt{9.81 \times 75} \text{ m/s}$$
$$= 27.1 \text{ m/s}$$
$$= 60.7 \text{ mi/h}$$

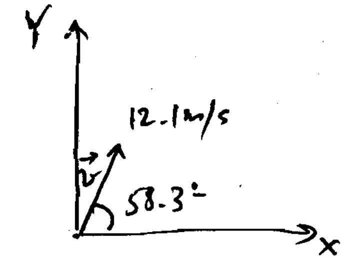
3.78

3.15 $$x_{Range} = \frac{v_0^2 \sin 2\theta}{g} \rightarrow \frac{\partial x_{Range}}{\partial \theta} = \frac{v_0^2}{g} 2 \cos 2\theta = 0$$

$$\rightarrow \cos 2\theta = 0^\circ \rightarrow 2\theta = 90^\circ \rightarrow \theta = 45^\circ$$

3.66





Along y-direction: $v_y = 0$

(3) $$\frac{v_y^2 - v_{oy}^2}{y - y_0} = -2g \rightarrow +v_{oy}^2 = +2g(y-y_0)$$

$$v_{oy} = \sqrt{2g(y-y_0)} = \sqrt{2g \times 8.6 \sin 39°} = 10.3 \text{ m/s}$$

(Motion along y-axis is a constant deceleration due to gravity) $\rightarrow a_y = -g$.

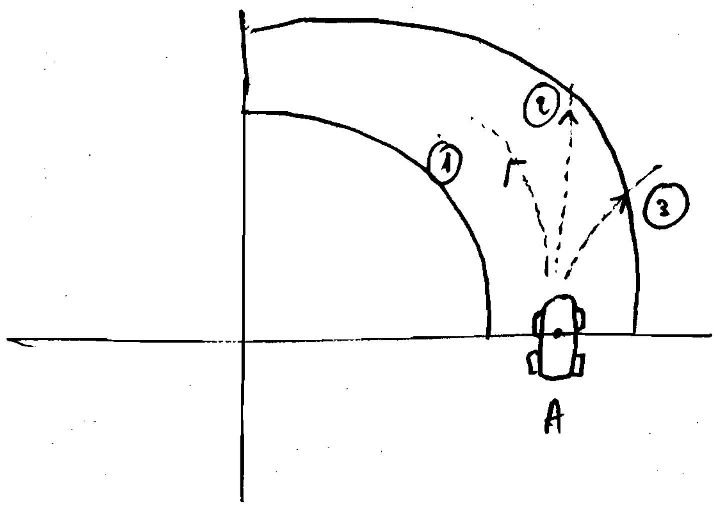
Along x-direction: constant velocity: $a_x = 0$

$$\rightarrow v_{ox} = \frac{x - x_0}{t} = \frac{8.6 \cos 39°}{\frac{10.3}{9.81}} = 6.36 \text{ m/s}$$

$$\underset{0}{v_y} = v_{oy} - gt \rightarrow t = \frac{v_{oy}}{g}$$

$$\vec{v} = \underbrace{6.36\,\hat{i} + 10.3\,\hat{j}}_{1^{st}\text{ quadrant}} \text{ m/s} \longrightarrow \text{polar} \begin{cases} \sqrt{6.36^2 + 10.3^2} = 12.1 \text{ m/s} \\ \tan^{-1}\frac{10.3}{6.36} = 58.3° \end{cases}$$

Ch. 4 Force and Motion:



icy road, downslope
A → 2 thanks to the force of friction b/w tire & road (toward center of curvature)

We need a force to change motion or to change velocity (a vector that includes magnitude and direction). Force is also a vector. Sometimes more than one force affect a same motion, in this case the net force (vector addition of forces) will change the motion.

1st Newton's Law: a body in uniform motion will stay in uniform motion, a body at rest will stay at rest, unless there is a net force acting on it.

2nd Newton's Law: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$

$\vec{p}$ = linear momentum $\equiv m\vec{v}$ (mass times velocity)

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

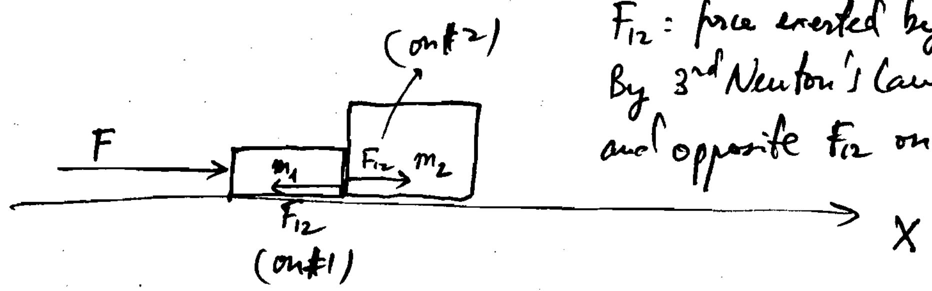
If the body mass is not changing as in a rocket (fuel is burned, a lot) the second term is negligible; on a daily basis

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Dimension of force: $[F] = \dfrac{[p]}{[t]} = \dfrac{[m][v]}{[t]} = \dfrac{M\frac{L}{T}}{T} = M\dfrac{L}{T^2}$

SI $\rightarrow$ unit of force is $kg\dfrac{m}{s^2} = N$ (Newton)

3rd Newton's Law: if A exerts a force on B, B exerts an equal and opposite force on A



F_{12}: force exerted by 1 on 2
By 3rd Newton's Law, 2 exerts equal and opposite F_{12} on 1

- Assume no friction b/w boxes & surface
- Weights of the boxes are ~~equal~~ compensated by the normal forces exerted by the ground (by 3rd Newton's Law)

* Acceleration for the system of two boxes: $a = \dfrac{F}{m_1 + m_2}$
* Focus box #1: call its acceleration $a_1 = \dfrac{F - F_{12}}{m_1}$
* Focus on box #2: call its acceleration $a_2 = \dfrac{F_{12}}{m_2}$

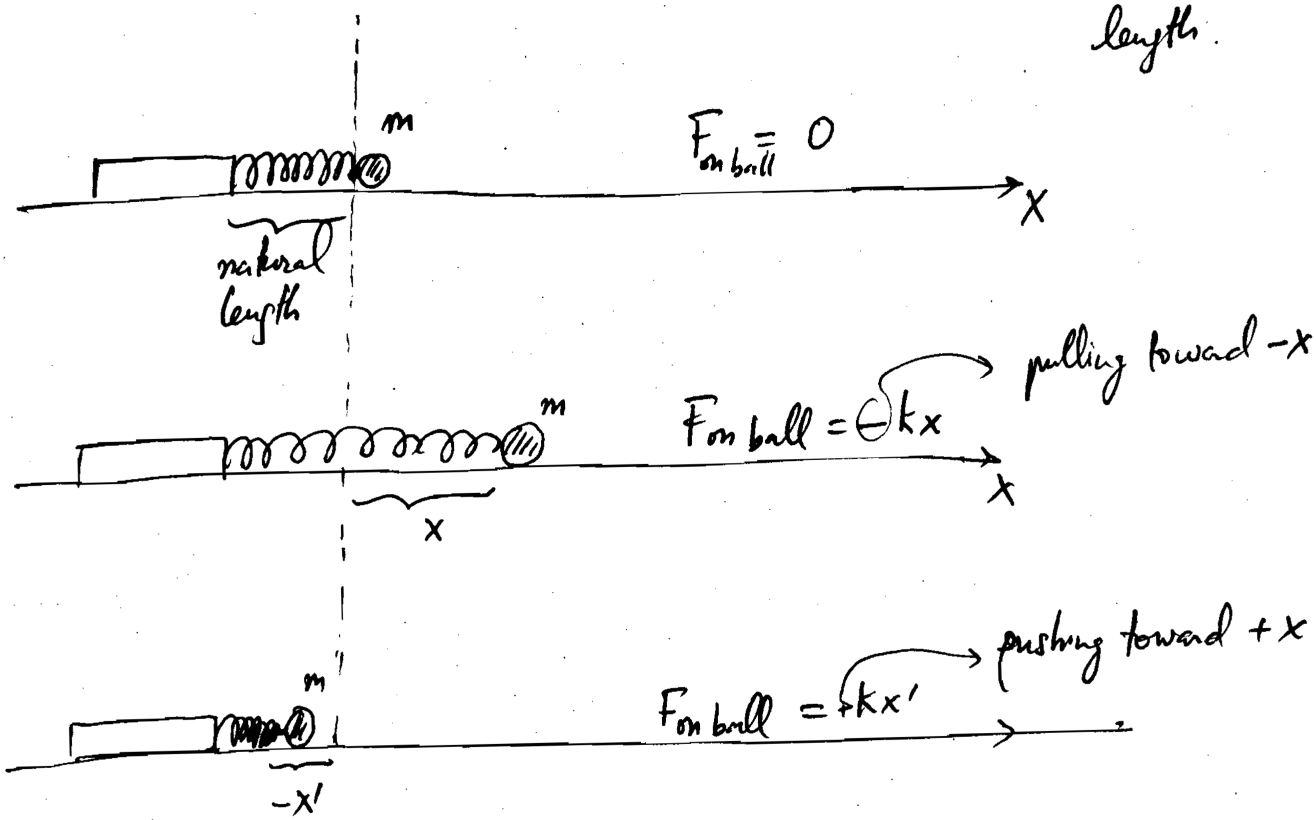
$$F - F_{12} = m_1 a_1$$
$$F_{12} = m_2 a_2$$

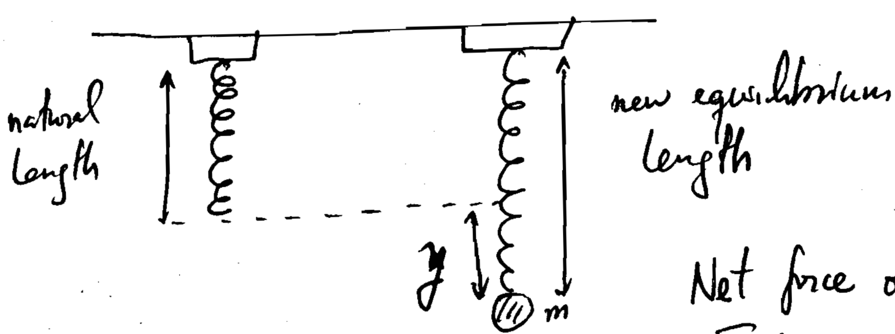
$$F = m_1 a_1 + m_2 a_2 \qquad = m_1 a + m_2 a$$

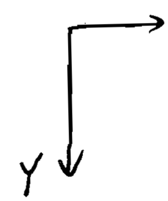
For any mass, if this is true $\rightarrow a_1 = a = a_2$.

Measuring forces:

Spring balance : Hooke's Law : $F = -kx$ $\begin{cases} k: \text{spring constant} \\ x: \text{elongation or displacement w.r.t. natural length.} \end{cases}$



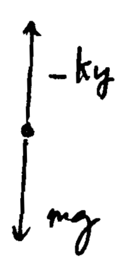




Net force on mass m is

$F_{net} = mg - ky$

Free-body diagram: mass is a dot



If mass stays still like that:

$mg = ky$

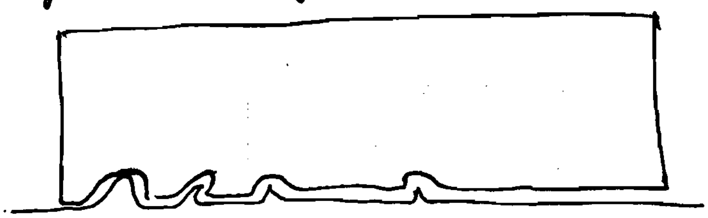
Fritional Forces: while a body is in contact with surfaces

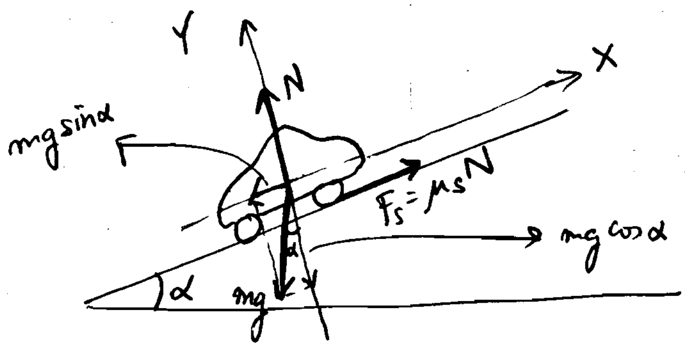
$F_s = \mu_s N$ Static friction: threshold force for some object to start moving

$F_k = \mu_k N$ Kinetic friction: while moving.

Difference: when pushing a heavy box: people ~~put~~ applying force on a box, suddenly they start moving fast. static friction is higher than kinetic friction

N normal force exerted by the surface





→ Car is parked

Forces acting on the car:

mg ✓

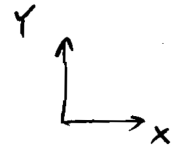
N ✓

F_s ✓

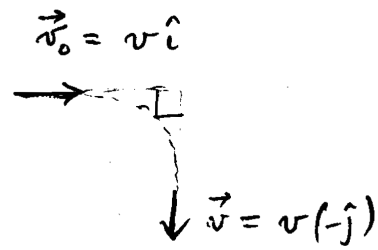
parked safe: $F_s = \mu_s N \geq mg\sin\alpha$

Net force along y is: 0 ⇒ $N = mg\cos\alpha$

→ $\mu_s \cancel{mg}\cos\alpha \geq \cancel{mg}\sin\alpha$ → $\boxed{\mu_s \geq \tan\alpha}$

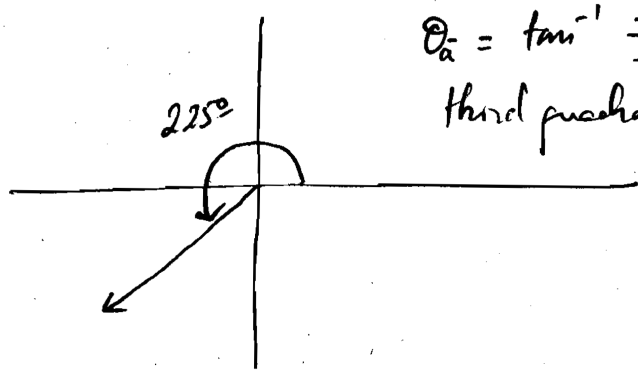


3.29



Average acceleration vector : $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v} - \vec{v}_0}{\Delta t} = \dfrac{-v\hat{j} - v\hat{i}}{\Delta t}$

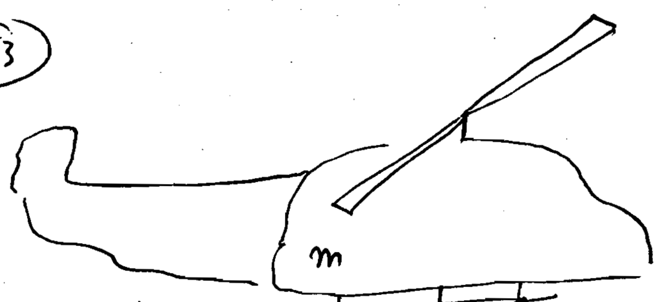
$\vec{\bar{a}} = \dfrac{v}{\Delta t}\,\underbrace{(-\hat{i} - \hat{j})}$ → direction 3rd quadrant,



$\theta_{\bar{a}} = \tan^{-1}\dfrac{-1}{-1} = 45°$ since it is in the third quadrant : $45° + 180 = 225°$.

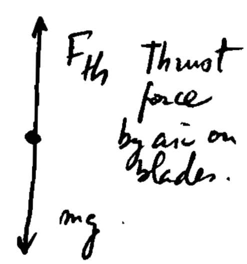
Southwest.

4.53



m = 4300kg

a) Hovering
a = 0



$F_{net} = ma \longrightarrow mg - F_{th} = 0$

Force exerted by blades on air $= F_{th}$ (by Newton's 3rd law).

$= mg = 4300 \times 9.81 = 42.1 \times 10^3 N$

b) dropping at 21 m/s ; speed decreasing (deceleration) at 3.2 m/s²
downward deceleration = upward acceleration.

$F_{net} = ma$

$F_{th} - mg = ma$ → Force exerted by blade on air $= F_{th} = m(g+a)$
$= 4300(9.81+3.2)$

c) rising at 17 m/s, speed increasing at 3.2 m/s^2
upward acceleration

$F_{net} = ma$: $F_{th} - mg = ma \Rightarrow F_{th} = m(g+a)$
$= 4300(9.81+3.2)$
$= 55.9 \times 10^3$ N

d) rising at steady 15 m/s : $a = 0$

$F_{th} = mg = 4300 \times 9.81 = 42.1 \times 10^3$ N

e) rising at 15 m/s, speed decreasing at 3.2 m/s^2
~~downw~~ downward acceleration

$F_{net} = ma$
$mg - F_{th} = ma \rightarrow F_{th} = m(g-a) = 4300(9.81 - 3.2)$
$= 28.4 \times 10^3$ N

3.52

$\vec{r} = 12t\,\hat{i} + (15t - 5t^2)\,\hat{j}$ m

a) $\vec{r}(t=2s) = 24\,\hat{i} + 10\,\hat{j}$, m

b) Average velocity from $t=0$ to $t=2s$

$$\overrightarrow{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(t=2s) - \vec{r}(t=0s)}{2-0} = \frac{24\,\hat{i} + 10\,\hat{j}}{2}$$

$$= 12\,\hat{i} + 5\,\hat{j} \text{ m/s}$$

c) Instantaneous velocity at $t = 2s$:

$$\vec{v} = \frac{d\vec{r}}{dt} = 12\,\hat{i} + (15 - 10t)\,\hat{j} \text{ m/s}$$

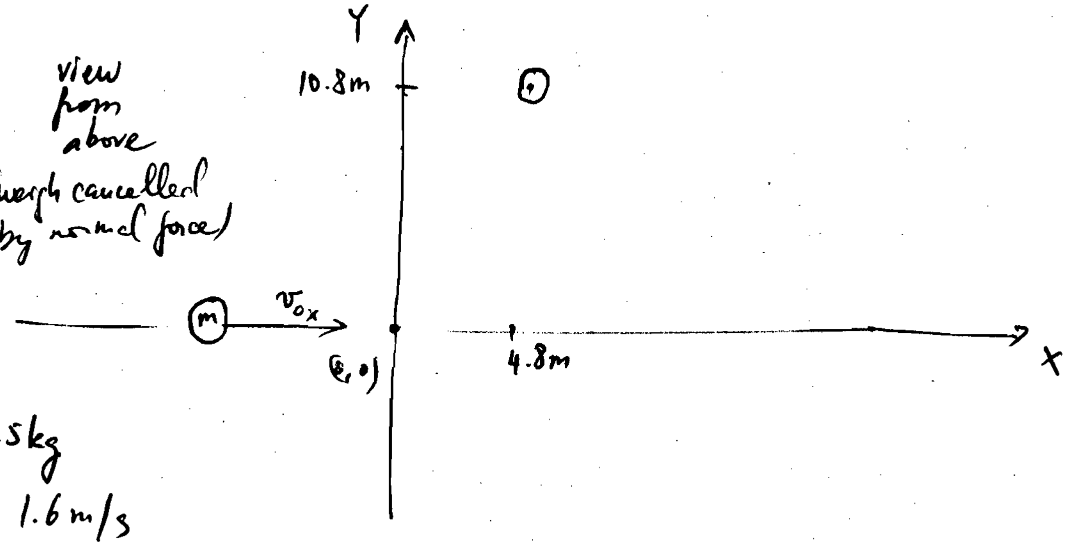
$$\vec{v}(t=2s) = 12\,\hat{i} - 5\,\hat{j} \text{ m/s}$$

$$\overrightarrow{\bar{v}} = \frac{1}{2}\int_0^2 \vec{v}\,dt = \frac{1}{2}\int_0^2 \left[12\,\hat{i} + (15-10t)\,\hat{j}\right]dt$$

$$= \frac{1}{2}\left[24\,\hat{i} + (30-20)\,\hat{j}\right] = 12\,\hat{i} + 5\,\hat{j} \text{ m/s (same)}$$

4.44

view from above (weight cancelled by normal force)



$m = 2.5\,kg$

$v_{0x} = 1.6\,m/s$

Forces $\vec{F}_1$ & $\vec{F}_2$ applied at origin during 3s ; $\vec{F}_1 = 15\hat{j}$ N ,
$\vec{F}_2$ is also along y

$$\vec{F}_{net} = m\vec{a} = ma_y\hat{j}$$

$(15 + F_2)\hat{j} = ma_y\hat{j}$ To find $F_2 \rightarrow$ Let's find a_y:

$$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = \frac{1}{2}a_yt^2 \rightarrow a_y = \frac{2(y-y_0)}{t^2} = \frac{2 \times 10.8}{3^2}$$

$$= 2.4\,m/s^2$$

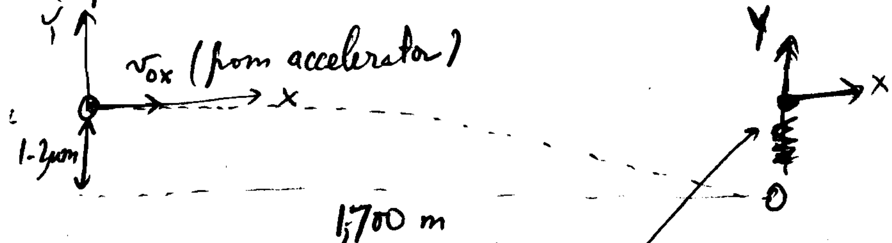
$$F_2 = ma_y - 15 = 2.5 \times 2.4 - 15 = -9N$$

$$\boxed{\vec{F}_2 = -9\hat{j}\ N}$$

3.43

Protons drop 1.2 μm over 1.7 km
↓
due to gravity (protons have mass) } Projectile motion

open roll of wire
(circular accelerator)



Approximate average speed:

→ Trajectory for projectile motion: $y = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2$

θ : angle of $\vec{v}_0 = 0$

$y = -\frac{g}{2v_0^2}x^2$

$\rightarrow v_0^2 = -\frac{gx^2}{2y}$

$v_0 = \sqrt{\frac{-9.81 \times 1700^2}{2(-1.2\times10^{-6})}} = 3.44\times10^{6}\,m/s$

$v_{0x} = 3.44 \times 10^6\ m/s$

→ Speed v_y acquired from gravity:

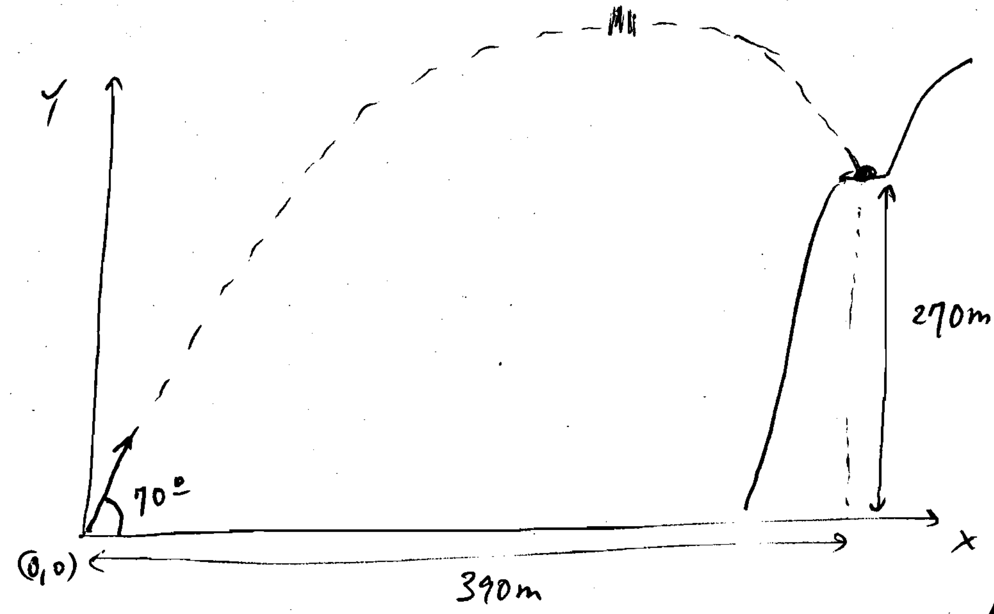
$v_y = 0 - gt = -9.81 \frac{1700\,\cancel{m}}{3.44\times10^6\,\frac{\cancel{m}}{s}} = -4.85\times10^{-3}\,m/s$

m/s²

→ Total approx average speed:

$v = \sqrt{(3.44\times10^6)^2 + (-4.85\times10^{-3})^2} \simeq \boxed{3.44\times10^6\ m/s}$

3.73



Projectile motion: we have a point along the trajectory:

Trajectory equation: $y = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2$

Know: $x = 390m$; $y = 270m$; $\theta = 70°$ → can find magnitude of initial velocity v_0

$$\rightarrow \frac{1}{v_0^2} = -(y - x\tan\theta)\frac{2\cos^2\theta}{gx^2}$$

$$v_0 = \frac{x}{\cos\theta}\sqrt{\frac{g}{2(x\tan\theta - y)}} = \frac{390}{\cos 70°}\sqrt{\frac{9.81}{2(390\tan 70° - 270)}}$$

$$\boxed{v_0 = 89.2\ m/s}$$

Alternative solution:

(1) v_{0x} : $v_x = v_{0x}$

v_{0y} : $v_y = v_{0y} + at$: $\dfrac{v_y^2 - v_{0y}^2}{y - y_0} = 2a$

(2) $x - x_0 = v_{0x} t + \cancel{\frac{1}{2}at^2} \to 0$ $\rightarrow$ $390 = v_{0x} t$

$y - y_0 = v_{0y} t + \frac{1}{2}at^2$ $\rightarrow$ $270 = v_{0y} t - \frac{1}{2}gt^2$

$\rightarrow \tan 70^\circ = \dfrac{v_{0y}}{v_{0x}} \rightarrow v_{0y} = v_{0x} \tan 70^\circ$

$$\begin{cases} 390 = v_{0x} t \longrightarrow t = 390/v_{0x} \\ 270 = v_{0x} \tan 70^\circ\, t - \frac{1}{2} g t^2 \end{cases}$$

$\hookrightarrow = (\tan 70^\circ) 390 - \frac{1}{2} 9.81 \dfrac{390^2}{v_{0x}^2}$

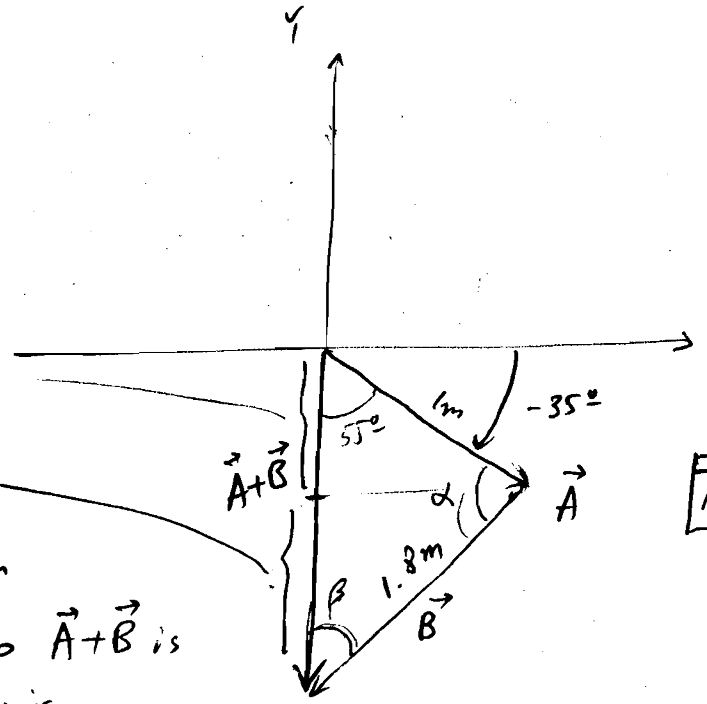
$1/v_{0x}^2 = \dfrac{-(270 - 390\tan 70^\circ) 2}{9.81 \times 390^2}$

$$v_{0x} = \sqrt{\frac{\cancel{(340\tan 70^\circ - 270)}\ 9.81 \times 390^2}{2(390\tan 70 - 270)}} = 30.5\, m/s$$

$\rightarrow v_{0y} = 30.5 \tan 70^\circ = 83.8\, m/s$

$\rightarrow v_0 = \sqrt{30.5^2 + 83.8^2} = 89.2\, m/s$.

3.49



$1\cos 55$

$B_y = 1.8\cos 27$

Third quadrant

$|\vec{B}| = 1.8m$

angle? so $\vec{A}+\vec{B}$ is along y axis

(either 3rd quadrant or 2nd quadrant)

$$\vec{C} = \vec{A}+\vec{B} = 0\,\hat{i} + C_y\,\hat{j}$$

$$= (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j}$$

$$B_x = -A_x = -1\cos 35° = -0.82$$

Since : $B_x = 1.8\sin\beta$

$\rightarrow \beta = \sin^{-1}\left(\frac{-0.82}{1.8}\right)$

$= 27.1°$

$\rightarrow \theta_B = \tan^{-1}\frac{B_y}{B_x} = \tan^{-1}\left(\frac{-1.8\cos 27.1°}{-0.82}\right)$

$= \tan^{-1}\frac{-1.6}{-0.82}$

$= 62.86° + 180°$

$= 242.86°$

Alternative :

Law sins :

$$\frac{\sin 55}{1.8} = \frac{\sin\alpha}{C_y} = \frac{\sin(180 - 55 - \alpha)}{1}$$

$$\frac{\sin 55°}{1.8} = \sin(125 - \alpha)$$

$$125 - \alpha = \sin^{-1}\left(\frac{\sin 55}{1.8}\right) = 27°$$

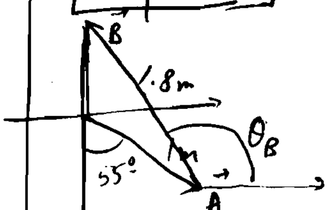
$$\alpha = 98° \rightarrow \beta = 27°$$

$$\rightarrow B_y = -1.8\cos 27° = -1.6$$

$$\theta_B = \tan^{-1}\frac{-1.6}{-0.82} + 180° = 242.86°$$

third quadrant

2nd quadrant



$B_x = -A_x = -0.82$

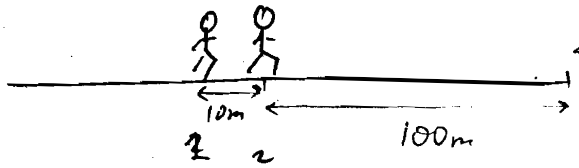
$B_x = 1.8\cos\theta_B$

$\rightarrow \theta_B = \cos^{-1}\frac{-0.82}{1.8}$

$= 117°$

2.82

$$v_2 = \frac{1609\,m}{360\,s} = 4.47\,m/s$$



1 needs cover 110m in $\frac{100}{4.47}$ s

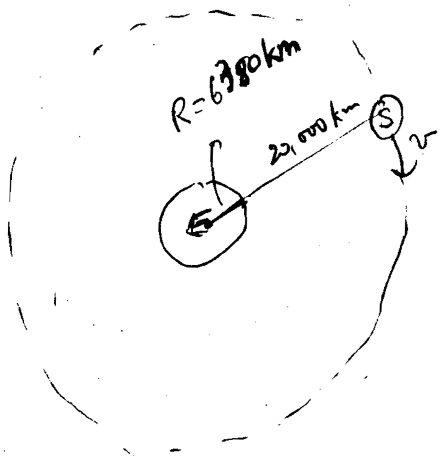
$v_{1_0} = v_2 = 4.47\,m/s$

$$\frac{v_1^2 - v_{1_0}^2}{110} = 2a \quad ; \quad x - x_0 = v_{1_0} t + \frac{1}{2} a t^2$$

$$110 = \cancel{4.47}\frac{100}{\cancel{4.47}} + \frac{1}{2} a \left(\frac{100}{4.47}\right)^2$$

$$\boxed{\frac{20 \times 4.47^2}{100^2} = a}$$

3.47



R= 26380 km

From above

$$g' = \frac{5.8}{100} 9.81\ m/s^2 = a$$

UCM: $a = \frac{v^2}{R} \rightarrow v = \sqrt{aR} = \sqrt{\frac{5.8}{100} 9.81 \times R}$

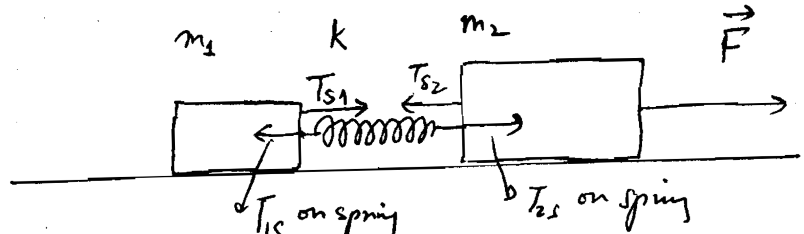
Orbital period = T to complete one circle

$$= \frac{2\pi \overbrace{(6380 + 20000)10^3}^{R}}{\sqrt{\frac{5.8}{100} 9.81\ R}}$$

$$= 2\pi \sqrt{\frac{26380 \times 10^3}{\frac{5.8}{100} 9.81}} \quad \cancel{s} \quad \frac{1h}{3600\,\cancel{s}}$$

$= 11.88\,hrs = 12\,hrs.$

4.51



horizontal, frictionless massless spring.

$m_1 = 2kg \longrightarrow T_{S1}$ (by spring on m_1) $\rightarrow F_{net\ on\ m_1} = T_{S1}$

$m_2 = 3kg \longrightarrow \vec{F}$ & $T_{S2} \longrightarrow F_{net\ on\ m_2} = F - T_{S2}$

$k = 140\ N/m$

$F = 15N$

Spring stretch or displacement from its natural length Δx. ?

→ Weights are compensated by normal forces

→ System of the two blocks go together → their accelerations are the same.

a) Net force on spring : $F_{net\ on\ spring} = T_{2S} - T_{1S} = m_s a = 0$

$T \equiv T_{2S} = T_{1S} \rightarrow$ This force acts on spring.

→ Find either one, then divide it by k to get Δx

b) Focus on m_1: $F_{net\ on\ m_1} = T_{S1} = m_1 a \rightarrow T = m_1 a$ (b)

→ need a.

c) Focus on m_2: $F_{net\ on\ m_2} = F - T_{S2} = m_2 a \rightarrow F - T = m_2 a$ (c)

(b)+(c) $F = (m_1 + m_2) a$

$$a = \frac{F}{m_1 + m_2} = \frac{15}{2+3} = 3 m/s^2 \rightarrow \text{(b)}\ T = m_1 a = 2 \times 3 = 6N$$

$$\rightarrow \Delta x = \frac{T}{k} = \frac{6\cancel{N}}{140\cancel{N}/m} = 0.0429m = 4.29\ cm.$$

Ch. 5 Using Newton's Laws:

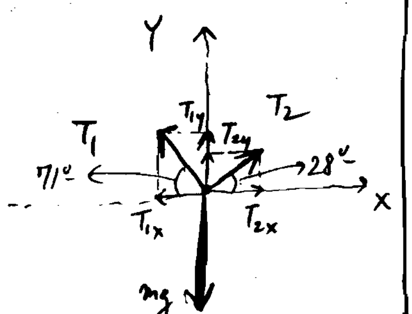
- Equilibrium
- Multiple objects
- Circular Motion
- Friction

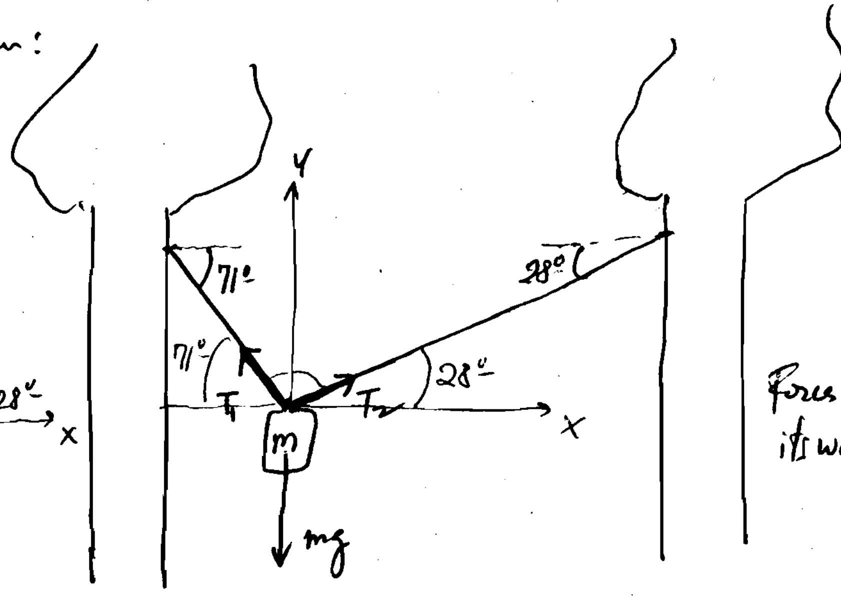
Common strategies:

1) Understanding the problem, making a sketch
2) Select a convenient coordinate system (most forces involved point along either axis x or y)
3) Draw a free-body diagram of forces involved for each object
4) Draw components of these forces along coordinate axes as needed
5) Write Newton's second law for each object using net forces
6) Solve equations to obtain numeric solutions with units.

- Equilibrium:

5-36





$m = 26\,kg$

Tension in each rope?

Forces on pack: its weight: $m\vec{g}$, $\vec{T}_1$, $\vec{T}_2$

1) Sketch ✓ 2) Coord system XY ✓ 3) Free-body ✓ 4) Components ✓ 5) Laws 6) Solve ✓

Step 5): Backpack : $\vec{F}_{net} = m\underset{0}{\underset{\downarrow}{\vec{a}}} = 0$

$$F_{net_x} = T_{2x} - T_{1x} = T_2\cos 28^\circ - T_1\cos 71 = 0 \quad (a)$$

$$F_{net_y} = T_{2y} + T_{1y} - mg = T_2\sin 28^\circ + T_1\sin 71^\circ - mg = 0 \quad (b)$$

Step 6) 2 equations with 2 unknowns T_1 & T_2

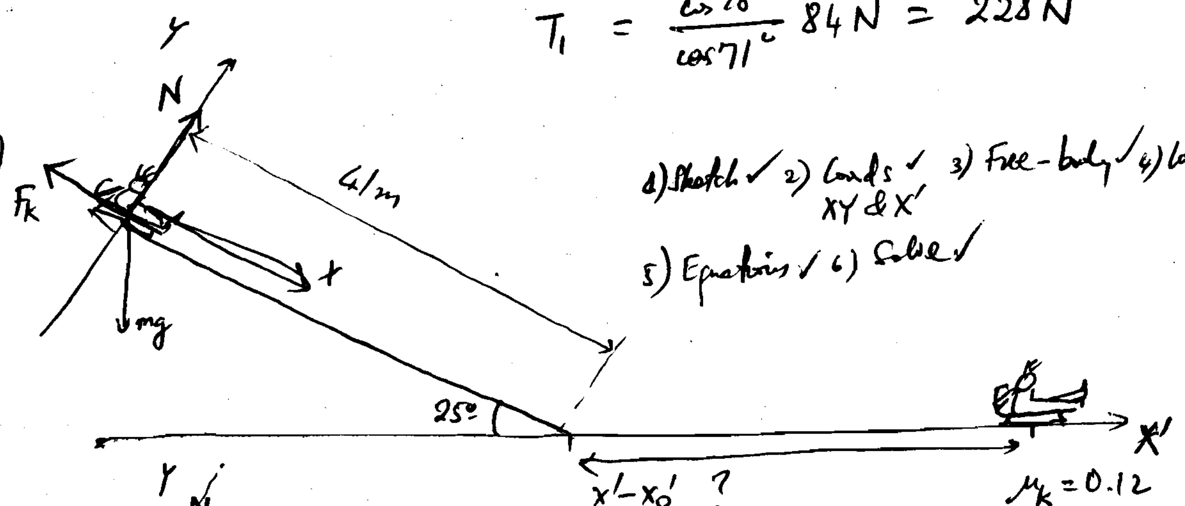
(a) $T_1 = \dfrac{\cos 28^\circ}{\cos 71^\circ} T_2$

(b) $T_2\sin 28^\circ + \dfrac{\cos 28^\circ \sin 71^\circ}{\cos 71^\circ} T_2 - mg = 0$

$$T_2 = \frac{mg}{\sin 28^\circ + \cos 28^\circ \tan 71^\circ} = 84\,N$$

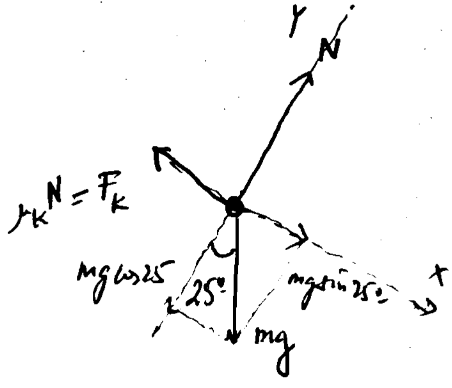
$$T_1 = \frac{\cos 28^\circ}{\cos 71^\circ} 84\,N = 228\,N$$

5.49



1) Sketch ✓ 2) Coords ✓ XY & X' 3) Free-body ✓ 4) Components

5) Equations ✓ 6) Solve ✓



Step 5): $\vec{F}_{net} = m\vec{a} \begin{cases} X: mg\sin 25^\circ - F_k = ma_x & (a) \\ Y: N - mg\cos 25^\circ = 0 & (b) \end{cases}$

$$\left.\begin{aligned} (a)\quad & mg\sin 25^\circ - \mu_k N = ma_x \\ (b)\quad & N - mg\cos 25^\circ = 0 \end{aligned}\right\} \text{2 equations and 2 unknowns } N \,\&\, a_x.$$

Plan: find a_x → find v at bottom of slope = v_0 for leveled portion → find $x' - x_0'$

Step 6:

(b) $N = mg\cos 25°$

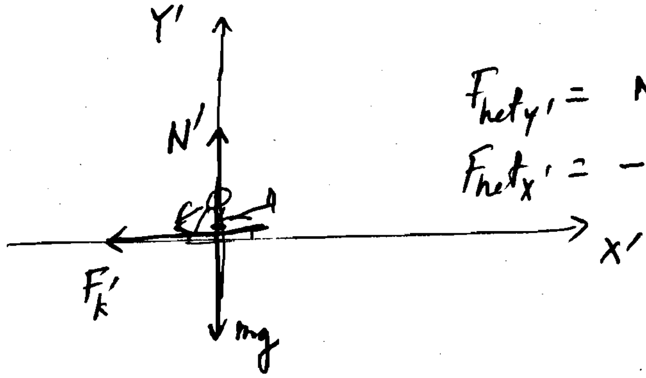
(a) $\cancel{m}g\sin 25 - \mu_K \cancel{m}g\cos 25 = \cancel{m}\, a_x$

$$a_x = g\,(\sin 25° - \mu_K \cos 25°) = 3.08\,m/s^2$$

→ Find v at bottom of slope:

(3) $$\frac{v^2 - 0^2}{41} = 2 \times 3.08 \;\to\; v = \sqrt{2 \times 3.08 \times 41} = 15.9\,m/s$$

→ To find $x' - x_0'$ we need also $a_{x'}$ along leveled portion:



$$F_{net_{y'}} = N' - mg = 0 \to N' = mg$$

$$F_{net_{x'}} = -\mu_k N' = ma_{x'} \to a_{x'} = \frac{-\mu_k N'}{m} = -\mu_k g$$

→ (3) $$\frac{0^2 - 15.9^2}{x' - x_0'} = -2\mu_K g \;\to\; x' - x_0' = \frac{15.9^2}{2 \times 0.12 \times 9.81} = 107\,m.$$