Dimensional analysis:

- Dimension of the speed \( [v] = \frac{[\Delta s]}{[\Delta t]} = \frac{L}{T} \) (length/time)

  "\( \Delta \)" = delta, "change of" or "increment of"

- Dimension of the acceleration \( [a] = \frac{[\Delta v]}{[\Delta t]} = \frac{L}{T^2} \)

- Dimension of energy \( [E] = \left[ \frac{1}{2} m v^2 \right] = [m][v]^2 = M \frac{L^2}{T^2} \)

  Numerical constants do not have dimension of 1.

Application: formula check:

\( v = \frac{1}{2} g h^2 \quad \Rightarrow \quad \left[ \frac{1}{2} g h^2 \right] = [g][h]^2 = \frac{L}{T^2} \cdot L^2 = \frac{L^3}{T^2} \) (No)

\( v = \sqrt{gh} \quad \Rightarrow \quad \left[ \sqrt{gh} \right] = \sqrt{[g][h]} = \sqrt{\frac{L}{T^2}} \cdot (L^1) = \frac{L^{\frac{3}{2}}}{T} \) (No)

g: acceleration of gravity;

h: height (length)
Units:

L: S.I. (system of international units): m (meter)
T: s (second)
M: kg (kilogram)

A (area): m²
V (volume): m³

Unit conversion:

1 kg = 10³ g

1 km = 10³ m; 1 light-year = 9.46 x 10¹⁵ m

1 km² = 10⁶ m²; 1 cm² = 10⁻⁴ m²; 1 mm² = 10⁻⁶ m²

1 km³ = 10⁹ m³; 1 cm³ = 10⁻⁶ m³; 1 mm³ = 10⁻⁹ m³

1 h = 3600 s; 1 day = 86400 s; etc.

\[
\begin{align*}
1 \text{mi} & = 1609 \text{ m} \\
1 \text{ft} & = 0.3048 \text{ m} \\
1 \text{in} & = 2.54 \text{ cm} = 2.54 \times 10^{-2} \text{ m} \\
1 \text{lb} & = 0.454 \text{ kg}
\end{align*}
\]
Accuracy & Significant Figures:

- **Scientific notation:**
  \[ 3105 \text{ m} = 3.105 \times 10^6 \text{ m} \]
  \[ 3000 \text{ s} = 3 \times 10^3 \text{ s} \]
  \[ \nu = \frac{3.105 \times 10^6 \text{ m}}{3 \times 10^3 \text{ s}} = 1.035 \times 10^3 \text{ m/s} \]

- **Accuracy:**
  3.1416 is more accurate than 3.15
  (number of decimal digits)

  **Addition & Subtraction:** accuracy → least accurate term
  \[ 3.1416 - 1.14 = 2.0016 = 2.00 \]

- **Significant figures:**
  \[ \frac{6370 \text{ 000}}{6370 \text{ 001}} \]
  Three
  Seven

  **Multiplication & Division:** keep smallest number of significant figures (except for numeric constants)

  Earth's circumference = \( 2\pi R_E = 2 \times 3.1416 \times 6.37\times10^6 \text{ m} \)
  \[ = 4.002398 \times 10^7 \text{ m} = 4.00 \times 10^7 \text{ m} \]
1.40  a) Volume of water flowing over Niagara Falls in 1 s?

Estimation: Use a rectangular slab of \( w, h, d \).

Volume: \( V = hwd \)

Flow rate: \( \frac{V}{t} = \frac{hwd}{t} \)

1) \( \frac{h}{t} \)

2) \( \frac{w}{t} \)

3) \( \frac{d}{t} \)

Water speed going down the Falls

\[ \frac{hdw}{t} = \frac{1 \text{ m} \times 10^3 \text{ m} \times 3 \text{ m}}{5 \text{ s}} = \frac{3 \times 10^3 \text{ m}^3}{5 \text{ s}} \]

5) Water would accumulate in Lake Erie if Falls is shut off.

- How fast: \( 3000 \times 1 \text{ s} \)
- How long for water level to rise 1 m?

\[ \frac{1 \text{ m}}{1000 \text{ m}} = \frac{3 \text{ m}}{5 \text{ s}} \]
How long: \[ \frac{V_{\text{ERIE}}}{3000 \text{ m}^3/\text{s}} = \frac{1 \text{ m} \times (\text{Area}_{\text{ERIE}})}{3000 \text{ m}^3/\text{s}} \]

\[ \text{Area}_{\text{ERIE}} = 75 \text{ km} \times 375 \text{ km} = 75 \times 375 \text{ km}^2 \]

\[ = \frac{1 \text{ m} \times 75 \times 10^3 \times 375 \times 10^3 \text{ m}^2}{3000 \text{ m}^3/\text{s}} = \frac{75 \times 375 \times 10^6}{3000} \text{ m}^3/\text{s} \]

\[ = 25 \times 375 \times 10^6 \text{ m}^3 = 9 \times 10^6 \text{ m}^3 \]

\[ \frac{1 \text{ day}}{84600 \text{ s}} = 100 \text{ day} \]
Chapter 2: Motion in a Straight Line

Average Motion:

\[ \text{speed} = \frac{\text{distance}}{\text{time}} \quad \rightarrow \quad \text{velocity} = \frac{\text{displacement}}{\text{time}} \]

\[ \text{W} \quad \text{10 mi} \quad \rightarrow \quad \text{E} \quad \text{2 mi} \]

Total time = 3 hr.

Speed = \( \frac{12 \text{ mi}}{h} \)

Velocity = \( \frac{8 \text{ mi}}{h} \)

\[ \rightarrow \quad \text{average velocity} \quad \bar{v} = \frac{\Delta x}{\Delta t} ; \quad \Delta \text{ delta} = \text{"change of"} \]

\[ \rightarrow \quad \text{instantaneous velocity} \quad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (\text{derivative of } x \text{ with } t) \]

\[ \text{Example:} \quad x = at^3 \quad \rightarrow \quad v = \frac{d}{dt} (at^3) = 3at^2 \]

\[ x = at^n \quad \rightarrow \quad v = nat^{n-1} \]

Units: \( \text{m/s} \), \( \frac{\text{km}}{\text{h}} \), \( \frac{\text{mi}}{\text{h}} \), \( \text{ft/s} \), ...

\[ \rightarrow \quad \text{Acceleration: change of velocity over time} \]

\[ x \quad \uparrow \]

\[ t \]
→ Average acceleration: $\bar{a} = \frac{\Delta v}{\Delta t}$

→ Instantaneous acceleration: $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

Units: $\frac{m}{s^2}$; $\frac{km}{s^2}$; $\frac{mi}{s^2}$; etc...

**Constant acceleration**

\[ a = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} \rightarrow v = v_0 + at \tag{1} \]

\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \rightarrow x = x_0 + \bar{v} \cdot t \tag{A} \]

Average velocity is also: \[ \bar{v} = \frac{1}{t_0} \int_{t_0}^{t} v \, dt = \frac{1}{t} \int_0^t (v_0 + at) \, dt \]

\[ = \frac{1}{t} \left[ \left. v_0 t + \frac{1}{2} at^2 \right|_0^t \right] = v_0 + \frac{1}{2} at \]

\[ = \frac{1}{2} v_0 + \frac{1}{2} at + \frac{1}{2} v_0 + \frac{1}{2} at \]

\[ = \frac{1}{2} \left[ (v_0 + at) + (v_0 + at) \right] \tag{B} \]

\[ = \frac{1}{2} \left[ v_0 + v \right] \]

\[ (B) \text{ into } (A) \]

\[ x = x_0 + \bar{v} \cdot t = x_0 + \frac{1}{2} (v_0 + v) \cdot t \]

\[ = x_0 + \frac{1}{2} \left( v_0 + v + at \right) \cdot t \]

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \tag{2} \]
Constant acceleration:
\[ v = v_0 + at \quad (1) \]
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \quad (2) \]

Can derive from (1) \[ \frac{v^2 - v_0^2}{x - x_0} = 2a \quad (3) \]

2.35

Car at \( v_0 = 50 \text{ m/s} \) goes to \( v = 0 \) in
\[ x - x_0 = 100 \text{ ft} \]
Find \( a \):

\[ v_0 = 50 \text{ m/s} \]
\[ a = ? \]
\[ v = 0 \]
\[ x - x_0 = 100 \text{ ft} \]

No time involved, by (3):
\[ 2a = \frac{0 - 22.35^2}{30.48} \]
\[ a = -8.19 \text{ m/s}^2 \]

\[ v_0 = 50 \text{ m/s} \cdot \frac{1609 \text{ m}}{3600 \text{ s}} \cdot \frac{1 \text{ m/s}}{1 \text{ ft}} = 22.35 \text{ m/s} \]
\[ x - x_0 = 100 \text{ ft} \cdot \frac{0.3048 \text{ m}}{1 \text{ ft}} = 30.48 \text{ m} \]

2.61

a)
\[ a = \text{constant} \]
\[ v_0 = 53 \text{ m/s} \]
\[ v = 53 \text{ m/s} \]
\[ x - x_0 = 140 \text{ m} \]
\[ t = 3.6 \text{ s} \]

(1) \[ v_0 = v - at \]

(2) \[ x - x_0 = v_0 t + \frac{1}{2} at^2 \]
Eliminating \( v_0 \):
\[ x - x_0 = (v - at) t + \frac{1}{2} at^2 = vt - at^2 + \frac{1}{2} at^2 \]
\[ = vt - \frac{1}{2} at^2 \]
\[ \Rightarrow a = -2 \frac{(x - x_0 - vt^2)}{t^2} = \frac{-2(140 - 53 \times 3.6)}{3.6^2} = 7.8 \text{ m/s}^2 \]
(1) \( v_0 = v - at = 53 - 7.83 \times 3.6 = 24.8 \text{ m/s} \)

b) How far did it travel from rest to the end of 140m interval?

\[ \begin{align*}
& v' = 0 \\
& v = 24.8 \text{ m/s} \\
& 140 \text{ m} \\
& 3.65 \\
& v = 53 \text{ m/s}
\end{align*} \]

\[ \text{(3) } \frac{v^2 - v_0^2}{2a} = x - x_0 \]

\[ x - x_0 = \frac{53^2 - 0}{2 \times 7.83} = 179.37 \text{ m} \]

Make sure

Lesser than 140m

\( AB = 39.37 \text{ m} \)

Kingfisher at 5m above the fish tries to catch it before the fish enters the water.

What \( v_0 \) should the kingfisher start with to get the fish?

- Calculate how long it will take for the fish to reach water (\( x-x_0 = 30 \text{ m} \))
- the kingfisher will need to cover 35m in that same time.

\[ \begin{align*}
& x-x_0 = v_0 t + \frac{1}{2} a t^2 \\
& \text{Fish: } 30 = 0 + \frac{1.981}{2} t^2 \\
& t = \sqrt{\frac{60}{1.981}} = 2.49 \text{ s} \\
& \text{Kingfisher: } 35 = v_0 \times 2.49 + \frac{1.981}{2} \times 2.49^2 \\
& v_0 = \frac{35}{2.49} = 12 \text{ m/s}
\end{align*} \]
$v_0 = 85 \text{ km/h}$, $\frac{100}{x} \text{ km/s} = 0.218 \text{ m/s}$
$v_0' = 25 \text{ km/h} = 6.94 \text{ m/s}$

$a = -2.1 \text{ m/s}^2$

"How soon they will collide?"

- Position of faster train at certain time is $x$
- Position of slower train at that time is $x'$

They will collide when $x = x'$

\[
\begin{align*}
&x_0 + v_0 t + \frac{1}{2} a t^2 = x_0' + v_{0'} t + \frac{1}{2} a' t^2 \\
&22.2 t - \frac{2.1}{2} t^2 = 50 + 6.94 t \\
&-1.05 t^2 + 15.56 t - 50 = 0 \\
&1.05 t^2 - 15.56 t + 50 = 0
\end{align*}
\]

\[
t = \frac{15.56 \pm \sqrt{15.56^2 - 4 \times 50 \times 1.05}}{2 \times 1.05}
\]

\[
t = \{4.78, 10.5\}
\]

- What a would avoid collision? $15.56^2 < 4 \times 50 \times 1.05$

\[
\frac{2 \times 15.56^2}{200} < a \quad \text{or} \quad \frac{8.42}{200} < a
\]
Back to collision scenario after 4.7s: at what relative speed they will collide?

Faster train: \[ v = v_0 + at \]
\[ = 22.2 + (-2.4) \times 4.7 = 11.76 \text{ m/s} \]

Slower train: \[ v = 6.46 \text{ m/s} \]
\[ \text{rel. speed at collision is 11.82 m/s} \]
Ch 4  Motion in More than One Dimension

Shown experiments in movies to confirm x- and y-component of motion are independent.

Kinematic equations:  
(1) \[ \vec{v} = \vec{v}_0 + \vec{a}t \]
(2) \[ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \]

Projectile motion:  (ball & car going on horizontal track at constant speed. Then ball got tossed up vertically)

\[ \begin{align*}
\sin \theta &= \sin \theta \\
\cos \theta &= \cos \theta \\
\end{align*} \]

\[ \begin{align*}
\vec{v}_x &= \vec{v}_0 \cos \theta \\
\vec{v}_y &= \vec{v}_0 \sin \theta - gt \\
\end{align*} \]

\[ \begin{align*}
x &= x_0 + \vec{v}_0 (\cos \theta) t \\
y &= y_0 + \vec{v}_0 (\sin \theta) t - \frac{1}{2} gt^2 \\
\end{align*} \]

\( (x_0, y_0) = (0, 0) \)  (first origin of coordinates, or initial position)

\( \vec{r} = \frac{x}{\vec{v}_0 \cos \theta} \)

\[ \begin{align*}
y &= \frac{\vec{v}_0 \sin \theta}{\vec{v}_0 \cos \theta} x - \frac{1}{2} \frac{\vec{v}_0 \sin \theta}{\vec{v}_0 \cos \theta} x = \frac{x}{2 \vec{v}_0^2 \cos \theta} \left( \vec{v}_0 \sin \theta - \vec{v}_0 \cos \theta \right) \\
\text{trajectory is a parabola (inverted)}
\end{align*} \]
Ch 3: Motion in 2 & 3 dimensions

2D: the position is determined by 2 quantities:
\[
\begin{align*}
(x, y) &\quad \text{Cartesian components or coordinates} \\
(r, \theta) &\quad \text{Polar components or coordinates}
\end{align*}
\]

Notation:
- Position vector: \( \overrightarrow{r} = (x, y) = (r, \theta) \)
  - Direction is important
- Velocity vector: \( \overrightarrow{v} = (v_x, v_y) = (v, \theta_v) \)
- Acceleration vector: \( \overrightarrow{a} = (a_x, a_y) = (a, \theta_a) \)
Changing from Cartesian to Polar coordinates:

\[ \vec{r} = (x, y) \rightarrow (r = \sqrt{x^2 + y^2}, \ \theta = \tan^{-1} \frac{y}{x}) \]

![Graph showing conversion from Cartesian to Polar coordinates]

Reversing:

\[ \vec{r} = (r, \theta) \rightarrow (x = r\cos \theta, \ y = r\sin \theta) \]

Unit vectors: Vectors with magnitude \( \frac{1}{\text{a unit of length}} \)

- \( \hat{i} \): along \( X \) \[ \hat{i} = (1, 0) = (1, 0^\circ) \]
- \( \hat{j} \): along \( Y \) \[ \hat{j} = (0, 1) = (1, 90^\circ) \]

\[ \vec{r} = (x, y) = x\hat{i} + y\hat{j} \]
\[ \vec{v} = (v_x, v_y) = v_x\hat{i} + v_y\hat{j} \]

In preparation to work with equations of motions (kinematic equations) in 2D, let's look at addition & subtraction of vectors.
Addition & Subtraction of Vectors:

Mathematically: use Cartesian coordinates:

\[
\vec{A} = A_x \hat{i} + A_y \hat{j} \quad ; \quad \vec{B} = B_x \hat{i} + B_y \hat{j}
\]

\[
\vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}
\]

\[
= (3 - 1) \hat{i} + (2 + 1) \hat{j} = 2 \hat{i} + 3 \hat{j}
\]

\[
\vec{A} - \vec{B} = (A_x - B_x) \hat{i} + (A_y - B_y) \hat{j}
\]

\[
= (3 + 1) \hat{i} + (2 - 1) \hat{j} = (4 \hat{i} + 1 \hat{j})
\]
Relative Motion

Person 1 is moving at vel. \( \vec{v}' \) w.r.t. walkway, since walkway moves at \( \vec{V} \) w.r.t. ground, person 1 vel relative to the ground is
\[ \vec{v}_1 = \vec{v}' + \vec{V} \]

Person 2 is moving at \( \vec{v}' \) w.r.t. ground
\[ \vec{v}_2 = \vec{v}' \]

Water flows at \( \vec{V} \)

For boat to go from A to B, where should \( \vec{v}' \) point?
1) To B
2) Left of B
3) Right of B

\( \vec{v}' \) = vel of water relative to ground
\( \vec{v} \) = vel of boat relative to water
Equations of Motion in 2D:

Motion along independent perpendicular directions are independent.

Can release spring w/ a button, ball is a marble, neglect air resistance, v is constant (vel. of car) and ball along x

If button is pressed, ball gets launched, car continue going at v. Ball gets moving along y - direction.

When ball falls back down
1) Falls ahead of car
2) Falls back to journey
3) Falls behind car

Bell motion along x was not affected!

It just has additional motion along y, but still moving along x at same v as the car

1D: \[
\begin{align*}
\vec{v} &= \vec{v}_0 + \vec{a} t \\
x &= x_0 + v_{x0} t + \frac{1}{2} a_x t^2
\end{align*}
\]  

2D: \[
\begin{align*}
\vec{v} &= \vec{v}_0 + \vec{a} t \\
x &= x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \\
y &= y_0 + v_{y0} t + \frac{1}{2} a_y t^2
\end{align*}
\]
In 3D: add a third component $z$. Some 2 kinematic equations.

Example:

![Diagram showing vector components]

$\vec{v}_i = 2100 \text{ km } \hat{i} (\text{ East})$

$\vec{v}_f = -1800 \text{ km } \hat{j} (\text{ South})$

Average acceleration vector:

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{0.5} = \frac{-500 \hat{i} - 583.3 \hat{j}}{150}$

$|\vec{v}_i| = \frac{2100 \text{ km}}{\text{ hr}} \frac{1000 \text{ m}}{1 \text{ km}} \frac{1 \text{ hr}}{3600 \text{ s}} = 583.3 \text{ m/s}$

$|\vec{v}_f| = \frac{1800 \text{ km}}{\text{ hr}} = 500 \text{ m/s}$

$\vec{a} = -3.3 \hat{j} - 3.9 \hat{i}$ (underlined) $\rightarrow$

3rd quadrant

$\alpha = \sqrt{(-3.3)^2 + (-3.9)^2} = 5.1 \text{ m/s}^2$

$\theta_\alpha = \tan^{-1} \left( \frac{-3.3}{-3.9} \right) = 40.5^\circ$

Actual direction:

$180 + 40.5^\circ = 220.5^\circ$
**Diver #2:** (3) \( \frac{v^2 - v_0^2}{x - x_0} = 2a \rightarrow v^2 = 2g(x - x_0) \)

\[ v_2 = \sqrt{2 \times 9.81 \times 3} = 7.69 \text{ m/s} \]

**Diver #1:**

\[ v = v_0 - gt = 0 \rightarrow t = \frac{v_0}{g} \]

\[ x - x_0 = v_0 t - \frac{1}{2} gt^2 = \frac{v_0^2}{2g} = \frac{1.8^2}{2 \times 9.81} \approx 0.15 \text{ m} \]

\[ v_2 = \sqrt{2 \times 9.81 \times (3 + 0.15)} = 7.88 \text{ m/s} \]

\[ v_1^e = 2g(x - x_0) + v_0^2 \]

\[ v_2 = \sqrt{2 \times 9.81 \times 3 + 1.8^2} = 7.88 \text{ m/s} \]

**Diver #1 will hit water first.**

\[ t_2 = \frac{v_2 - v_0}{g} = \frac{7.67 - 0}{9.81} = 0.7825 \text{ s} \]

\[ t_1 = \frac{v_2 - v_0}{g} = \frac{7.88 - 1.8}{9.81} = 0.68 \]