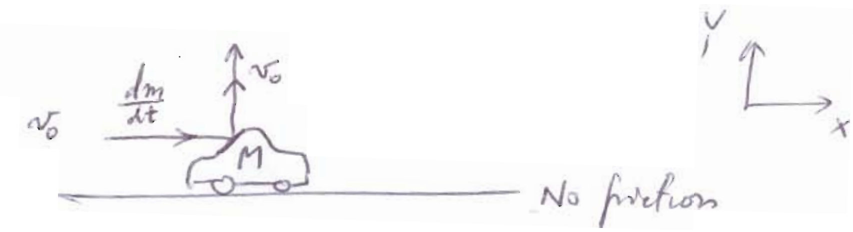
10.32/



a) Acceleration of the car?

No net external force on system (water & car):

$$\vec{F}_{net} = 0 = \frac{d\vec{P}_{Total}}{dt} \rightarrow \vec{P}_{Total} = \text{conserved}.$$

$$\underset{\substack{\downarrow \\ \text{water coming} \\ \text{car at rest}}}{\vec{P}_1} = \underset{\substack{\downarrow \\ \text{water leaving} \\ \text{car moving forward}}}{\vec{P}_2}$$

$$\frac{d}{dt}\left[m v_0 \hat{i} + M \cdot 0 = m v_0 \hat{j} + M \vec{v}_c \right]$$

$$\frac{dm}{dt} v_0 \hat{i} = \frac{dm}{dt} v_0 \hat{j} + M \underbrace{\frac{d\vec{v}_c}{dt}}_{\vec{a}_c} \rightarrow \vec{a}_c = \frac{1}{M}\frac{dm}{dt} v_0 (\hat{i} - \hat{j})$$

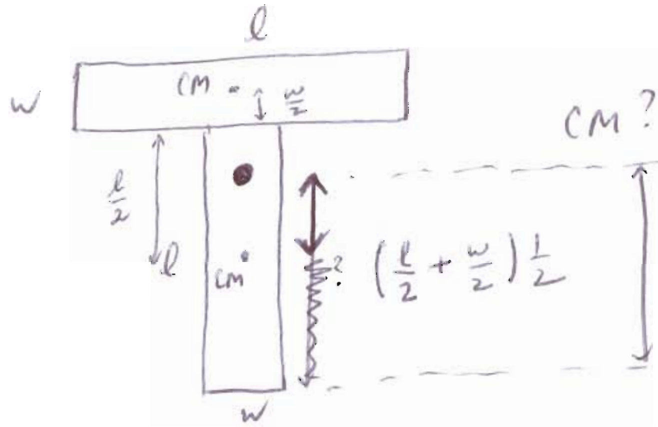
→ Car will not move downward, just forward:

$$\boxed{a_x = \frac{1}{M}\frac{dm}{dt} v_0}$$

b) What is the max speed reached by car?
(Source of jet of water is fixed)

When car reaches v_0: no further push by water
→ no further acceleration.

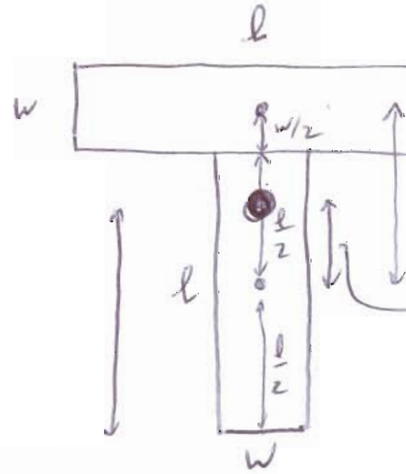
10.12 /



$\left(\frac{l}{2}+\frac{w}{2}\right)\frac{1}{2}$

CM ?

$\frac{l+w}{4} + \frac{l}{2} = \frac{3l}{4} + \frac{w}{4}$



$\frac{w}{2} + \frac{l}{2}$ (distance b/w two CM's)

$\frac{1}{2}\left(\frac{w}{2}+\frac{l}{2}\right)$

$\frac{1}{4}(w+l) + \frac{l}{2}$

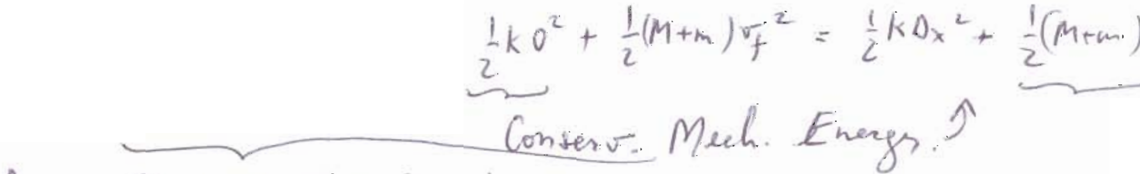
$= \frac{3l+w}{4}$

↳ Total CM w.r.t. bottom.

10.2a / $k = 3.2\times10^5\ N/m$ $M = 11000\,kg$ $m = 9400\,kg$.

They stay together after collision.

a) Compression of spring ?



$$\underbrace{\tfrac{1}{2}k0^2 + \tfrac{1}{2}(M+m)v_f^2}_{} = \tfrac{1}{2}k\Delta x^2 + \tfrac{1}{2}(M+m)0^2$$

Conserv. Mech. Energy

Conser. of momentum: $M\cdot 0 + m\,8.5 = (M+m)\,v_f$

$$v_f = \frac{m}{M+m} 8.5 = \frac{9400}{11000+9400} 8.5 \cong 3.92 \ m/s$$

$$\rightarrow \quad \Delta x = \sqrt{\frac{M+m}{K}} \, v_f = \sqrt{\frac{20400}{3.2 \times 10^5}} \, 3.92 = 0.99 \, m$$

Ch. 11 Collisions b/w two objects

Collisions: $\vec{F}_{net,\,external} = 0 = \frac{d\vec{p}}{dt} \rightarrow \vec{P}_{Total} = $ conserved.

Internal force: Impulse $\vec{I} = \int_1^2 \vec{F}\,dt = \int_1^2 \frac{d\vec{p}}{dt}\,dt = \vec{P}_2 - \vec{P}_1$

$\overline{F} = \frac{\Delta P}{\Delta t} = \frac{I}{\Delta t}$ (av. force in collisions) internal

Unit is same as for $\vec{p}$ (kg m/s or N.s)

Inelastic collision: two objects stick together after collision $\underbrace{}_{\vec{v}_{1f} = \vec{v}_{2f} \equiv \vec{v}_f}$

$\boxed{m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1+m_2)\vec{v}_f}$ (KE of these two is NOT conserved, some converted to modify their internal structure)

Elastic collision: KE of both objects is conserved.

$$\boxed{\begin{aligned} m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \\ \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 &= \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \end{aligned}}$$

1D: v_{1f} & v_{2f} & 2 eqs.

2D: $v_{1fx}, v_{1fy}, v_{2fx}, v_{2fy}$ & 3 eqs. → need some extra info.

1D ↓ (no need for arrows)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1+m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} + \frac{m_2 - m_1}{m_1+m_2} v_{2i}$$

1D

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \rightarrow \text{(a)}\; m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \rightarrow \text{(b)}\; m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$\frac{\text{(b)}}{\text{(a)}}: \boxed{v_{1i} + v_{1f} = v_{2i} + v_{2f}} \leftarrow$$

Elastic 1D collision

or $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

10.47

Before

$m_1 = 50kg$ $m_2 = 70kg$

$v_1 = 0$ $v_2 = 0$

After.

v_{1f} Δx ? $v_{2f} = 2.8 m/s$

after 3s

$F_{net\ external} = 0$

→ Conserv Mom.

No friction.

$$\Delta x = \underbrace{(v_{2f} - v_{1f})}_{v_{relative_f}} \cdot 3s$$

$$m_1 0 + m_2 0 = 0 = 50 \times v_{1f} + 70 \times 2.8$$

$$\rightarrow v_{1f} = -\frac{70 \times 2.8}{50} m/s = -3.92 m/s$$

$$\Delta x = (2.8 - (-3.92)) \cdot 3s = 20.16 m.$$

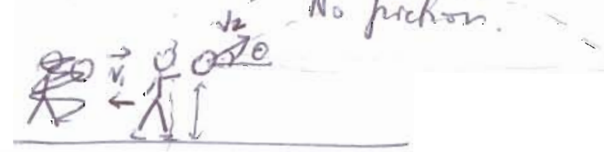
10.61

Before

m_2

m_1

After.



$= 12 m/s$

No friction.

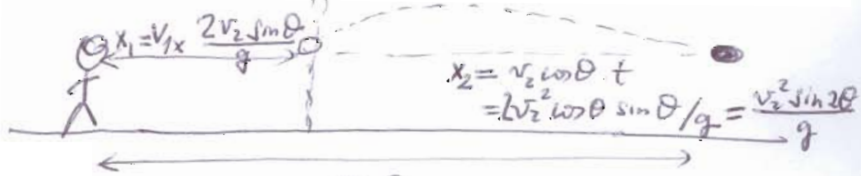
$\vec{v}_1$ v_2 θ

$$0 = v_2 \sin\theta - g t_{up} \rightarrow t_{up} = \frac{v_2 \sin\theta}{g}$$

$$0 = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$\boxed{0 = m_1 V_{1x} + m_2 V_{2x}}$$

$$V_{2x} = -\frac{m_1}{m_2} V_{1x} \text{ or } V_{1x} = -\frac{m_2}{m_1} V_{2x}$$



$x_1 = V_{1x} \frac{2 v_2 \sin\theta}{g}$

$x_2 = v_2 \cos\theta\ t$

$= 2 v_2^2 \cos\theta \sin\theta / g = \frac{v_2^2 \sin 2\theta}{g}$

15.2m

$$x_1 + x_2 = \boxed{15.2 m = -V_{1x} \frac{2 V_{2y}}{g} + \frac{2}{g} V_{2x} V_{2y}}$$

$$\left.\begin{aligned} V_{2x} &= v_2 \cos\theta \\ V_{2y} &= v_2 \sin\theta \end{aligned}\right\} \rightarrow v_{2y} = v_{2x} \tan\theta$$

$$\frac{g}{2}(15.2 m) = V_{1x} V_{2x} \tan\theta + v_{2x}^2 \tan\theta$$

$$= (-V_{1x} + V_{2x}) v_{2x} \tan\theta$$

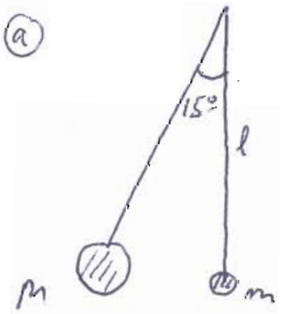
$$\boxed{\frac{9.81 \times 15.2}{2}} = \left(+ \frac{m_2}{m_1} + 1 \right) v_{2x}^2 \tan\theta = \left(1 + \frac{4.5}{65}\right) 12^2 \cos^2\theta \frac{\sin\theta}{\cos\theta}$$

$$= \boxed{\left(1 + \frac{4.5}{65}\right) 144 \underbrace{\cos\theta \sin\theta}_{\frac{1}{2}\sin 2\theta}}$$

$$\rightarrow \underbrace{\frac{9.81 \times 15.2}{\cancel{2}} \cancel{2} \frac{1}{\left(1 + \frac{4.5}{65}\right)144}}_{0.968} = \sin 2\theta$$

$$\rightarrow \theta = 75.57/2 = 37.78°$$

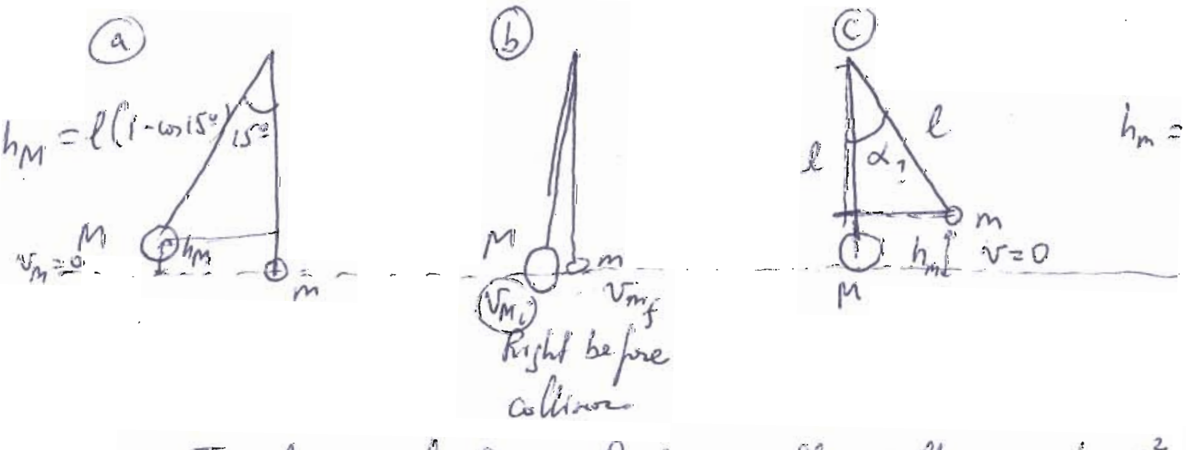
(11.45) (a)



$M = 0.39\ kg$ $\quad m = 0.14\ kg$

$l = 0.5m$

<u>Elastic collision</u> → max angle by smaller bob ?



$h_m = l - l\cos\alpha = l(1-\cos\alpha)$

1) To get α: find speed of m after collision $\frac{1}{2} m v_{m_f}^2 = m g h_m = m g l (1-\cos\alpha)$

2) Use elastic collision to get v_m : 1D formula:

$$v_{m_f} = \frac{2M}{M+m} v_{M_i} + \frac{m-M}{m+M} \underbrace{v_{m_i}}_{0} = \frac{2 \times 0.39}{0.39 + 0.14} v_{M_i}$$

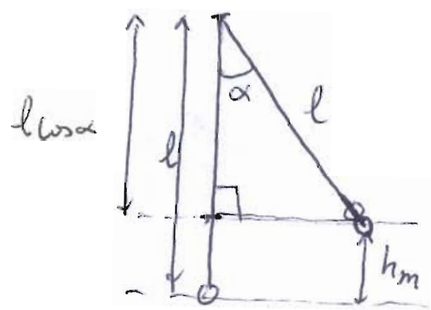
3) $\cancel{M} g\, l (1-\cos 15°) = \frac{1}{2} \cancel{M} v_{M_i}^2$

3) $\rightarrow \quad v_{m_i} = \sqrt{2gl(1-\cos 15^\circ)} = \sqrt{2\times 9.81\times 0.5\,(1-\cos 15^\circ)} = 0.58\,m/s$

2) $\quad v_{mf} = \dfrac{2\times 0.39}{0.53}\,0.58 = 0.85\,m/s$

1) $\quad 1-\cos\alpha = \dfrac{V_{mf}^2}{2gl} \rightarrow \cos\alpha = 1-\dfrac{V_{mf}^2}{2gl} = 1-\dfrac{0.85^2}{2\times 9.81\times 0.5} = 0.9264$

$\rightarrow \alpha = \cos^{-1} 0.9264 = 22.11^\circ$



$h_m = l(1-\cos\alpha)$

(11.54)

$I = \bar{F}.\Delta t$

Snow ball to fore head
- During $54\,ms = \Delta t$
- $\begin{cases} v_{ball} = 13\,m/s \longrightarrow 0 \\ m_{ball} = 0.33\,kg \end{cases}$

$\vec{F} = \dfrac{d\vec{p}}{dt} \rightarrow \vec{F} = \dfrac{\Delta p}{\Delta t} = \dfrac{m(V_i - V_f)}{\Delta t} = \dfrac{0.33\times(13-0)}{54\times 10^{-3}}$

$= 79.4\,N$

(11.44) $\quad m_1 = 3.2\,kg;\ v_{1i} = 1\,m/s \qquad m_2 = 0.35\,kg;\ v_{2i} = 0$ Elastic Collision.

v_{1f} & v_{2f} ?
Direction of $\vec{v}_{1f}$?
The final angle for m_2 is given : now we have 3 unknowns and 3 equations !

Conservation of momentum

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad \begin{cases} x: & m_1 \cdot 1 + m_2 \cdot 0 = m_1 v_{1fx} + m_2 v_{2fx} \\ y: & 0 = m_1 v_{1fy} + m_2 v_{2fy} \end{cases}$$

$$\rightarrow \begin{cases} m_1 = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos 45° & (1) \\ 0 = m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin 45° & (2) \end{cases}$$

Conservation K.E.

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\cancel{\tfrac{1}{2}} m_1 + 0 = \cancel{\tfrac{1}{2}} m_1 v_{1f}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2f}^2 \quad (3)$$

$\cos 45 = \sin 45$!

$$(1) - (2) \rightarrow \cancel{m_1} = \cancel{m_1} v_{1f} (\cos\theta_1 - \sin\theta_1)$$

$$v_{1f} = \frac{1}{\cos\theta_1 - \sin\theta_1}$$

$$\cos^2\theta_1 + \sin^2\theta_1 = 1$$

$$(1): \quad m_1 - m_2 v_{2f} \frac{1}{\sqrt{2}} = m_1 v_{1f} \cos\theta_1$$

$$(2) \quad - m_2 v_{2f} \frac{1}{\sqrt{2}} = m_1 v_{1f} \sin\theta_1$$

$$(1)^2 + (2)^2 \Rightarrow m_1^2 + \frac{1}{2} m_2^2 v_{2f}^2 - m_1 m_2 v_{2f} \sqrt{2} + \frac{1}{2} m_2^2 v_{2f}^2 = m_1^2 v_{1f}^2$$

$$\rightarrow (a) \quad \boxed{m_1^2 - m_1 m_2 v_{2f} \sqrt{2} = m_1^2 v_{1f}^2 - m_2^2 v_{2f}^2}$$

$$(3) \rightarrow \quad m_1 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

$$(b) \quad \boxed{m_1^2 = m_1^2 v_{1f}^2 + m_1 m_2 v_{2f}^2}$$

$$(a) - (b) \quad . \quad + m_1 \cancel{m_2} v_{2f} \sqrt{2} = + m_2^{\cancel{2}} v_{2f}^2 + m_1 \cancel{m_2} v_{2f}^2$$

$$m_1 v_{2f} \sqrt{2} = m_2 v_{2f}^2 + m_1 v_{2f}^2$$

$\rightarrow \quad (m_1+m_2)\, v_{2f}^{\cancel{2}} = m_1 \cancel{v_{2f}} \sqrt{2}$

$$v_{2f} = \frac{m_1\sqrt{2}}{m_1+m_2} = \frac{3.2\sqrt{2}}{3.2+0.35} = \boxed{1.27 \text{ m/s}}$$

3 $\rightarrow \quad v_{1f}^2 = \frac{1}{m_1}\left(m_1 - m_2 v_{2f}^2\right) = 1 - \frac{m_2}{m_1} v_{2f}^2$

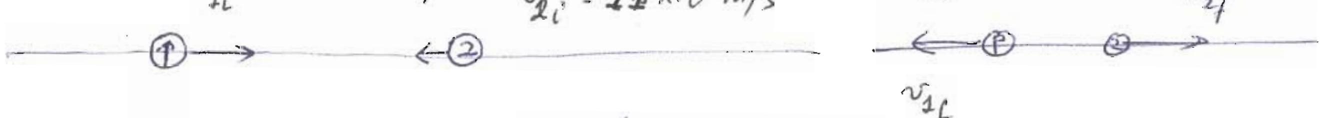
$$v_{1f} = \sqrt{1-\left(\frac{0.35}{3.2}\, 1.27^2\right)} = \boxed{0.908 \text{ m/s}}$$

2 $\quad \sin\theta_1 = -\frac{m_2}{m_1}\frac{v_{2f}}{v_{1f}}\frac{1}{\sqrt{2}} = -\frac{0.35}{3.2}\,\frac{1.27}{0.908}\,\frac{1}{\sqrt{2}}$

$$\boxed{\theta_1 = -6.21^\circ}$$

11.35 ID elastic collision $\quad m_1 = m_2$

$v_{1i} = 6.9\times10^6$ m/s $\quad v_{2i} = 11\times10^6$ m/s $\quad v_{1f} = ?\quad v_{2f}\ ?$



$$v_{1f} = \frac{2m}{2m} v_{2i} = 11\times10^6 \text{ m/s (left)}$$

$$v_{2f} = \frac{2m}{2m} v_{1i} = 6.9\times10^6 \text{ m/s (right)}$$

11.20 | Baseball is caught → inelastic collision!

a) Conserv. of momentum: $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$, with $v_{1i} = 0$

$m_1 = 70\,kg$

$m_2 = 0.15\,kg$

$m_1;\ V_{1i} = 0 \quad m_2,\ v_{2i} = 23\,m/s$

$$v_f = \frac{m_2}{m_1 + m_2} v_{2i} = \frac{0.15}{70.15}\,23\,m/s = 0.0492\,m/s$$

b) $\Delta t = 36 \times 10^{-3}\,s$

$$\overline{F} = \frac{\Delta p_{ball}}{\Delta t} = \frac{m_2 (23 - 0.0492)}{36 \times 10^{-3}} = 95.6\,N$$

11.30 | $m_1 = 1300\,kg$; $v_{1i} = 10\,km/h = \frac{10}{3.6}\,m/s = 2.7\,m/s$ ‖ $m_2 = 1600\,kg$

$v_{2i} = 6.6\,\frac{km}{h} = 1.8\,m/s$

$k = 28000\,\frac{N}{m}$

Δx?

Inelastic collision b/c of spring (which will take away part of initial KE → KE not conserved → can't be elastic!)

Find the KE stolen by the spring bumper → then Δx from

$$\frac{1}{2} k \Delta x^2 = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 - \frac{1}{2}(m_1 + m_2) v_f^2$$

?

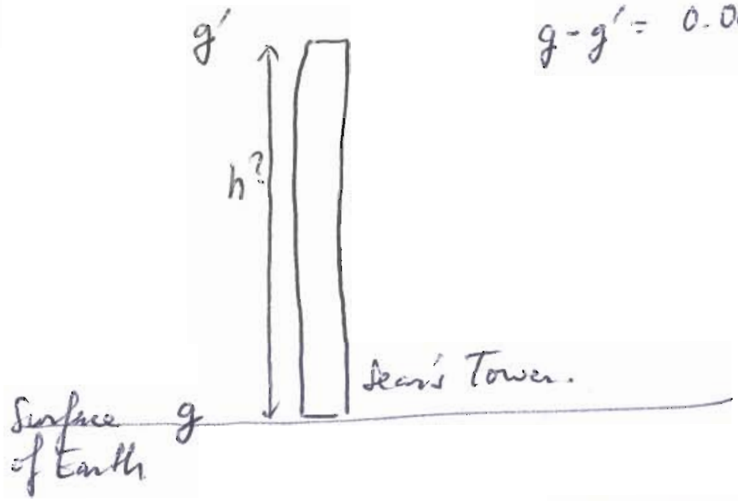
$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

$$v_f = \frac{1300 \times 2.7 + 1600 \times 1.8}{2900} = 2.203\,m/s$$

$\rightarrow \frac{1}{2} 28000\,\Delta x^2 = 1300 \times 2.7^2 + 1600 \times 1.8^2 - 2900 \times 2.203^2$

$\Delta x = 0.15\,m$

9.9



$$g - g' = 0.00136 \ m/s^2$$

On surface of Earth : $F = \boxed{G\dfrac{Mm}{R_E^2} = mg}$

Top Sears Tower : $F' = \boxed{G\dfrac{Mm}{(R_E+h)^2} = mg'}$

$$\rightarrow g - g' = GM\left[\frac{1}{R_E^2} - \frac{1}{(R_E+h)^2}\right]$$

$$g - g' = GM\left[\frac{(R_E+h)^2 - R_E^2}{R_E^2(R_E+h)^2}\right] = GM\left[\frac{\cancel{R_E^2} + h^2 + 2R_E h - \cancel{R_E^2}}{R_E^2(R_E+h)^2}\right]$$

$$= GM\frac{h(h+2R_E)}{R_E^2(R_E+h)^2} \simeq GM\frac{h(2R_E)}{R_E^4} = \underbrace{\frac{GM}{R_E^2}}_{g}\frac{2h}{R_E}$$

$$h + 2R_E \cong 2R_E$$
$$R_E + h \simeq R_E$$

$R_E = 6370000 \ m$

$$\rightarrow g - g' = g\frac{2h}{R_E} \rightarrow h = \frac{g-g'}{g}\frac{R_E}{2} = \frac{0.00136}{9.81}\frac{6370000}{2}$$

$$= 442 \ m.$$