ch 1: Doing Physics

Dimensional analysis:

→ Dimension of the speed $[v] = \frac{[\Delta s]}{[\Delta t]} = \frac{L \text{ (length)}}{T \text{ (time)}}$

"Δ" = delta, "change of" or "increment of"

→ Dimension of the acceleration $[a] = \frac{[\Delta v]}{[\Delta t]} = \frac{\frac{L}{T}}{T} = \frac{L}{T^2}$

→ Dimension of energy: $[E] = [\frac{1}{2}mv^2] = [m][v]^2$

- Numerical constants have dimension of 1

$$= M\left(\frac{L}{T}\right)^2 = M\frac{L^2}{T^2}$$

Application: formula check:

$v = \frac{1}{2}gh^2 \Rightarrow [\frac{1}{2}gh^2] = [g][h]^2 = \frac{L}{T^2}L^2 = \frac{L^3}{T^2}$ (No)

$v = \sqrt{gh} \Rightarrow [\sqrt{gh}] = \sqrt{[g][h]} = \left(\frac{L}{T^2}L\right)^{1/2} = \left(\frac{L^2}{T^2}\right)^{\frac{1}{2}} = \frac{L}{T}$ (Yes)

g: acceleration of gravity;
h: height (length)

- **Units**:

L: S.I. (system of international units) : m (meter)

T: " " : s (second)

M: " " : kg (kilogram) $\rightarrow 10^3$

A (area): " " : m^2

V (volume): " " : m^3

Unit conversion: $1\,kg = 10^3\,g$ (g → gram)

$1\,nm = 10^{-9}\,m$ ("nanometer"); $1\,\mu m = 10^{-6}\,m$ ("micron");

$1\,km = 10^3\,m$ (km → kilometer) ; 1 light-year $= 9.46 \times 10^{15}\,m$

$1\,km^2 = 10^6\,m^2$; $1\,cm^2 = 10^{-4}\,m^2$; $1\,mm^2 = 10^{-6}\,m^2$
($1\,cm = 10^{-2}\,m$)

$1\,km^3 = 10^9\,m^3$; $1\,cm^3 = 10^{-6}\,m^3$; $1\,mm^3 = 10^{-9}\,m^3$

$1\,h = 3600\,s$; $1\,day = 86400\,s$; etc.

$$\begin{cases} 1\,mi = 1609\,m & \text{(mi → mile)} \\ 1\,ft = 0.3048\,m \\ 1\,in = 2.54\,cm = 2.54 \times 10^{-2}\,m \\ 1\,lb = 0.454\,kg \end{cases}$$

Accuracy & Significant figures:

→ Scientific notation: $3\,105\,000\ m = \underbrace{3.105}_{\text{number} < 10} \times 10^{6}\ m$

$3000\ s = 3 \times 10^{3}\ s$

$$v = \frac{3.105 \times 10^{6}\ m}{3 \times 10^{3}\ s} = \frac{3.105}{3} \times 10^{6-3} = 1.035 \times 10^{3}\ \frac{m}{s}$$

→ Accuracy: 3.1416 is more accurate than 3.15
↳ (# of decimal digits)

Addition & substraction: accuracy → least accurate term

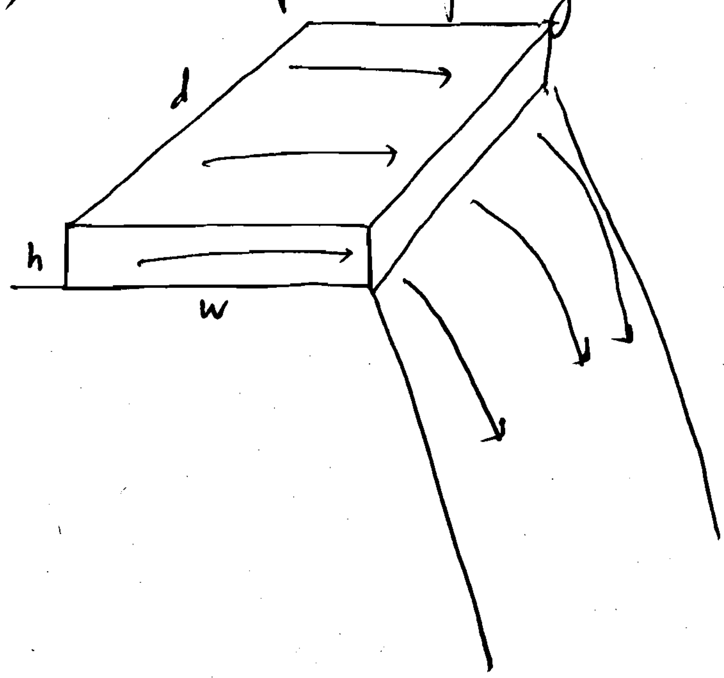
$3.1416 - 1.14 = 2.0016 = 2.00$

→ Significant figures: $\underbrace{6\,370\,000}_{\text{three}}$ $\underbrace{6\,370\,001}_{\text{seven}}$

Multiplication & division: keep smallest number of significant figures (except for numeric constants)

Earth's circumference $= 2\pi R_E = 2 \times 3.1416 \times 6.37 \times 10^{6}\ m$
$= 4.002398 \times 10^{7} = 4.00 \times 10^{7}\ m$

1.40 a) Volume of water flowing over Niagara Falls in 1 s?



Estimation: use a rectangular slab of w, h, d

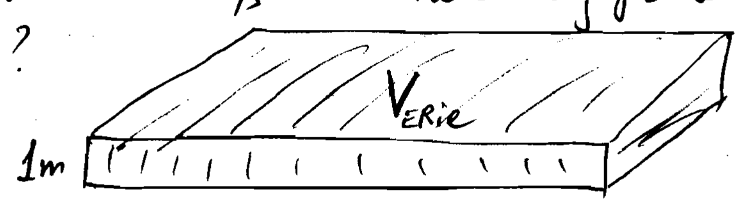
$Vol = hwd$

Flow rate $= \dfrac{vol}{t} = \dfrac{hwd}{t}$ $\begin{cases} 1)\ \dfrac{h}{t} wd \\ 2)\ h \dfrac{w}{t} d \\ 3)\ hw \dfrac{d}{t} \end{cases}$ we can estimate the water speed going down the Falls

$\to hd \dfrac{w}{t} = 1m \times 10^3 m \times 3 \dfrac{m}{s} = 3 \times 10^3 \dfrac{m^3}{s}$

h: 1m (common sense), d: 1000m, $\dfrac{w}{t}$: $3 \dfrac{m}{s}$

b) Water would accumulate in Lake Erie if Falls is shut-off
→ how fast: 3000 m³/s : How long for water level to rise 1m?



(5)

How long: $\dfrac{V_{ERIE}}{3000\,\frac{m^3}{s}} = \dfrac{1m \times Area_{ERIE}}{3000\,\frac{m^3}{s}}$.

$Area_{ERIE} = 75\,km \times 375\,km = 75 \times 375\,km^2$

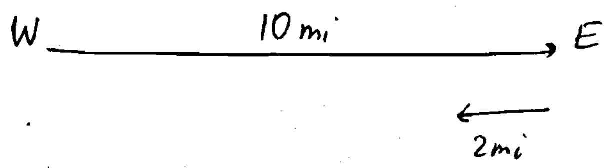
$= \dfrac{1\cancel{m} \times 75 \times 10^3 \cancel{m} \times 375 \times 10^3 \cancel{m}}{3000\,\frac{\cancel{m^3}}{s}} = \dfrac{75 \times 375 \times 10^3}{3}\,s$

$= 25 \times 375 \times 10^6\,s = 9 \times 10^6\,\cancel{s} \cdot \dfrac{1\,day}{84600\,\cancel{s}} = 100$ days.

Ch 2: Motion in a Straight Line

Average Motion:

→ $\text{speed} = \dfrac{\text{distance}}{\text{time}}$ → $\text{velocity} = \dfrac{\text{displacement}}{\text{time}}$



Total time = 1hr.

$\text{speed} = 12\,\dfrac{mi}{h}$ $\text{velocity} = \dfrac{8\,mi}{h}$

→ average velocity: $\bar{v} = \dfrac{\Delta x}{\Delta t}$; "Δ" delta = "change of"

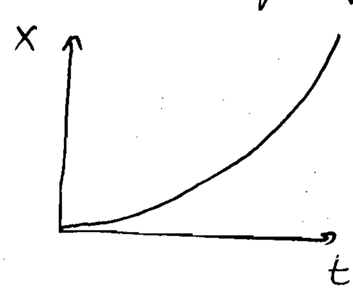
→ instantaneous velocity $v = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ (derivative of x wrt t)

↳ example: $x = at^3 \rightarrow v = \dfrac{d}{dt}(at^3) = 3at^2$

$x = at^n \longrightarrow v = nat^{n-1}$

units: $\dfrac{m}{s}$; $\dfrac{km}{h}$; $\dfrac{mi}{h}$, etc...

→ Acceleration: change of velocity over time



→ Average acceleration : $\bar{a} = \frac{\Delta v}{\Delta t}$

→ Instantaneous acceleration $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

Units : $\frac{m}{s^2}$; $\frac{km}{s^2}$; $\frac{mi}{s^2}$; etc...

$\boxed{\text{Constant acceleration}}$ $\quad a = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} \rightarrow \boxed{v = v_0 + at}$ (1)

$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \longrightarrow x = x_0 + \bar{v} t$ (A)

Average velocity is also : $\bar{v} = \frac{1}{t-0}\int_0^t v\, dt$

$= \frac{1}{t}\int_0^t (v_0 + at)\, dt$

$= \frac{1}{t}\left[v_0 t + \frac{1}{2}at^2 \right]_0^t = v_0 + \frac{1}{2}at$

$= \frac{1}{2}v_0 + \frac{1}{2}a.0 + \frac{1}{2}v_0 + \frac{1}{2}at$

$= \frac{1}{2}\left[(v_0 + a.0) + (v_0 + at) \right]$

$= \frac{1}{2}\left[v_0 + v \right]$ (B)

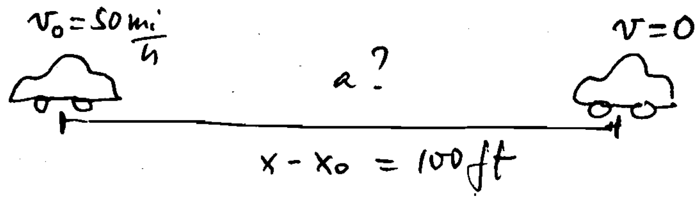
(B) into (A) $\quad x = x_0 + \bar{v} t = x_0 + \frac{1}{2}(v_0 + v)t$

$= x_0 + \frac{1}{2}(v_0 + v_0 + at)t$

$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$ (2)

$$\boxed{\begin{array}{ll} \underline{\text{Constant acceleration:}} & v = v_0 + at \quad (1) \\ & x = x_0 + v_0 t + \frac{1}{2}at^2 \quad (2) \\ \text{can derive from (1) \& (2):} & \frac{v^2 - v_0^2}{x - x_0} = 2a \quad (3) \end{array}}$$

2.35 Car at $v_0 = 50 \frac{mi}{h}$ goes to $v = 0$ in $x - x_0 = 100\,ft$, find a:

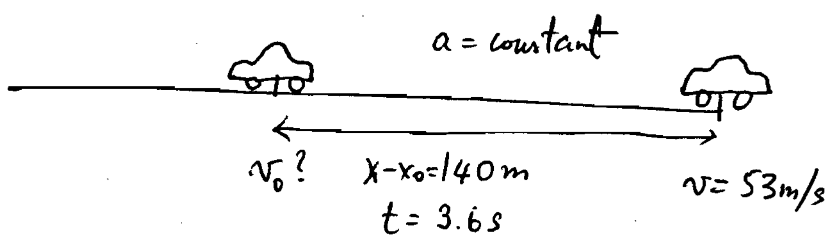


No time involved, try (3): $2a = \frac{0 - 22.35^2}{30.48} \rightarrow \boxed{a = -8.192 \frac{m}{s^2}}$

$$v_0 = 50 \frac{\cancel{mi}}{\cancel{h}} \cdot \frac{1\cancel{h}}{3600\,s} \cdot \frac{1609\,m}{1\,\cancel{mi}} = 22.35\,m/s$$

$$x - x_0 = 100\,ft \cdot \frac{0.3048\,m}{1\,ft} = 30.48\,m$$

2.61 a)



(1) $v_0 = v - at$

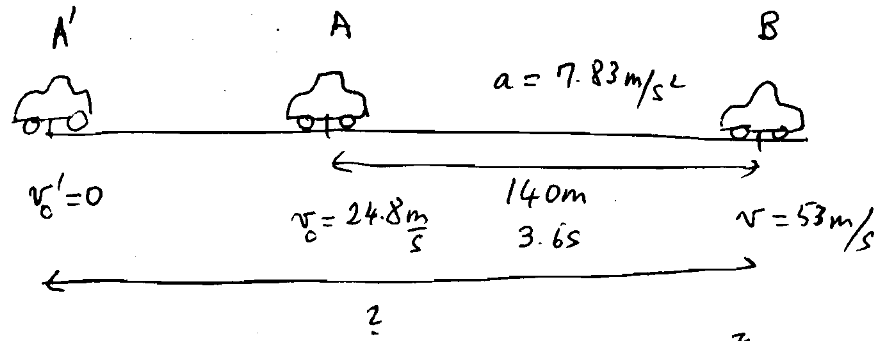
(2) $x - x_0 = v_0 t + \frac{1}{2}at^2$, eliminating v_0:

$$x - x_0 = (v - at)t + \frac{1}{2}at^2 = vt - at^2 + \frac{1}{2}at^2$$
$$= vt - \frac{1}{2}at^2$$

$$\rightarrow a = -\frac{2(x - x_0 - vt)}{t^2} = -2\frac{(140 - 53 \times 3.6)}{3.6^2} = +7.83\frac{m}{s^2}$$

(1) $v_0 = v - at = 53 - 7.83 \times 3.6 = 24.8\ m/s$

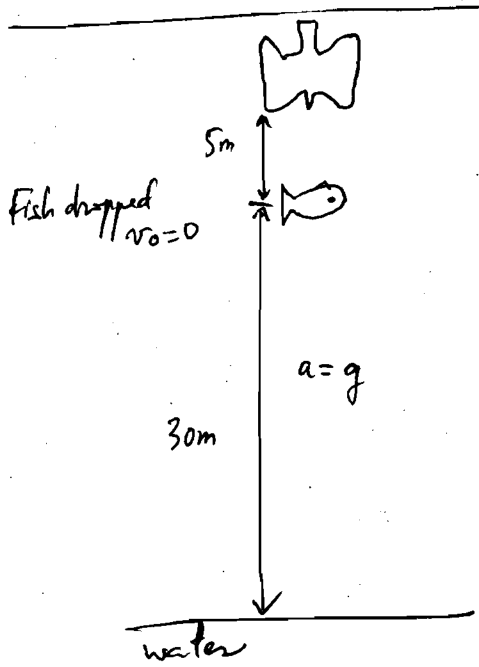
b) How far did it travel from rest to the end of 140m interval?



(3) → $\dfrac{v^2 - v_0'^2}{\underbrace{x - x_0'}_{b/w\ A'\&B}} = 2a \rightarrow x - x_0' = \dfrac{53^2 - 0}{2 \times 7.83} = 179.37m$

Makes sense, larger than 140m

$A'A = 39.37m$



Kingfisher at 5m above the fish tries to catch it before the fish enters the water:

What v_0 should the kingfisher start with to get the fish?

→ Calculate how long it will take for the fish to reach water ($x - x_0 = 30m$)

→ then the kingfisher will need to cover 35m in that same time.

(2) $x - x_0 = v_0 t + \frac{1}{2} g t^2$

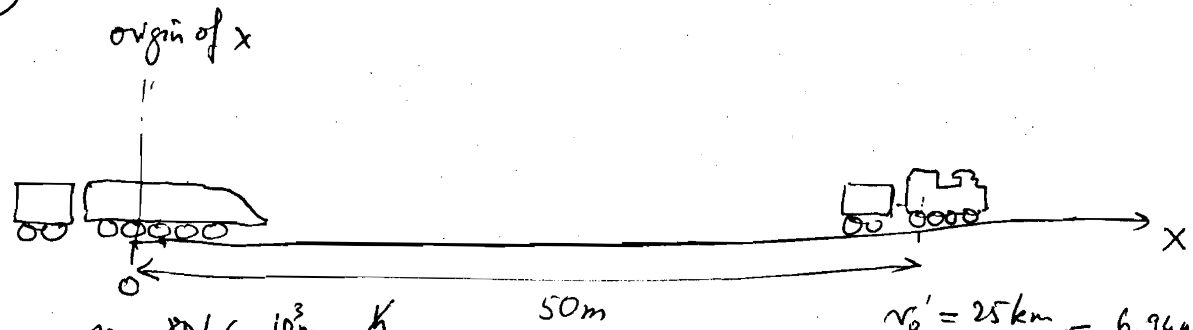
Fish: $30m = 0 + \frac{1}{2} 9.81 t^2$

$\rightarrow t = \sqrt{\dfrac{60}{9.81}} = 2.47s$

Kingfisher: $35m = v_0 \times 2.47 + \frac{1}{2} 9.81 \times 2.47^2$

$\rightarrow v_0 = \dfrac{5}{2.47} = 2\ m/s$

2.73



$v_0 = 80\,\frac{\cancel{km}}{\cancel{h}} \cdot \frac{10^3 m}{\cancel{km}} \cdot \frac{\cancel{h}}{3600\,s} = 22.2\,\frac{m}{s}$ $\qquad v_0' = 25\,\frac{km}{h} = 6.94\,m/s$

$a = -2.1\,m/s^2$

How soon they will collide?

→ Position of faster train at certain time is x

→ " " slower train at " " is x'

They will collide when: $x = x'$

(2) $\cancel{x_0} + v_0 t + \frac{1}{2} a t^2 = x_0' + v_0' t + \frac{1}{2} \cdot 0 t^2$

$$22.2t - \frac{2.1}{2} t^2 = 50 + 6.94t$$

$$-1.05t^2 + 15.56t - 50 = 0$$

$$1.05t^2 - 15.56t + 50 = 0$$

$$t = \frac{15.56 \pm \sqrt{15.56^2 - 4 \times 50 \times 1.05}}{2.1}$$

$$t = \begin{cases} 10\,s \\ 4.7\,s \end{cases}$$

→ 4.7s

→ What a would avoid collision? $\quad 15.56^2 < 4 \times 50 \times \frac{|a|}{2}$

$$\frac{2 \times 15.56^2}{200} < a \quad \text{or} \quad \boxed{2.42\,m/s^2}$$

Back to collision scenario after 4.7s : at what relative speed they will collide ?

Faster train : $v = v_0 + at$

$= 22.2 + (-2.1) \times 4.7 = 11.76\ m/s$

Slower train : $v = 6.94\ m/s$

rel. speed at collision is $4.82\ m/s$

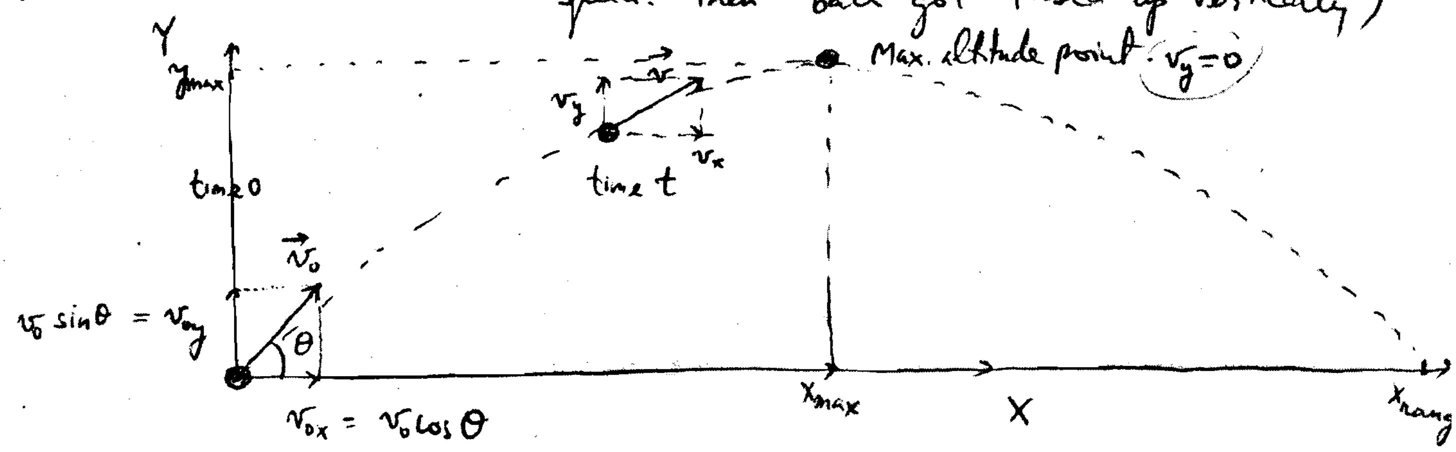
Ch 4 Motion in More than One Dimension

Shown experiments in movies to confirm x- & y- components of motion are independent.

Kinematic equations (constant acceleration)

$$\begin{cases} (\mathrm{I}) & \vec{v} = \vec{v}_0 + \vec{a}t \\ (\mathrm{II}) & \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \end{cases}$$

Projectile motion: (ball & car going on horizontal track at constant speed. Then ball got tossed up vertically)



$$(\mathrm{I}) \begin{cases} v_x = v_{0x} = v_0 \cos\theta \\ v_y = v_{0y} - gt = v_0 \sin\theta - gt \end{cases}$$

$$(\mathrm{II}) \begin{cases} x = x_0 + v_0(\cos\theta)t \\ y = y_0 + v_0(\sin\theta)t - \frac{1}{2}gt^2 \end{cases}$$

gravity is along y-direction only and opposite to the motion (before ball reaches highest point.)

$(x_0, y_0) = (0, 0)$ (Place origin of coordinates on initial position)

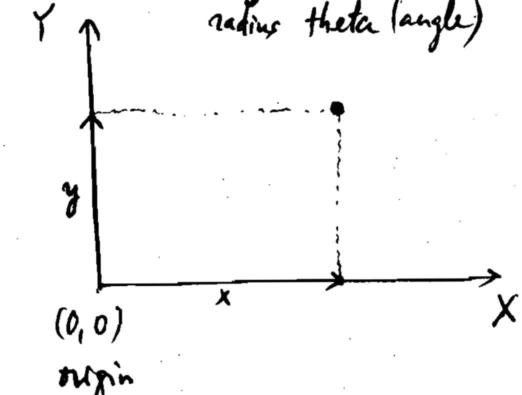
$$(\mathrm{II}) \begin{cases} t = \dfrac{x}{v_0 \cos\theta} \\ \boxed{y = \dfrac{v_0 \sin\theta}{v_0 \cos\theta} x - \dfrac{1}{2} g \dfrac{x^2}{v_0^2 \cos^2\theta} = x\tan\theta - \dfrac{g}{2} \dfrac{x^2}{v_0^2 \cos^2\theta}} \end{cases}$$

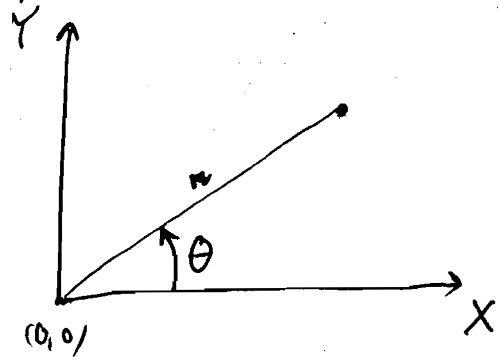
Trajectory is a parabola (inverted)

Ch 3. Motion in 2 & 3 dimensions

2D: the position is determined by 2 quantities:

$$\begin{cases} (x, y) \rightarrow & \text{Cartesian components or coordinates} \\ (\underset{\text{radius}}{r}, \underset{\text{theta (angle)}}{\theta}) \rightarrow & \text{Polar components or coordinates} \end{cases}$$

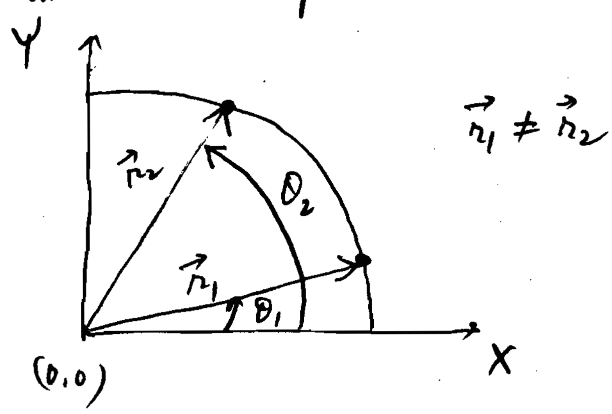




Notation:

Position vector : $\vec{r} = (x, y) = (r, \theta)$

direction is important

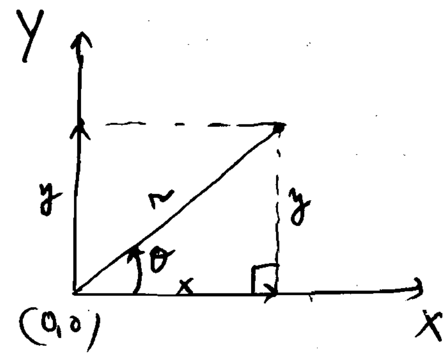


Velocity vector : $\vec{v} = (v_x, v_y) = (v, \theta_v)$

Acceleration vector : $\vec{a} = (a_x, a_y) = (a, \theta_a)$

Changing from Cartesian to Polar coordinates:

$$\vec{r} = (x, y) \longrightarrow \left(r = \sqrt{x^2+y^2} \ , \ \theta = \tan^{-1}\frac{y}{x} \right)$$



Reversing: $\vec{r} = (r, \theta) \longrightarrow \left(x = r\cos\theta \ , \ y = r\sin\theta \right)$

Unit vectors: vectors with magnitude 1 (or radius or length)

$\hat{i}$: along X $\rightarrow$ $\hat{i} = (1, 0) = (1, 0°)$

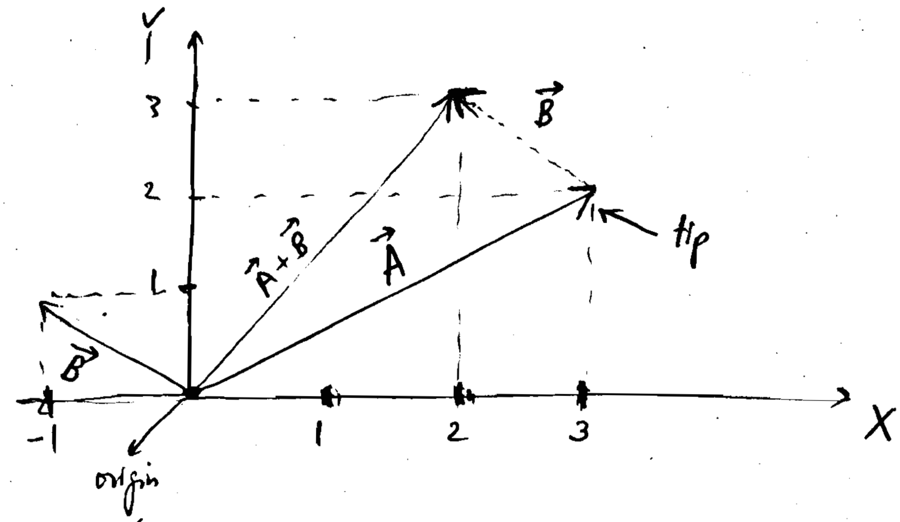
$\hat{j}$: along Y $\rightarrow$ $\hat{j} = (0, 1) = (1, 90°)$

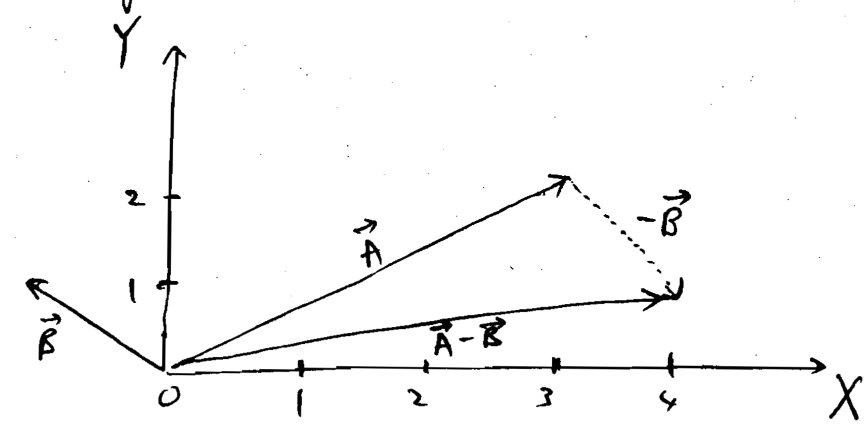
$\vec{r} = (x, y) = x\hat{i} + y\hat{j}$

$\vec{v} = (v_x, v_y) = v_x\hat{i} + v_y\hat{j}$

In preparation to work with equations of motions (Kinematic equations) in 2D → let's look at addition & substraction of vectors :

Addition & Substraction of vectors :





Mathematically : use Cartesian coordinates :

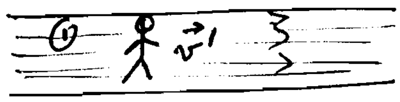
$$\vec{A} = A_x \hat{i} + A_y \hat{j} \; ; \quad \vec{B} = B_x \hat{i} + B_y \hat{j}$$

$$\vec{A}+\vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$
$$= (3-1)\hat{i} + (2+1)\hat{j} = 2\hat{i} + 3\hat{j}$$

$$\vec{A}-\vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$
$$= (3+1)\hat{i} + (2-1)\hat{j} = (4\hat{i} + 1\hat{j})$$

Relative Motion:

1D



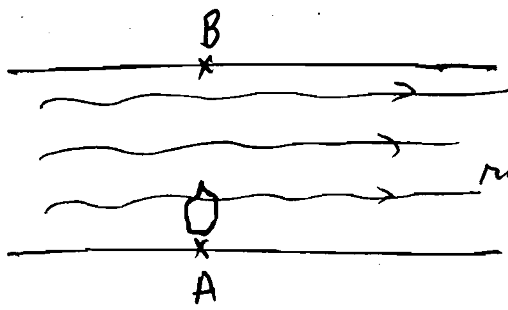
automatic walkway moving at $\vec{V}$

Person ① is moving at vel. $\vec{v}'$ w.r.t. walkway, since walkway movs at $\vec{V}$ w.r.t. to ground. Person ① vel. relative to the ground is $\vec{v}_{①} = \vec{v}' + \vec{V}$

Person ② is moving at $\vec{v}'$ w.r.t ground

$\vec{v}_{②} = \vec{v}'$

2D

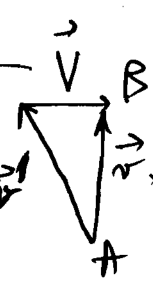


river water flows at $\vec{V}$

For boat to go from A to B : where should $\vec{v}'$ point

1) To B
2) Left of B ✓ → so $\vec{v} = \vec{v}' + \vec{V}$ would point A → B
3) Right of B

$\vec{V}$ vel. of water relative to ground ←

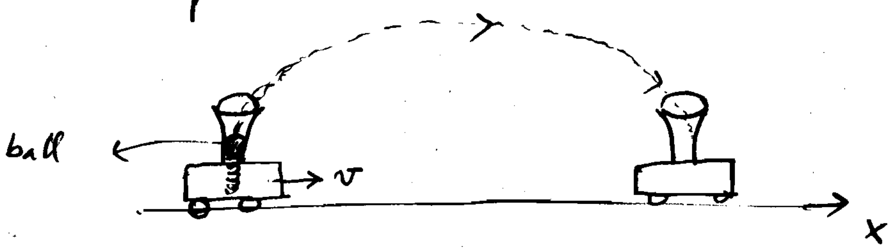


vel. of boat relative to water ← $\vec{v}'$

$\vec{v}$ → vel. of boat relative to ground.

Equations of Motion in 2D :

Motion along ~~independent~~ perpendicular directions are independent. ✓



Can release spring w/ a button; ball is a marble, neglect air resistance.
v is constant (vel. of car and ball along x)

If button is pressed, ball gets lauched, car continue going at v. Ball gets moving along y-direction.

When ball falls back down
1) Falls ahead of car
2) Falls back to funnel ✓
3) Falls behind car

Ball motion along x was not affected!
It just has additional motion along y, but still moving along x at same v as the car

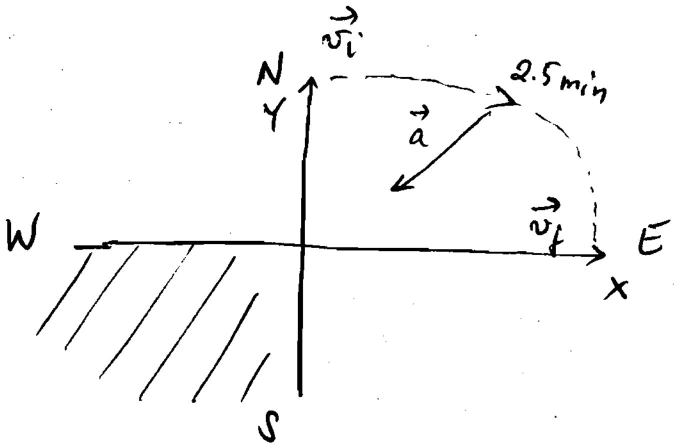
$$1D: \begin{cases} v = v_0 + at & (1) \\ x = x_0 + v_0 t + \frac{1}{2}at^2 & (2) \end{cases} \longrightarrow 2D: \begin{cases} v_x = v_{0x} + a_x t & (1) \\ v_y = v_{0y} + a_y t & \\ x = x_0 + v_{0x} t + \frac{1}{2}a_x t^2 & (2) \\ y = y_0 + v_{0y} t + \frac{1}{2}a_y t^2 & \end{cases}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$
$$\vec{v} = v_x\hat{i} + v_y\hat{j} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$
$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

or
$$\begin{cases} \vec{v} = \vec{v}_0 + \vec{a}t & (1) \\ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 & (2) \end{cases}$$

In 3D: add a third component z. Same 2 kinematic equations.

Example:



$\vec{v_i} = 2100 \frac{km}{h}\, \hat{i}$ (East)

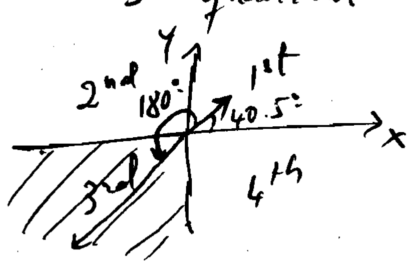
$\vec{v_f} = -1800 \frac{km}{h}\, \hat{j}$ (South)

Average acceleration vector: $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v_f} - \vec{v_i}}{\Delta t} = \frac{-500\hat{j} - 583.3\hat{i}}{150}$

$|\vec{v_i}| = 2100 \frac{km}{h} \cdot \frac{10^3 m}{km} \frac{h}{3600 s} = 583.3\ m/s$

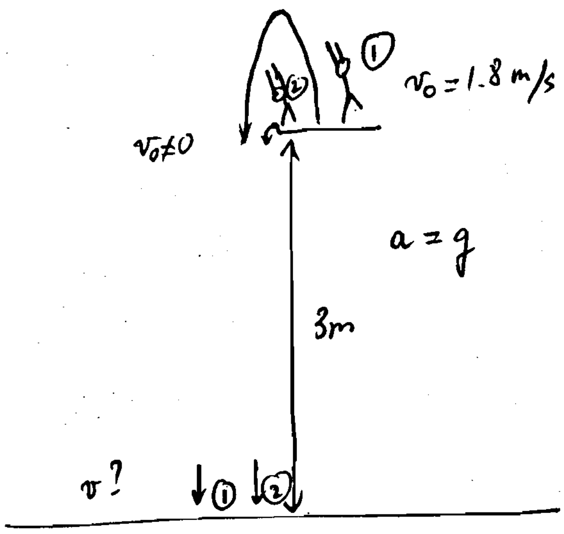
$|\vec{v_f}| = 1800 \frac{km}{h} = 500\ m/s$

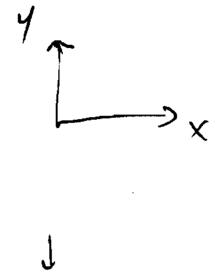
$\vec{a} = \underbrace{-3.3\hat{j} - 3.9\hat{i}}_{3^{rd}\ \text{quadrant}}$ (Cartesian) $\rightarrow$ $\begin{cases} a = \sqrt{(-3.9)^2 + (-3.3)^2} \\ \quad = 5.1\ m/s^2 \\ \theta_a = \tan^{-1} \frac{-3.3}{-3.9} = 40.5° \end{cases}$



Actual direction:
$180 + 40.5° = 220.5°$.

2.69

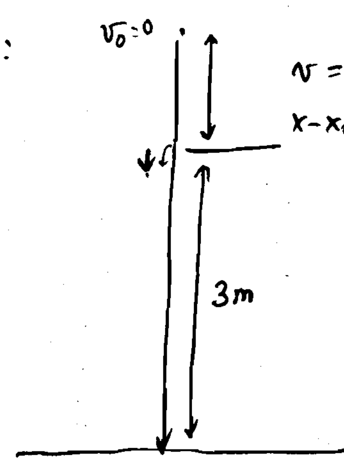




Diver # 2 : (3) $\rightarrow \dfrac{v^2 - v_0^2}{x - x_0} = 2a \rightarrow v^2 = 2g(x - x_0)$

$$v_{②} = \sqrt{2 \times 9.81 \times 3} = 7.67 \text{ m/s}$$

Diver # 1 :

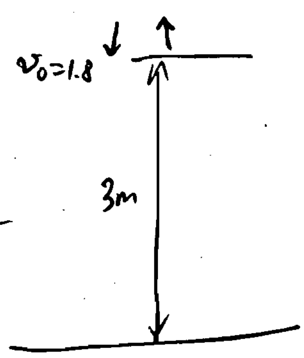


$v = v_0 - gt = 0 \rightarrow t = \dfrac{v_0}{g}$

$x - x_0 = v_0 t - \frac{1}{2} g t^2 = \frac{1}{2} \dfrac{v_0^2}{g} = \frac{1}{2} \dfrac{1.8^2}{9.81} = 0.17\text{m}$

$$v_1 = \sqrt{2 \times 9.81 \times (3 + 0.17)} = 7.88 \text{m/s}$$

or



$$v_1^2 = 2g(x - x_0) + v_0^2$$

$$v_1 = \sqrt{2 \times 9.81 \times 3 + 1.8^2} = 7.88 \text{ m/s}$$

→ Diver #1 will hit water first

$t_2 = \dfrac{v_2 - v_0}{g} = \dfrac{7.67 - 0}{9.81} = 0.782\text{s}$; $t_1 = \dfrac{v_1 - v_0}{g} = \dfrac{7.88 - 1.8}{9.81} = 0.62\text{s}$