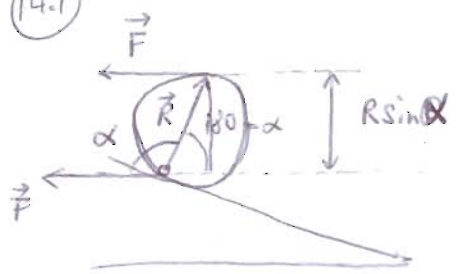
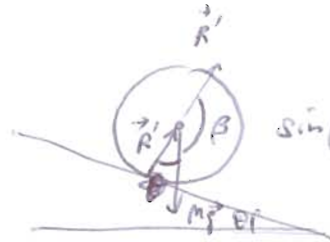
14.1

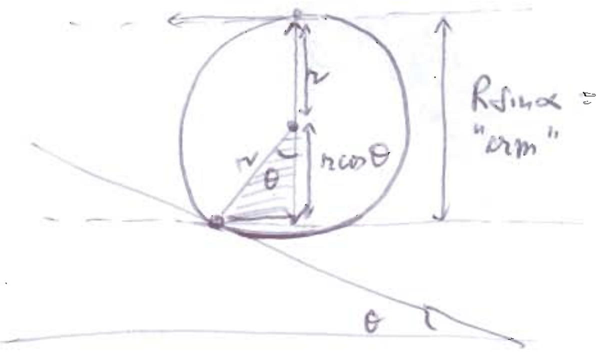


$\vec{R} \times \vec{F} = \underline{RF \sin \alpha}$

Radius of disc is r



$\sin\beta = \sin(180-\beta)$
$= \sin\theta$



$R \sin\alpha = r + r\cos\theta$
"arm"

15.2) 2) Focus on m_1: $kx - \mu_s m_2 g = m_1 a$ (x-direction)

$\downarrow$ same as m_2 (going together before m_2 starts to slip)

($m_2 g$ also acts on m_1 but it is along Y-direction)

$$\mu_s m_2 g = kx - m_1 a \qquad \left(\omega = \sqrt{\frac{k}{m_1+m_2}} \Rightarrow k = \omega^2(m_1+m_2) \right)$$

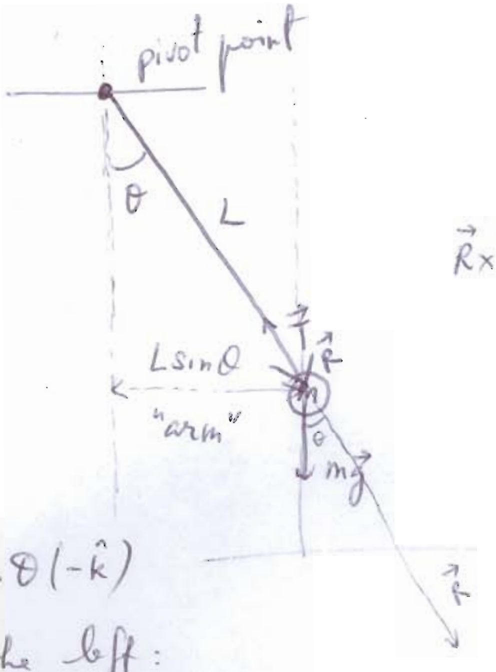
$$= \omega^2(m_1+m_2)\underset{X_{max}}{A} - m_1 \underset{a_{max}}{A\omega^2}$$

$$\mu_s \cancel{m_2} g = \omega^2 \cancel{m_2} A \rightarrow \mu_s = \frac{\omega^2 A}{g} = \frac{10 \times 0.6}{9.81} = 6.11$$

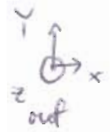
Ch. 15 Oscillatory Motion

Examples

1) Pendulum :



$\vec{R} \times \vec{T} = RT\sin 180 = 0$



$\vec{\tau}_{net,ext} = \vec{\tau}_{mg} = Lmg\sin\theta(-\hat{k})$, $= mgL\sin\theta(-\hat{k})$ or mg"arm"$(-\hat{k})$

(when bob swing to the left:
$\vec{R}$ will be to the left of $m\vec{g}$: ⇒
torque will change direction to point out of screen $(\hat{k})$)

Analog of Newton's law: $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$, $I = mL^2$

$$-\cancel{m}g\cancel{L}\sin\theta = \cancel{m}L^{\cancel{2}}\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Small angle approximation: θ small ; $\sin\theta \simeq \theta$ (Taylor's expansion of $\sin\theta$)

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta}$$

This allows me to get angular freq. easily

Good for ANY SHM

Oscillatory motion: Simple harmonic motion (SHM) can be described with a harmonic: sine or cosine

$$\theta = \underbrace{\theta_M}_{\text{amplitude}} \underbrace{\cos(\omega t)}_{\text{runs b/w } 1 \,\&\, -1}$$

ω: angular frequency

$$\frac{d\theta}{dt} = -\theta_M \omega \sin(\omega t) \; ; \quad \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(-\theta_M \omega \sin \omega t\right) = -\theta_M \omega^2 \cos \omega t$$

Pendulum: $\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{L}\theta$

$$+\cancel{\theta_M}\omega^2 \cancel{\cos\omega t} = +\frac{g}{L}\cancel{\theta_M}\cancel{\cos\omega t} \rightarrow \boxed{\omega = \sqrt{\frac{g}{L}}}$$

2) Torsional pendulum

Twisting

$\tau = -\kappa\theta$

κ: Kappa, the torsional constant

(Analog of Hooke's Law for spring)

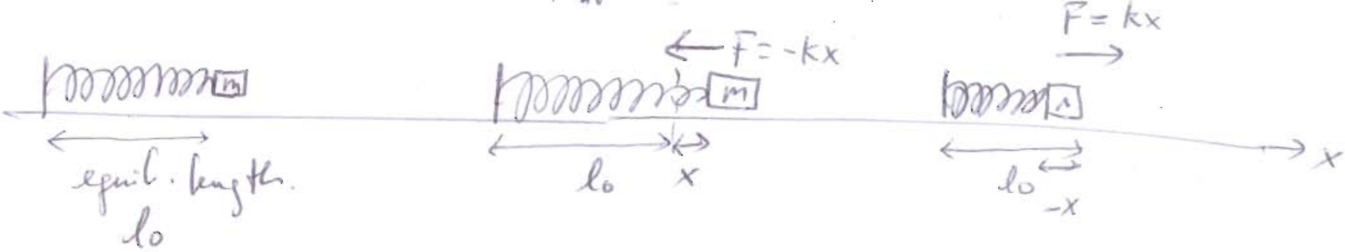
$$\tau = I\alpha$$

$$-\kappa\theta = I\frac{d^2\theta}{dt^2} \rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta} \Rightarrow \boxed{\omega = \sqrt{\frac{\kappa}{I}}}$$

3) Spring : $\boxed{F = -kx}$

spring constant

displacement from equilibrium.

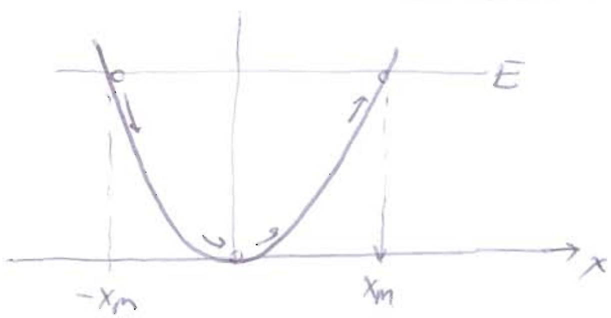


Newton's law $\quad F = ma = m \dfrac{d^2x}{dt^2}$

$$-kx = m \frac{d^2x}{dt^2} \rightarrow \boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

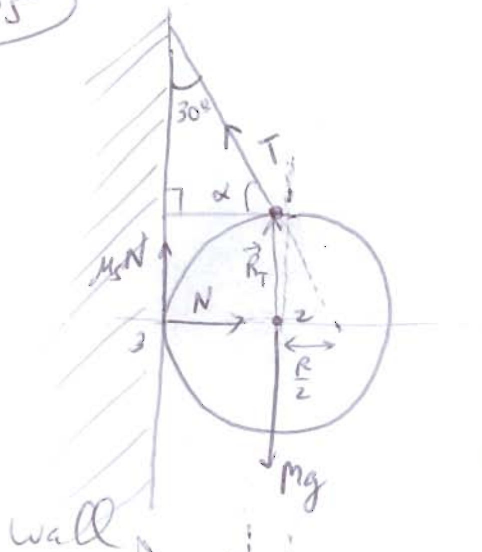
$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

4)



5) X & Y of circular motion :

14-25



Solid sphere, M

μ_s?

$\vec{\tau}_{net,ext} = 0$ (depends on pivot point → different # of torques)

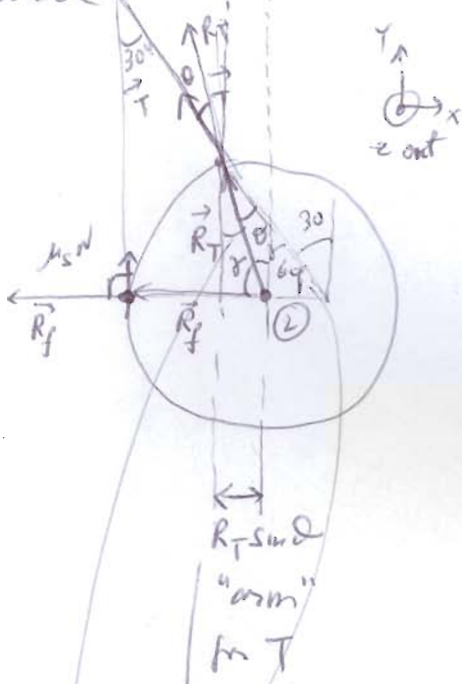
pivot

① : $\vec{\tau}_{Mg} + \vec{\tau}_N + \vec{\tau}_{\mu_s N} = 0$

② : $\boxed{\vec{\tau}_T + \vec{\tau}_{\mu_s N} = 0}$

x : $\boxed{N - T\cos\alpha = 0}$

y = $\mu_s N + T\sin\alpha - Mg = 0$



$TR_T \sin\theta\, \hat{k} - \mu_s N\, \hat{k} = 0$

$$N = T\cos\alpha = T\cos(90 - 30°) = T\sin 30° = \frac{T}{2}$$

$$T = 2N$$

$2N R_T \sin\theta - \mu_s N = 0$

$\mu_s = \overbrace{2R_T \sin\theta}^{\text{arm for tension } T}$

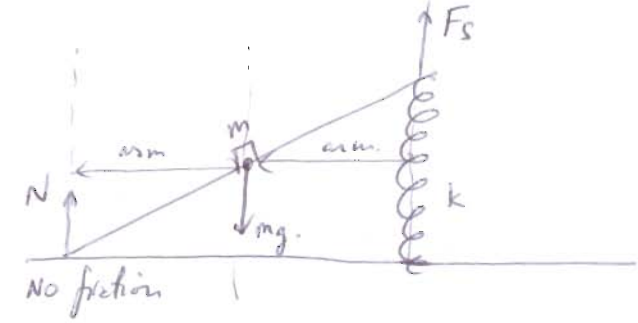
+ radius × cos γ = radius × cos(60 + θ)

$30 - \theta \quad \rightarrow \quad \gamma = 180 - 90 - (30 - \theta) = 90 - 30 + \theta = 60 + \theta$

$\gamma + \theta + 60 + 90 = 180 \quad \rightarrow \quad \gamma + \theta = 30°$

$\theta + \gamma + 90 + 60 = 180$

14.51



$\vec{F}_{net} = 0 \quad -N + F_s - mg = 0$

$\vec{\tau}_{net} = 0 \quad \rightarrow \quad N \text{ arm} = F_s \text{ arm}$

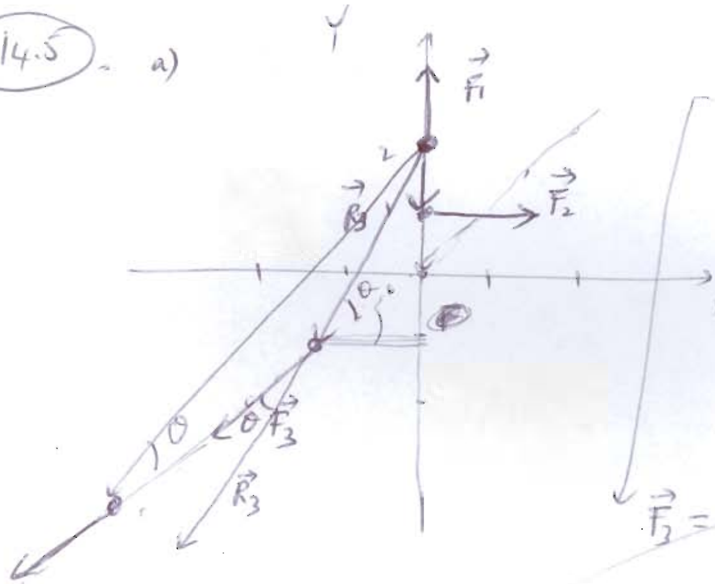
CM of board as pivot

$2F_s = mg \rightarrow F_s = \frac{mg}{2}$

$kx = \frac{mg}{2}$

$\boxed{x = \frac{mg}{2k}}$

14.5 a)



1) $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

2) $\vec{\tau_1} + \vec{\tau_2} + \vec{\tau_3} = 0$

pivot at $(0, 2) \rightarrow \vec{\tau_1} = 0$

$\underbrace{1 . F_2 \hat{k} + R_3 F_3 \sin\theta \, (-\hat{k}) = 0}$

$F_2 = F_3 = F \rightarrow R_3 \sin\theta = 1$

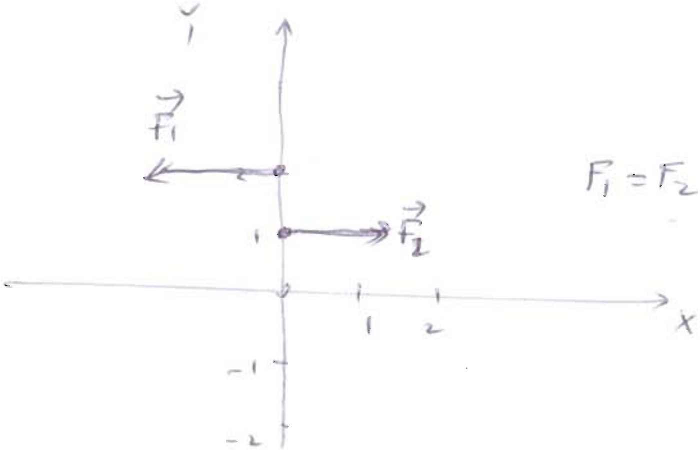
$\vec{F_3} = -\vec{F_1} - \vec{F_2}$

$= -F\hat{j} - F\hat{i} = F(-\hat{i} - \hat{j})$

(3rd quadrant.)
along diagonal.

Not unique answer: any pt along diagonal of 3rd quad is good: $(-1, -1)$ or $(-3, -3)$

b)



$F_1 = F_2$

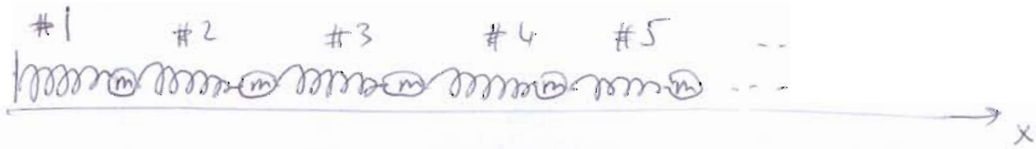
$\vec{F}_1 + \vec{F}_2 = 0 \rightarrow$ No $\vec{F}_3$!

Total energy is constant: when x is max $v=0$ and $KE=0$, but $U_{elastic}$ is $\frac{1}{2}kA^2$; when x is 0, v is max but $U_{elastic}=0$

Ch 16 Wave Motion

<u>Wave:</u> propagation of a local disturbance (oscillation), not of matter.

Local distance = osc. of spring #1:



It takes some time for the osc. in spring #1 to <u>propagate</u> to spring #5. Propagation is going at the "wave speed" $v=\frac{\lambda}{T}$

λ: wavelength ; $\underbrace{T\text{: period.}}$

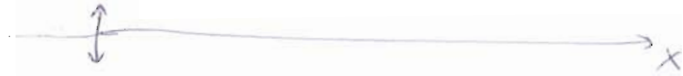
"lambda"

separation b/w two consecutive time cycles

As the oscillation is propagated in space, the variation in space is also periodic or harmonic: the separation b/w two consecutive spatial cycles is a wavelength → dimension of length.

Two types of waves:
- Longitudinal: disturbance or oscillation is parallel to the direction of propagation.
- Transverse: when oscillation is perpendicular to direction of propagation.

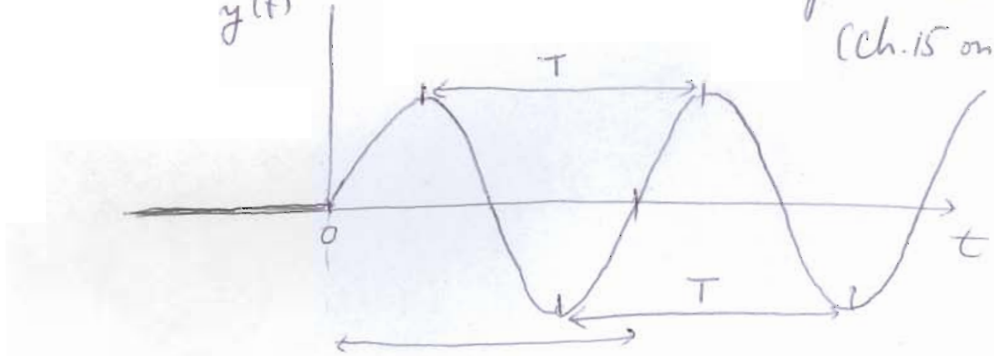
Transverse wave: propagates along a rope



if we move the left end up & down, this disturbance (along Y axis) is propagated along x-axis.

Disturbance at left end:

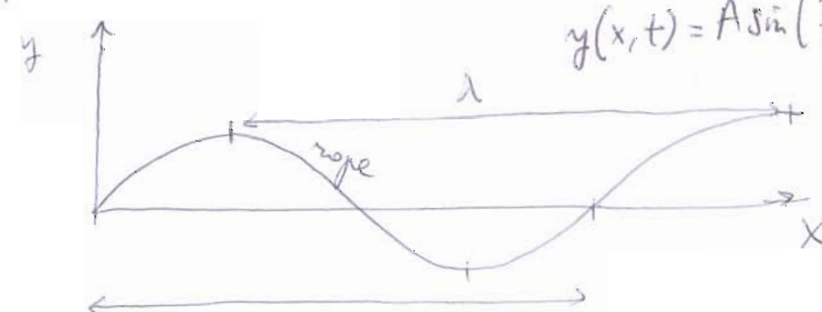
$y(t) = A \sin(\omega t)$
(Ch. 15 on Oscillations)



one cycle in time:
or one period T $\longrightarrow$ $\omega = \frac{2\pi}{T}$

Snapshot of rope at a later time

$y(x,t) = A \sin\left(\underbrace{\frac{2\pi}{\lambda}}_{k} x - \underbrace{\frac{2\pi}{T}}_{\omega} t\right)$ — k: wave number

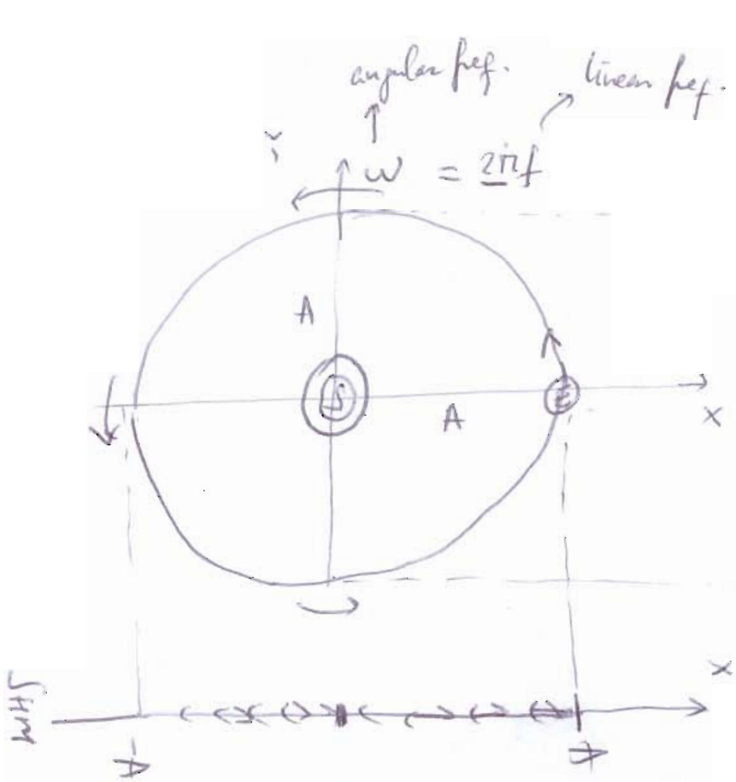


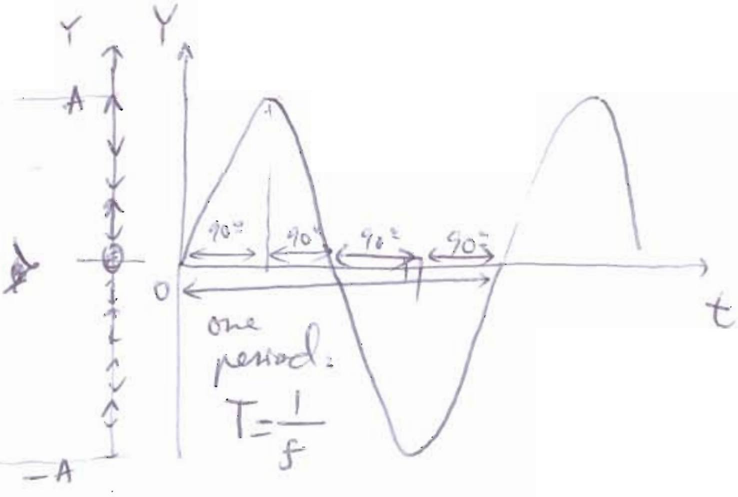
one cycle in space
or one wavelength λ

$y(x,t) = A \sin(kx - \omega t)$

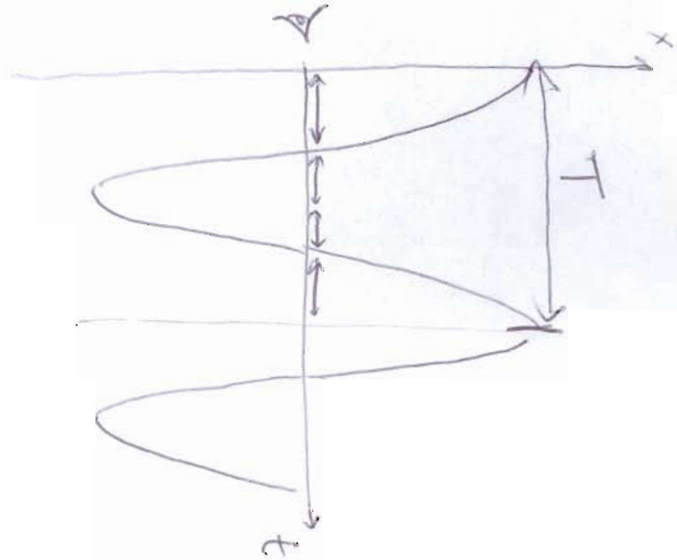
- A → amplitude
- k → wave number $k \equiv \frac{2\pi}{\lambda}$
- ω → ang. freq. $\omega = \frac{2\pi}{T}$

wave speed $v = \frac{\lambda}{T}$





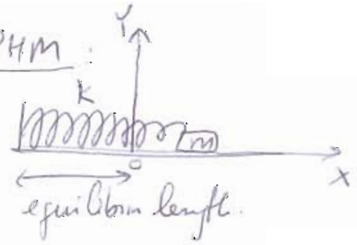
SHM $\quad Y(t) = A \sin \omega t =$



$\longrightarrow \quad X(t) = A \cos \omega t = A \sin(\omega t + 90°)$

X & Y are shifted by $\frac{\pi}{2}$ or 90°
($\frac{1}{4}$ of a period)

Energy in SHM:

Spring: 

$\omega = \sqrt{\frac{k}{m}}$

$\omega^2 = \frac{k}{m}$

$x = A \cos \omega t$
displacement w.r.t. equilibrium length.

Total energy $E = KE + U_{elastic} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

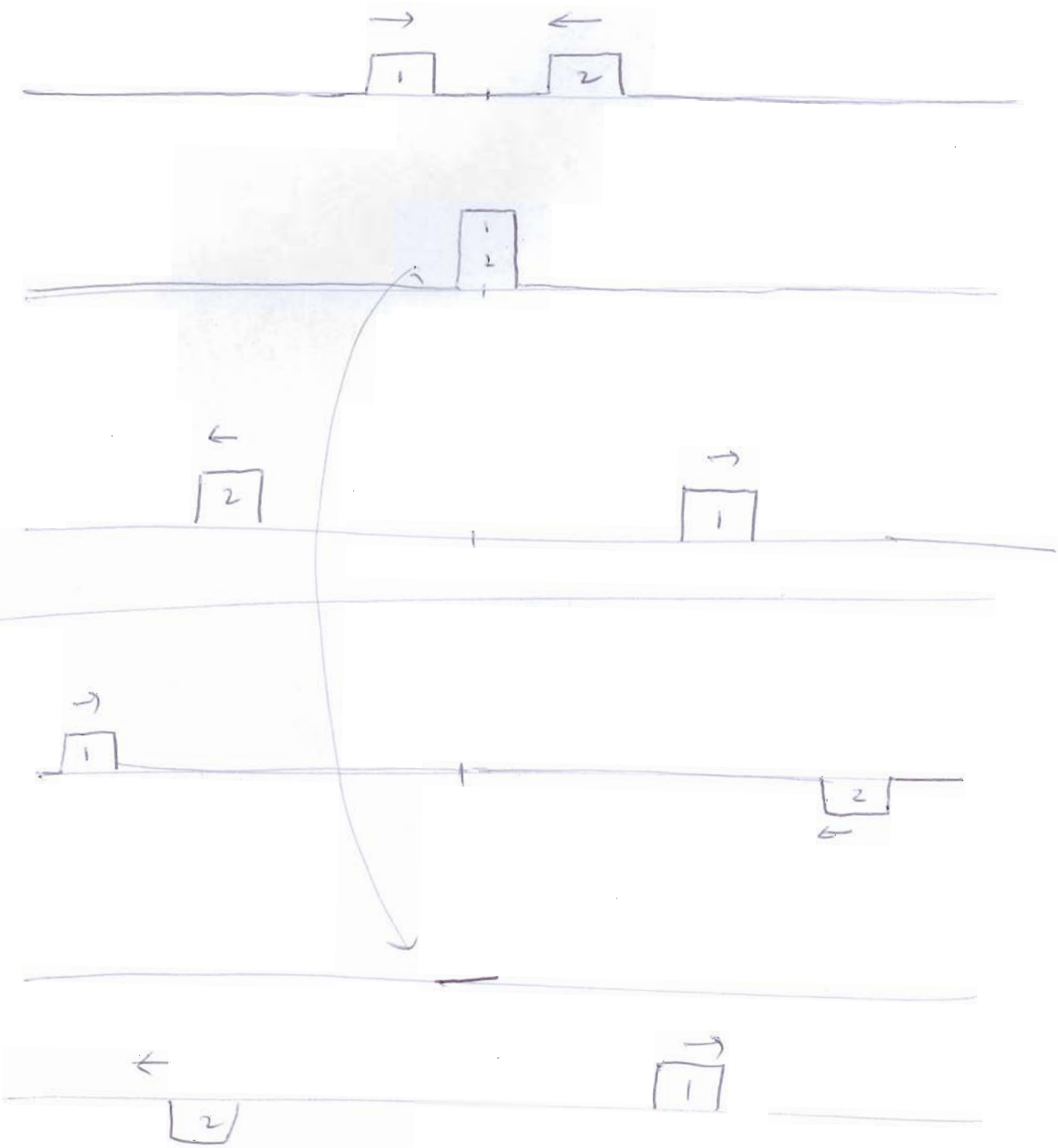
$v = \frac{dx}{dt} = -A\omega \sin \omega t$

$\rightarrow E = \frac{1}{2} \cancel{m} A^2 \underbrace{\omega^2}_{\frac{k}{m}} \sin^2 \omega t + \frac{1}{2} k A^2 \cos^2 \omega t = \frac{1}{2} k A^2 \underbrace{(\sin^2 \omega t + \cos^2 \omega t)}_{1}$

$\longrightarrow$ Total energy $= \frac{1}{2} k A^2$ (Not time-dependent)

Wave superposition

moving these rectangular pulses at same speed



Quantitative expression for wave superposition: or interference.

$$y(x,t) = A\cos(kx - \omega t)$$

At $x=0$ $\quad y(0,t) = A\cos\omega t$

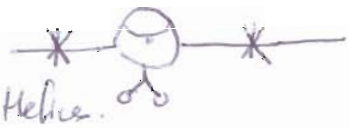
Two waves at same location $x=0$: $\begin{cases} \omega_1 : & y_1(0,t) = A\cos\omega_1 t \\ \omega_2 : & y_2(0,t) = A\cos\omega_2 t \end{cases}$

$$y_1 + y_2 = A(\cos\underbrace{\omega_1 t}_{\alpha} + \cos\underbrace{\omega_2 t}_{\beta}) = 2A\underbrace{\cos\left(\frac{\omega_1-\omega_2}{2}t\right)}_{\text{Beat}}\cos\left(\frac{\omega_1+\omega_2}{2}t\right)$$

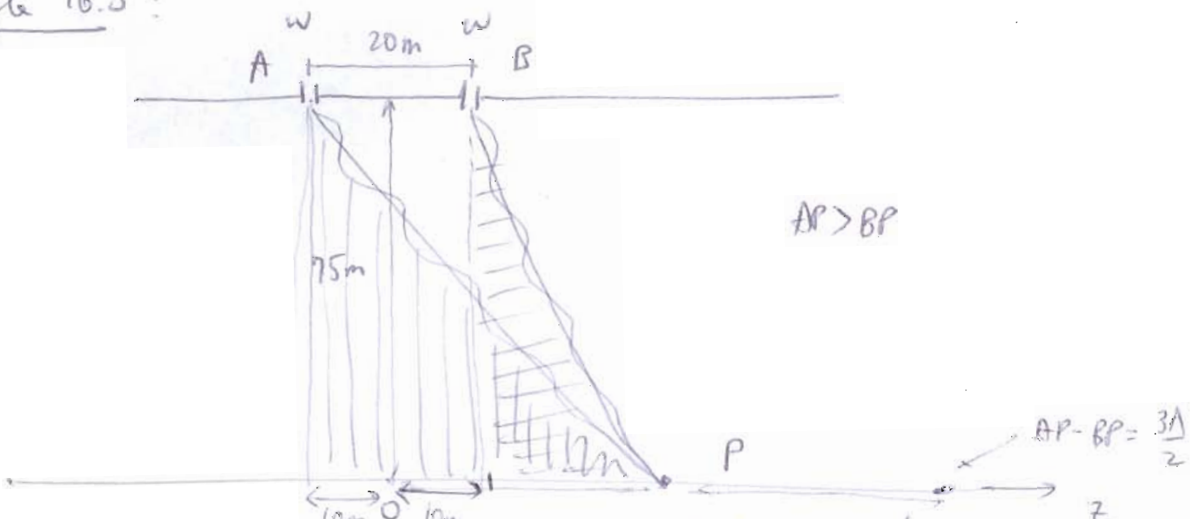
$\cos\alpha + \cos\beta =$
$2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\omega_1 = 7000\ s^{-1}$
$\omega_2 = 7010\ s^{-1}$

Beat phenomenon: bi motor airplane



Example 16.5:



z_1

waves arrive here at different phase: $\boxed{AP - BP = \frac{\lambda}{2}}$

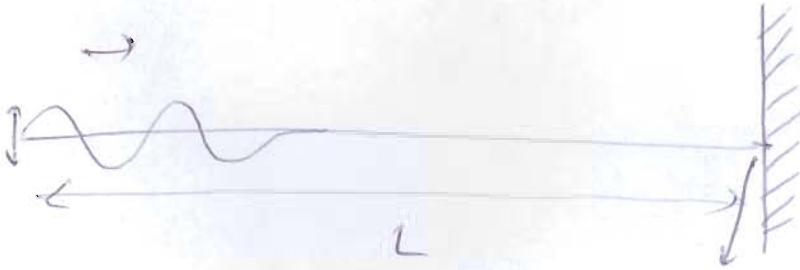
cancellation ↓ calm water.

$$AP - BP = \sqrt{75^2 + (z_1+10)^2} - \sqrt{75^2 + (z_1-10)^2} = \frac{\lambda}{2}$$

for 1st point of calm water

$\rightarrow z_1 = 33\,m.$ $(\lambda = 16\,m)$

Standing waves.



wave is reflected here and comes back left.

$$y_T(x,t) = y_1(x,t) + y_2(x,t)$$

$$= \underbrace{A\cos(kx - wt)}_{\text{propagating along } +x} - \underbrace{A\cos(kx + wt)}_{\text{propagating along } -x}$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$y_T(x,t) = +2A\underbrace{\sin(kx)}_{\text{Nodes when this is } 0}\sin(wt)$$

Nodes when this is 0

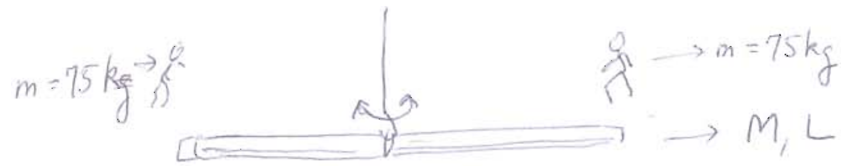
$$\sin kL = 0 \rightarrow kL = n\pi$$

$$\frac{2\pi}{\lambda}L = n\pi$$

$$\rightarrow \boxed{\lambda = \frac{2L}{n}} \quad n = 1, 2, 3, \text{etc.}$$

15.26

Torsional oscillations: thin beam $L = 8m$, M?



Before $\omega = \sqrt{\frac{K}{I}}$ ← torsional constant, ← moment of inertia

After (with the workers on the beam) $\omega' = \sqrt{\frac{K}{I'}}$

$$\omega' = 0.8\omega \rightarrow \left(\frac{\omega'}{\omega}\right)^2 = 0.8^2 = \frac{I}{I'} = \frac{\frac{1}{12}ML^2}{\frac{1}{12}ML^2 + 2m\left(\frac{L}{2}\right)^2}$$

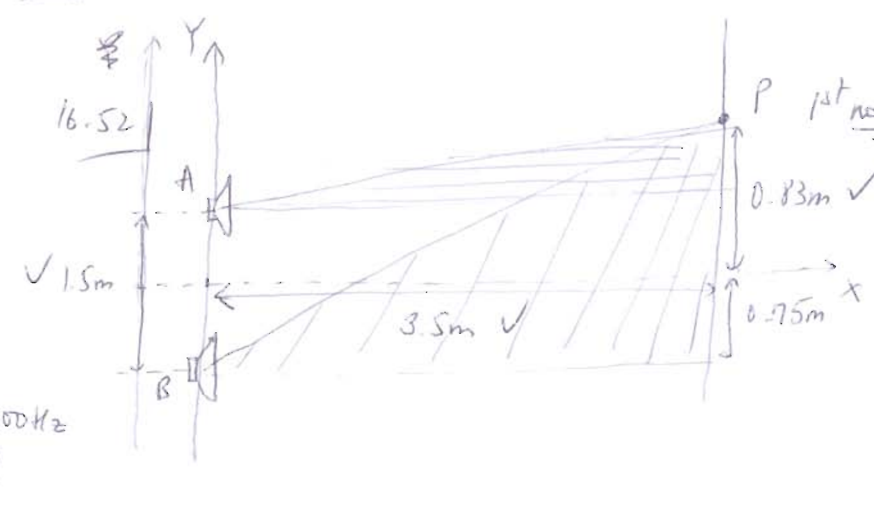
$$0.8^2 = \frac{\frac{1}{12}M}{\frac{1}{12}M + 2\times\frac{75}{4}} \rightarrow \frac{0.8^2}{12}M + 0.8^2\times\frac{75}{2} = \frac{1}{12}M$$

$$\frac{1}{12}M(1-0.8^2) = 0.8^2\times\frac{75}{2}$$

$$M = \frac{12\times0.8^2\times75}{(1-0.8^2)\,2}$$

$$= 800\text{ kg.}$$

16.52



$f = 500\,Hz$

v?

$$\boxed{BP - AP = \frac{\lambda}{2}}$$

(troughs on peaks → cancellation)

Strategy: find λ from location of 1st node P then

$$v = \frac{\lambda}{T} = \lambda f$$

$$BP - AP = \frac{\lambda}{2} \Rightarrow \underbrace{\sqrt{3.5^2 + (0.83+0.75)^2} - \sqrt{3.5^2 + (0.83-0.75)^2}}_{1}$$

$$\rightarrow v = \lambda f = 2 \cdot (1) \cdot 500 = 339 \text{ m/s}$$

16.56

$$y(x,t) = 23 \cos(\underbrace{0.025}_{k = \frac{2\pi}{\lambda}} x - \underbrace{350}_{\omega = \frac{2\pi}{T}} t) \quad \left\{ \begin{array}{l} x, y \text{ in mm} \\ t \text{ in seconds} \end{array} \right.$$

Cable $\mu = 410 \text{ g/m} = 0.41 \text{ kg/m}$

a) Amplitude $= A = 23$ mm

b) Wavelength $= \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.025} = 251$ mm

c) Frequency: $f = \frac{\omega}{2\pi} = \frac{350 \text{ s}^{-1}}{2\pi} = 55.7$ Hz

d) Wave speed: $v = \frac{\lambda}{T} = \lambda f = 0.251 \text{ m} \times 55.7 \text{ s}^{-1}$

$$= 14 \text{ m/s}$$

$$v = \frac{\omega}{k} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} \rightarrow v = \frac{350 \text{ s}^{-1}}{0.025 \text{ mm}^{-1}} = \frac{350}{0.025} \frac{\text{mm}}{\text{s}}$$

$$= \frac{350}{25} \text{ m/s} = 14 \text{ m/s}$$

e) Power carried by wave: $\bar{P} = \frac{1}{2}\mu \omega^2 A^2 v$

$$= \frac{1}{2} \times 0.41 \times 350^2 \times 0.023^2 \times 14$$

$$= 186 \text{ W}$$

15.44

Mass-spring system: $\underset{(\text{Total})}{E} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{const.}$

$$v = \frac{dx}{dt}$$

$$\left(15.4 : \underbrace{m\frac{d^2x}{dt^2}}_{F_{net}} = -kx\right)$$

$$\frac{d}{dt}\left(E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2\right)$$

$$0 = \frac{1}{\cancel{2}}m\cancel{2}v\frac{dv}{dt} + \frac{1}{\cancel{2}}k\cancel{2}x\underbrace{\frac{dx}{dt}}_{v} \rightarrow m\cancel{v}\frac{dv}{dt} = -kx\cancel{v}$$

$$\boxed{m\frac{d^2x}{dt^2} = -kx}$$