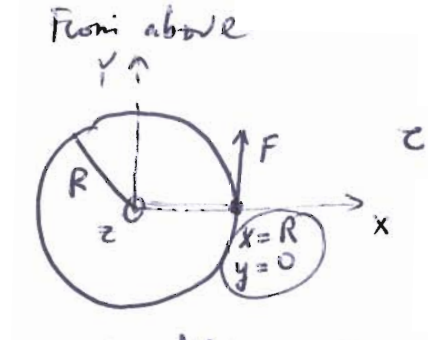
13.8 (a)



$\tau = 1.2\,N.m$; $R = 0.35\,m$

Axis of rotation = z-axis

$\vec{\tau} = \vec{r} \times \vec{F}$: for a rotation along z → we need τ_z ⇒

$\tau_z = 1.2\,N.m$ or ($\vec{r}$ & $\vec{F}$ on the XY plane)

(any z component of $\vec{F}$ will not turn the disk w.r.t to the z axis!)

$\vec{r} = x\hat{i} + y\hat{j}$; $\vec{F} = F_x\hat{i} + F_y\hat{j}$ → $\vec{r}\times\vec{F} = xF_x\underbrace{(\hat{i}\times\hat{i})}_{0} + xF_y\underbrace{(\hat{i}\times\hat{j})}_{1\hat{k}}$

$\vec{r}\times\vec{F} = (xF_y - yF_x)\,\hat{k}$ $\qquad yF_x\underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}} + yF_y\underbrace{(\hat{j}\times\hat{j})}_{0}$

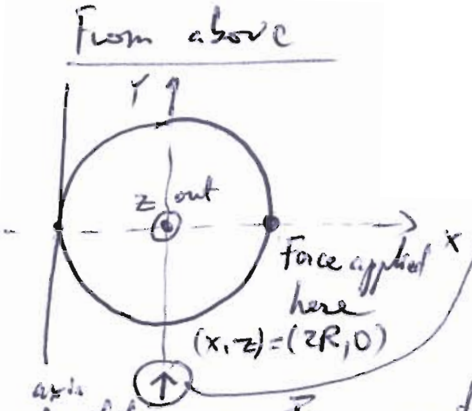
$$\rightarrow \tau_z = xF_y - y\underbrace{F_x}_{0}$$

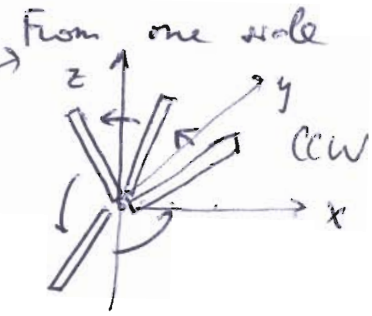
$$= RF_y \rightarrow F_y = \frac{\tau_z}{R} = \frac{1.2\,N.\cancel{m}}{0.35\,\cancel{m}} = 3.43\,N$$

(If rotation is to be CCW → $\vec{F} = 3.43\,\hat{j}$ N

" " " CW → $\vec{F} = -3.43\,\hat{j}$ N)

(b)





axis of rotation → For a rotation along Y-axis → need τ_y

$\tau_y = 1.2\,N.m$ or ($\vec{r}$ & $\vec{F}$ on XZ plane)

$$\vec{r} = x\hat{i} + z\hat{k} \;; \quad \vec{F} = F_x\hat{i} + F_z\hat{k} \rightarrow \vec{r}\times\vec{F} = xF_x\underbrace{(\hat{i}\times\hat{i})}_{0} + xF_z\underbrace{(\hat{i}\times\hat{k})}_{-\hat{j}} + zF_x\underbrace{(\hat{k}\times\hat{i})}_{\hat{j}} + zF_z\underbrace{(\hat{k}\times\hat{k})}_{0}$$

$$\vec{r}\times\vec{F} = (-xF_z + zF_x)\,\hat{j}$$

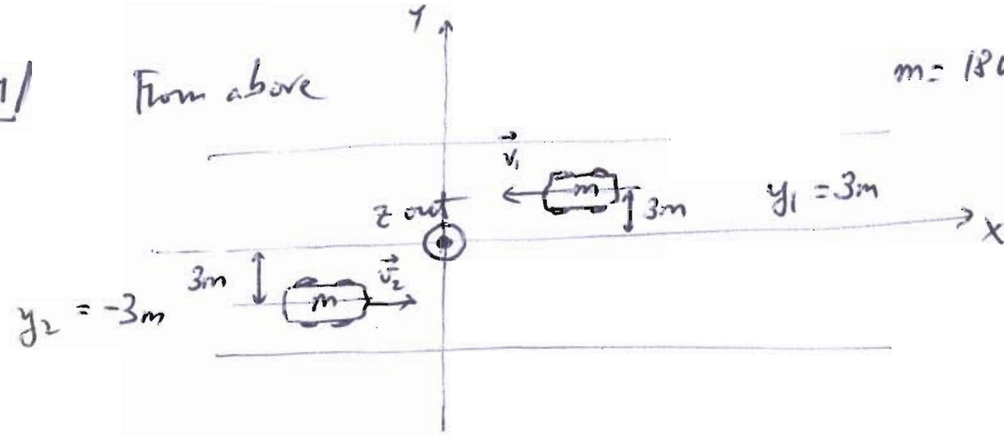
$$\rightarrow \tau_y = -xF_z + \underset{\substack{\downarrow \\ 0}}{z}F_x = -2RF_z \rightarrow F_z = \frac{\tau_y}{-2R} = \frac{1.2\ \mathrm{N\cdot m}}{-0.35\times 2\ \mathrm{m}}$$

$$= -1.71\ \mathrm{N}$$

(If rotation is CCW on XZ plane → $\vec{F} = 1.71\,\hat{k}$ N
CW " " " $\vec{F} = +1.71\,\hat{k}$ N)

13.27/ From above

$m = 1800\,kg$; $v_1 = v_2 = 90\,\frac{km}{h} = v$



$90\,\frac{km}{h} = 25\,m/s$

$$\vec{L}_{Total} = \vec{L}_1 + \vec{L}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2$$

w.r.t. a point along X

$\vec{r}_i, \vec{p}_i$ on XY plane

$\rightarrow \vec{r}_i \times \vec{p}_i$ along z axis

$$\left.\begin{array}{l} \vec{r}_1 = x_1\hat{i} + y_1\hat{j} \\ \vec{p}_1 = -mv\,\hat{i} \end{array}\right\} \quad \vec{r}_1 \times \vec{p}_1 = (x_1\hat{i} + y_1\hat{j}) \times (-mv)\,\hat{i} = -mvy_1 \underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}}$$

$$= mvy_1\,\hat{k}$$

$$\left.\begin{array}{l} \vec{r}_2 = x_2\hat{i} + y_2\hat{j} \\ \vec{p}_2 = mv\,\hat{i} \end{array}\right\} \quad \vec{r}_2 \times \vec{p}_2 = (x_2\hat{i} + y_2\hat{j}) \times (mv)\,\hat{i} = mvy_2 \underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}}$$

$$= -mvy_2\,\hat{k}$$

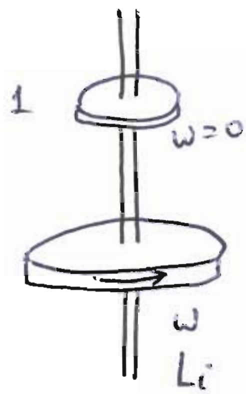
$$\rightarrow \vec{L}_{Total} = mv\,(y_1 - y_2)\,\hat{k} = 1800 \times 25\,(3-(-3))\,\hat{k}$$

$$= 18 \times 15 \times 10^3\,\hat{k}\ \ \frac{kg\,m^2}{s} = Js$$

$$= 27 \times 10^4\ Js$$

13.33

$\vec{\tau}_{net_{ext}} = 0$



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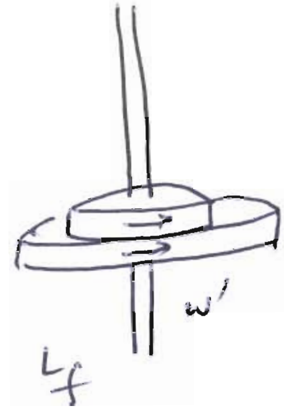


stick together ("inelastic collision")

$R_1 = 2.3\,cm\,;\; m_1 = 0.29\,kg$

$R_2 = 3.5\,cm\,;\; m_2 = 0.44\,kg$

$\omega_i = 180\,rpm$

Disks $= I_{\substack{central \\ axis}} = \frac{1}{2} m R^2$

(a) $L_i = I_2 \omega \quad = \quad L_f = (I_1 + I_2)\omega'$

$$\omega' = \frac{I_2}{I_1 + I_2}\,\omega = \frac{1}{\frac{I_1}{I_2} + 1}\,\omega = \frac{1}{\frac{m_1 R_1^2}{m_2 R_2^2} + 1}\, 180\,rpm$$

$\omega' = 142\,rpm.$

(b) Fraction of initial KE lost into friction.

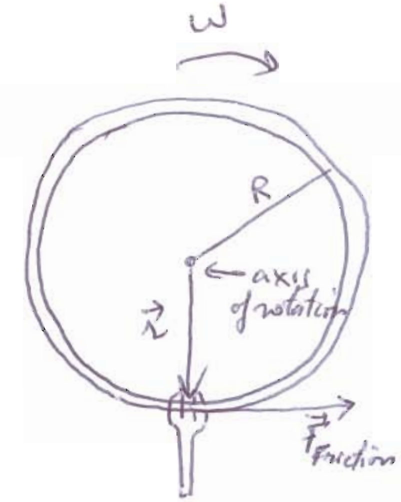
$$\frac{KE_i - KE_f}{KE_i} = \frac{\cancel{\frac{1}{2}}(\cancel{\frac{1}{2}} m_2 R_2^2)\omega^2 - \cancel{\frac{1}{2}}\left[\cancel{\frac{1}{2}} m_1 R_1^2 + \cancel{\frac{1}{2}} m_2 R_2^2\right]\omega'^2}{\cancel{\frac{1}{2}}\cancel{\frac{1}{2}} m_2 R_2^2 \omega^2}$$

Here $KE = \frac{1}{2} I \omega^2$ (just rotation, no translation of CM for neither (i) nor (f) situations)

$$= 1 - \left[\frac{m_1 R_1^2}{m_2 R_2^2} + 1\right]\left(\frac{\omega'}{\omega}\right)^2$$

$$= 1 - \left[\frac{0.29 \times 2.3^2}{0.44 \times 3.5^2} + 1\right]\left(\frac{142}{180}\right)^2 = 0.21 \text{ or } 21\%$$

12.43



$I = mR^2$

$R = 0.33m$, $\omega_0 = 230$ rpm

$m = 1.9kg$ (at rim) or at R from axis of rotation.

$\Delta t = 3.1s$ with $F_{Normal} = 2.7N$

$\mu_{wrench-tire} = 0.46$

ω_f ?

→ Bike wheel will slow down b/c friction b/w wrench and tire: $F_{friction} = \mu N$ (against motion)

$F_{friction}$ provides a torque w.r.t. axis of rotation:

$$\vec{\tau}_{friction} = \vec{r} \times \vec{F}_{friction} =$$

$$\tau_{friction} = R\mu F_{Normal}$$

Analog of Newton's law for rotation. $\tau = I\alpha$

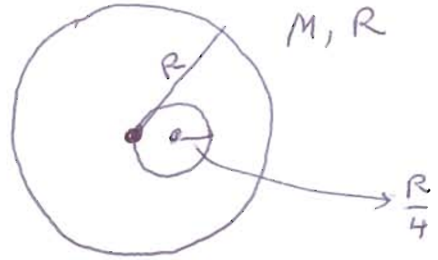
$$\rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\cancel{R}\mu F_N}{mR^{\cancel{2}}}$$

In our case, it's an angular deceleration: $\alpha = -\frac{\mu F_N}{mR}$

$$\rightarrow \quad \omega_f = \omega_0 + \underbrace{\alpha \Delta t}_{\frac{rad}{s}} = 230 \text{ rpm} - \underbrace{\frac{0.46 \times 2.7 \times 3.1}{1.9 \times 0.33} \frac{\cancel{rad}}{\cancel{s}} \cdot \frac{60\cancel{s}}{1 min} \cdot \frac{1 rev}{2\pi \cancel{rad}}}_{58.6 \quad rpm}$$

$$= 171 \text{ rpm}.$$

12.63



$I_{\substack{\text{central}\\ \text{axis}}}$?

$$I_{c.a.} = \underset{\downarrow \atop \frac{1}{2}MR^2}{I} - I_{Hole}$$

$$+ \; m\left(\frac{R}{4}\right)^2 = \frac{1}{2}m\left(\frac{R}{4}\right)^2 + m\left(\frac{R}{4}\right)^2 = \underbrace{\frac{3}{2}m\left(\frac{R}{4}\right)^2}$$

Now find m ?

whole disk: πR^2

small disk: $\pi \left(\frac{R}{4}\right)^2$

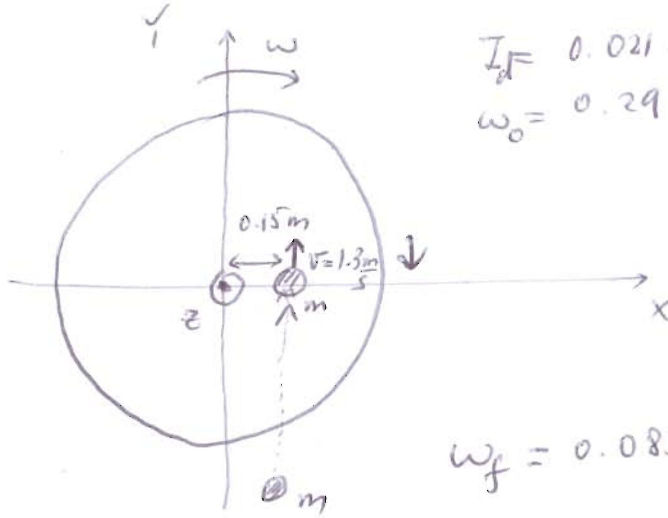
} if mass is uniformly distributed

$$\rightarrow \; m = \frac{\cancel{\pi}\left(\frac{R}{4}\right)^2}{\cancel{\pi}\,\cancel{R^2}} M = \frac{M}{16}$$

$$\rightarrow I_{\substack{remain\\ c.a.}} = \frac{1}{2}MR^2 - \frac{3}{2}\,\frac{M}{16}\,\frac{R^2}{16} = \frac{1}{2}MR^2\underbrace{\left(1 - \frac{3}{16^2}\right)}_{0.988}$$

$$= 0.494\; MR^2.$$

13.51



$I_d = 0.021\ kg\,m^2$
$\omega_0 = 0.29\ rad/s$

(radius along x, v_{clay} is ⊥ or along y)

$\omega_f = 0.085\ rad/s$

What is m?

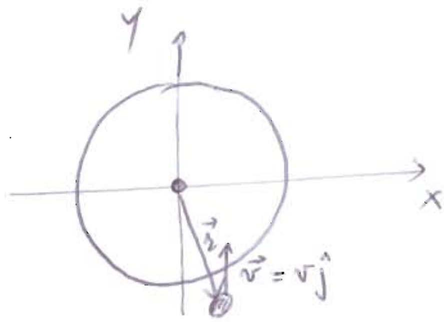
No net torque on system disk/clay ball $\rightarrow \vec{L}_0 = \vec{L}_f$

$\vec{L}_0$: clay ball has not landed

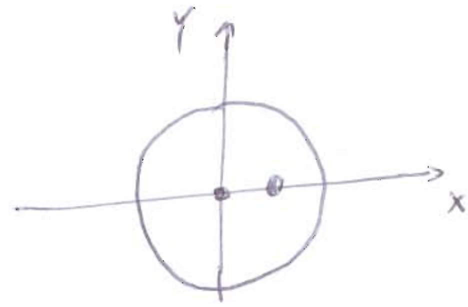
$\vec{L}_f$: disk & clay ball rotating together

higher inertia & lower ω_f

Initial



Final



$$\vec{L}_0 = I_d\vec{\omega}_0 + \underbrace{\vec{r}\times m\vec{v}}_{\underbrace{(x\hat{i}+y\hat{j})\times mv\hat{j}}_{xmv\underbrace{(\hat{i}\times\hat{j})}_{\hat{k}}}}$$

$I_d\omega_0(-\hat{k})$

$= (-I_d\omega_0 + xmv)\hat{k}$

$$= \qquad \vec{L}_f = I_{d+ball}\,\vec{\omega}_f$$

$\vec{\omega}_f = \omega_f(-\hat{k})$

$= (I_d + mx^2)\,\omega_f\,(-\hat{k})$

$= -(I_d + mx^2)\,\omega_f\,\hat{k}$

$$-I_d\omega_0 + xmv = -I_d\omega_f - mx^2\omega_f$$

$$I_d(-\omega_0 + \omega_f) = m(-xv - x^2\omega_f) = mx(-v - x\omega_f)$$

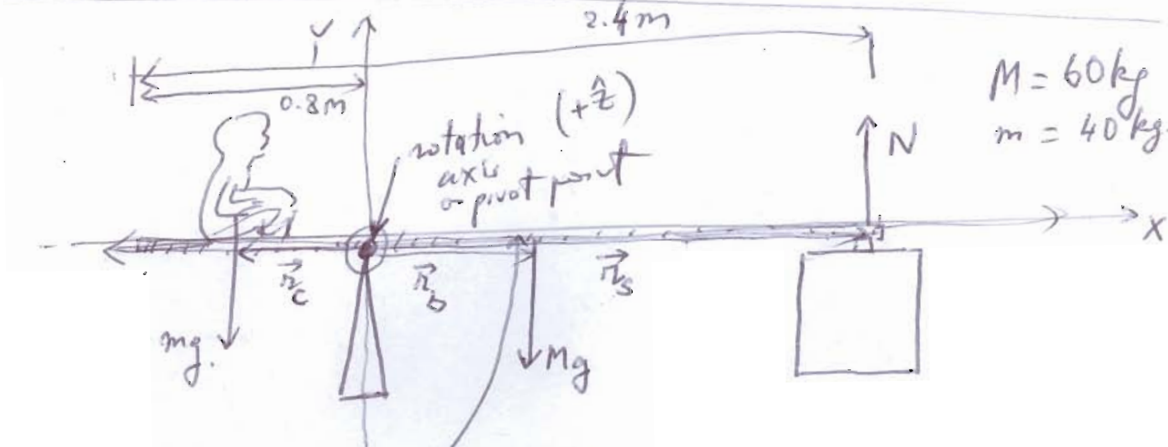
$$m = \frac{I_d(-\omega_0 + \omega_f)}{x(-v - x\omega_f)} = \frac{0.021(-0.29 + 0.085)}{0.15(-1.3 - 0.15 \times 0.085)} = 0.0218 \text{ kg}$$

or $\boxed{m = \cancel{40.8\,g}\ \ 21.8\,g}$

Ch. 14 Static Equilibrium:

1) $\sum_i \vec{F}_i = 0$; 2) $\sum_i \vec{\tau}_i = 0$

14.13



$M = 60$ kg
$m = 40$ kg.

Focusing on the bar. : 3 forces mg, Mg down and N up.

(a)

$$\sum_i \vec{\tau}_i = 0 = \vec{\tau}_c + \vec{\tau}_s + \vec{\tau}_b$$

We know:
$r_s = 1.6$ m
$r_b = 0.4$ m
$N = 100$ N

$$0 = r_c mg\,\hat{k} + r_s N\,\hat{k} - r_b Mg\,\hat{k}$$

$$r_c = \frac{r_b Mg - r_s N}{mg} = \frac{0.4 \times 60 \times 9.81 - 1.6 \times 100}{40 \times 9.81} \text{ m}$$

$r_c = 0.19$ m. or $(0.8 - 0.19 = 0.61$ m from left end!

(b) $N = 300$ N $\rightarrow$ $r_c = -0.62$ m $\rightarrow$ or $0.8 - (-0.62) = 1.42$ m from left end