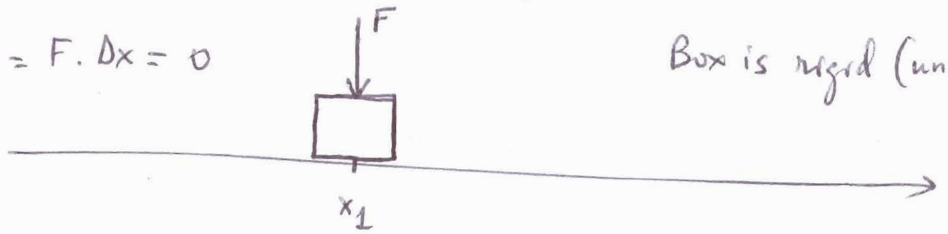
Ch. 7 Work, Energy, Power

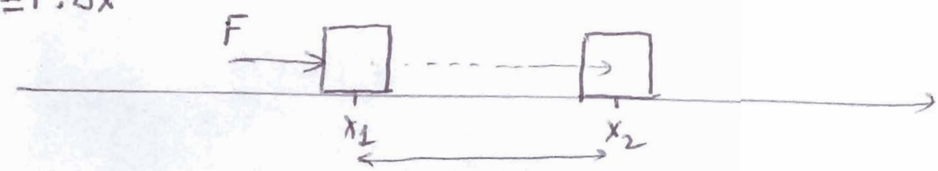
Work:

Work = $F \cdot \Delta x = 0$



Box is rigid (uncompressible)

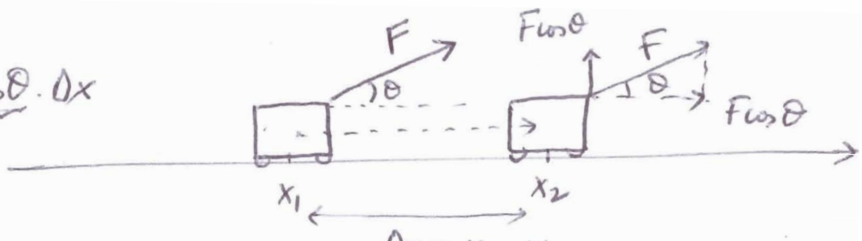
Work $= F \cdot \Delta x$



$\Delta x = x_2 - x_1$

Unit: (SI): N.m = J (Joule)

Work $= F\cos\theta \cdot \Delta x$



$\Delta x = x_2 - x_1$

Along Y direction Work $= F\sin\theta \cdot \underbrace{\Delta y}_{0} = 0$

More generally:

(Takes care of $\cos\theta$)

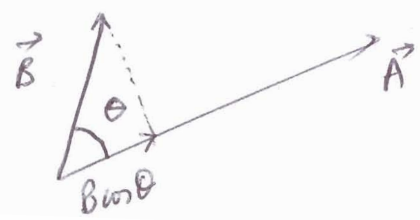
Work done $= \vec{F} \cdot \Delta\vec{r}$

Force applied

Scalar product b/w two vectors

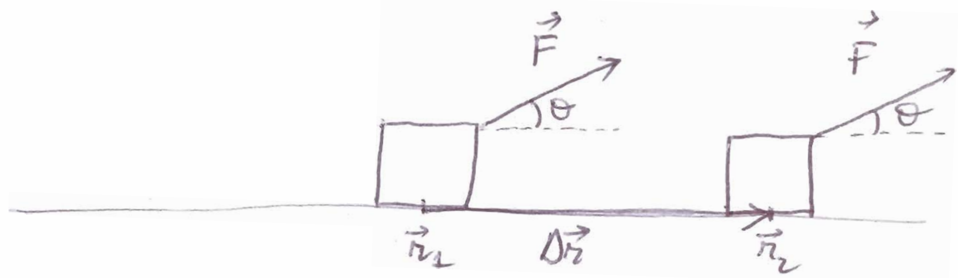
displacement vector

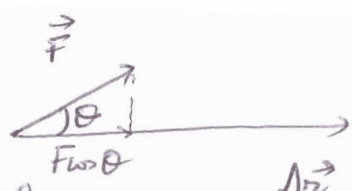
Scalar product b/w two vectors $\vec{A}$ & $\vec{B}$:
(a number)



$$\vec{A} \cdot \vec{B} \equiv \underbrace{AB\cos\theta}$$

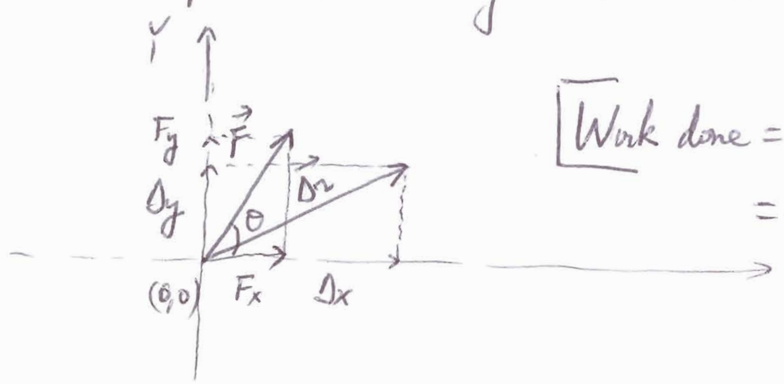
Projection of $\vec{B}$ onto the direction of $\vec{A}$





$$\vec{F} \cdot \Delta\vec{r} = F\cos\theta\, \Delta r$$

This displacement is along x-axis. But scalar product can handle displacement along ANY direction :



$$\boxed{\text{Work done} = \vec{F} \cdot \Delta\vec{r}}$$

$$= (F\cos\theta\, \hat{i} + F\sin\theta\, \hat{j}) \cdot (\Delta x\, \hat{i} + \Delta y\, \hat{j})$$

$$= F\cos\theta\, \Delta x + F\sin\theta\, \Delta y$$

a number

$\hat{i} \cdot \hat{i} = 1 \times 1 \times \cos 0 = 1$; $\hat{i} \cdot \hat{j} = 1 \times 1 \times \cos 90^\circ = 0$

$\hat{j} \cdot \hat{j} = 1$; $\hat{j} \cdot \hat{i} = 0$

$$W = \vec{F} \cdot \Delta\vec{r}$$

For $\boxed{\text{varying force}}$ with position we need to generalize this a little bit further:

$$\boxed{W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot \underbrace{d\vec{r}}_{\text{infinitesimal displacement vector}}}$$

$\boxed{\text{Work done in stretching a spring}}$:

Hooke's law: $\underset{\text{by spring}}{F} = -kx$: this force changes with displacement.

$$\underset{\text{by person}}{F} = kx$$

$$\rightarrow W = \int_0^x F\,dx = \int_0^x kx\,dx = K \underbrace{\int_0^x x\,dx}_{\left[\frac{x^2}{2}\right]_0^x = \frac{x^2}{2}} = \frac{1}{2}kx^2$$

$$\boxed{\int_a^b x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b}$$

$\boxed{\text{Kinetic Energy:}}$ using Newton's law: $\vec{F}_{net} = \frac{d\vec{p}}{dt} \underset{\text{assuming } m \text{ is constant.}}{=} m\frac{d\vec{v}}{dt}$

$$\Delta KE = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net} \cdot d\vec{r} = m\int_{\vec{r}_1}^{\vec{r}_2} \frac{d\vec{v}}{dt} \cdot d\vec{r} = m\int_{\vec{r}_1}^{\vec{r}_2} d\vec{v} \cdot \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}}$$

$$\underset{\text{linear motion}}{=} m\int_{\vec{r}_1}^{\vec{r}_2} v\,dv = \left[\frac{1}{2}mv^2\right]_{\vec{r}_1}^{\vec{r}_2} = \underbrace{\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}_{\text{we talked about it before.}}$$

linear motion $\underline{\underline{d\vec{v} \cdot \vec{v}}} = dv\,v\cos 0$

we ~~need~~ talked about it before.

Power = work or energy per unit time

Average Power $= \overline{P} = \dfrac{\Delta Work}{\Delta t}$

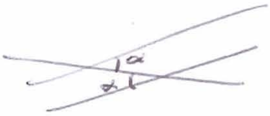
Instantaneous Power $= P = \dfrac{dWork}{dt}$

<u>Unit</u>: $\dfrac{J}{s} = W$ (watt)

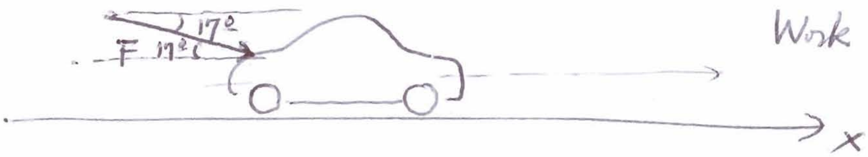
<u>Power & velocity</u>: $P = \dfrac{dWork}{dt} = \dfrac{d}{dt}(\vec{F}\cdot\Delta\vec{r}) = \vec{F}\cdot\dfrac{d\Delta\vec{r}}{dt} = \vec{F}\cdot\vec{v}$

constant force

<u>7.69</u>



$F = 470N$

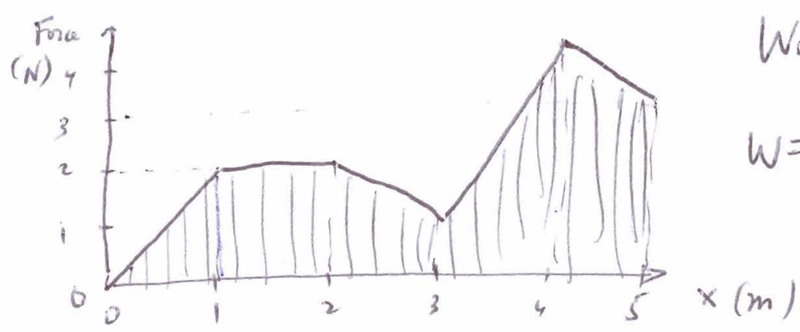


Work = 860 J ; Δx ?

$Work = \vec{F}\cdot\Delta\vec{r} = F\,\Delta x \cos 17^\circ \rightarrow \Delta x = \dfrac{Work}{F\cos 17^\circ}$

$\Delta x = \dfrac{860}{470\cdot\cos 17^\circ} = 1.9m$

<u>7.24</u>



Work ($x: 0 \rightarrow 5m$) ?

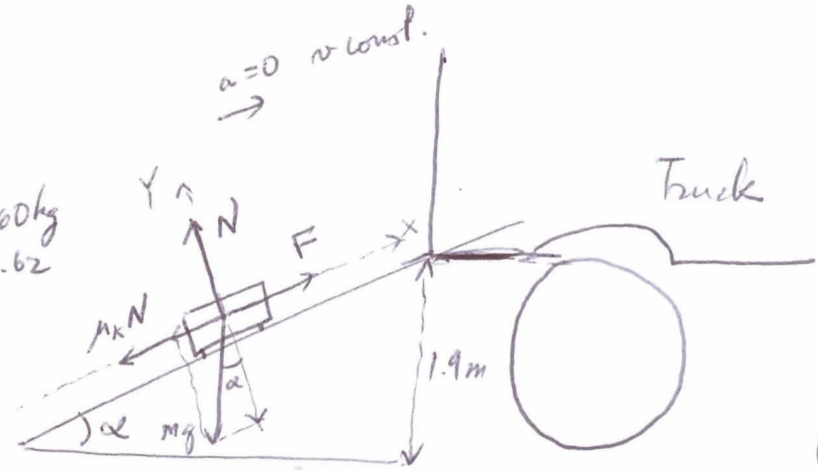
$W = \int_0^5 F.dx$ = area under the curve: can be done geometrically here

$W = \dfrac{1.2}{2} + 1.2 + \left(\frac{1}{2}+1\right) + \left(\frac{3}{2}+1\right) + \left(\frac{1}{2}+3\right) = 1 + 2 + \frac{3}{2} + \frac{5}{2} + \frac{7}{2} = 10.5\ J$

7.75

$m = 460\,kg$

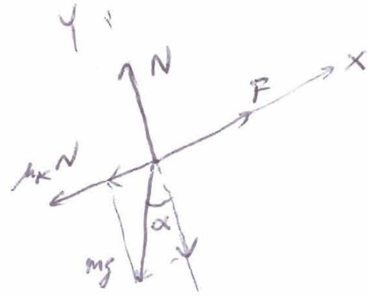
$\mu_k = 0.62$



Work done along ramp.
(const. force)

$W = F . d$

$\sin\alpha = \dfrac{1.9}{d}$



$$x: \quad F_{net_x} = F - mg\sin\alpha - \mu_k N = 0$$
$$y: \quad F_{net_y} = N - mg\cos\alpha = 0$$
$$\Big\} \; F = mg\sin\alpha + \mu_k mg\cos\alpha$$

$$W = F . d = mg(\sin\alpha + \mu_k\cos\alpha) \cdot \frac{1.9}{\sin\alpha} \quad \begin{cases} \alpha = 15^\circ : W = 28.5\,kJ \\ \alpha = 30^\circ : W = 17.8\,kJ \end{cases}$$

$$\begin{cases} \alpha = 15^\circ & F = 3.88\,kN \\ \alpha = 30^\circ & F = 4.68\,kN \end{cases}$$

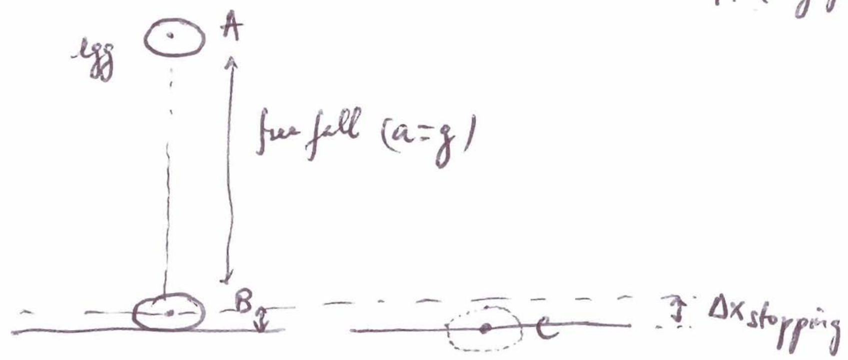
~~Work~~ If force required is not a problem (sufficient helping hands) steeper ramp required less total work (less distance travelled). But if force required is too high we need to use a longer ramp for a smaller angle, and need to do more work.

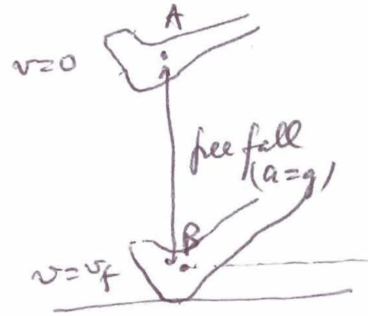
(7.49) Foot & leg $m = 8\,kg$

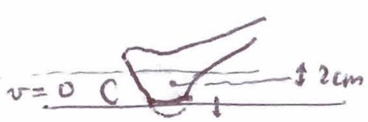
$\Delta x = 0.7\,m$ (free fall)

$\Delta x_{stopping} = 0.02\,m \leftarrow$

F. (by floor on foot ?)







AB : constant accel. $v: 0 \to v_f \longrightarrow v_f^2 - 0^2 = 2g\,\Delta x = 2 \times 9.81 \times 0.7$

BC : deceleration $v: v_f \to 0$

Work done to stop leg b/w B & C : $F_{floor} \cdot \Delta x_{stopping}$

Same as "Work" done by leg on floor : $\frac{1}{2} m v_f^2 - \frac{1}{2} m\, 0^2$

$$\rightarrow F_{floor} = \frac{m v_f^2}{2\Delta x_{stopping}} = \frac{8 \times 2 \times 9.81 \times 0.7}{2 \times 0.02}\,N = 2.75\,k\cancel{N} \cdot \frac{1\,lb}{4.448\,\cancel{N}}$$

$$= \frac{2750}{4.448}\,lb = 617.5\,lb$$

(7.58) $m = 75kg$ $v = 0 \xrightarrow{3.1s} v_f = 10 m/s$ Power output?

Power = Energy per unit time

Energy delivered by accelerating from 0 to 10m/s?

It's kinetic: $\frac{1}{2} m v_f^2$

Power out is this energy per time taken $P = \frac{\frac{1}{2} 75 \cdot 10^2}{3.1} = 1.21 kW$

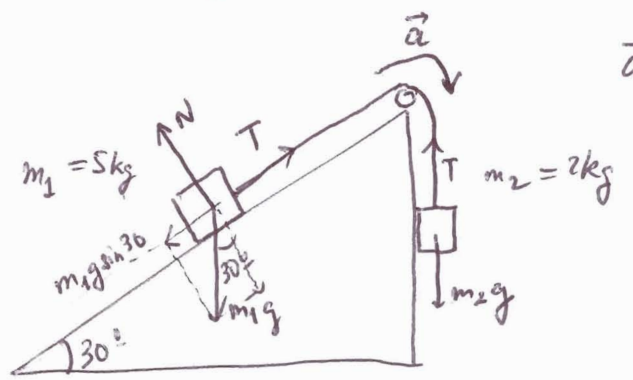
$P = 1210 \cancel{W} \cdot \frac{1 HP}{746 \cancel{W}} = 1.62 HP.$

(This works for cars too !)

(6.67)

$\vec{a}$ for m_2

surface & pulley frictionless



$m_1: \begin{cases} T - m_1 g \sin 30 = m_1 a \\ N - m_1 g \cos 30 = 0 \end{cases}$ $m_2 \; m_2 g - T = m_2 a$

$T = m_2 (g - a)$

$m_2 g - m_2 a - m_1 g \sin 30 = m_1 a$

$$\boxed{a = \frac{(m_2 - m_1 \sin 30) g}{m_1 + m_2}}$$

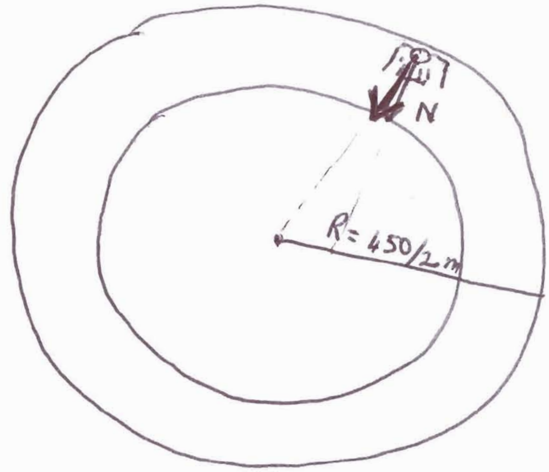
$$= \frac{(2 - 2.5) 9.81}{7}$$

$$\boxed{a = -0.7 \, m/s^2}$$

indicates the system accelerates in the CCW direction (m2 up; m1 down)

6.75

(From above)



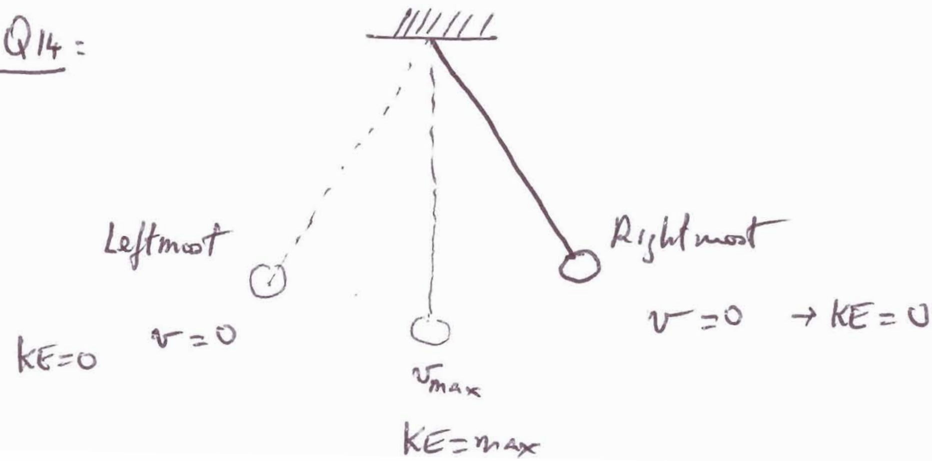
N: provides radial acceleration

n Rev. per min: ? $a = g$

$$a = \frac{v^2}{R} \qquad N = ma = \cancel{m}\frac{v^2}{R} = \cancel{m}g \rightarrow v = \sqrt{gR}$$

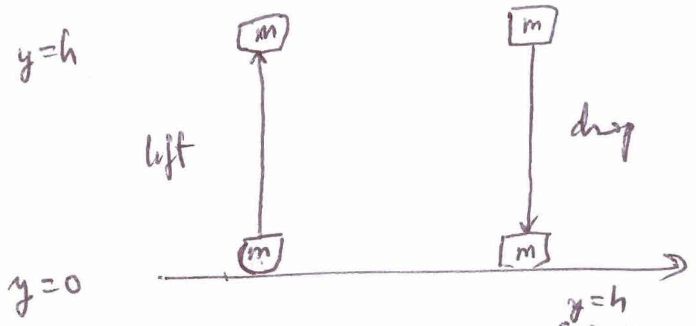
$$v = \sqrt{9.81 \times 225} = 46.98\ \cancel{m}/\cancel{s} \cdot \frac{1\,\text{rev}}{2\pi \cdot 225\ \cancel{m}} \cdot \frac{60\ \cancel{s}}{1\,\text{min}} = 1.99\ \text{rpm}$$

Ch 7 Q14:



KE changes!

Ch. 8 Conservation of Energy



lifting { Work done by lifter: $\int_{y=0}^{y=h} \vec{F}\cdot d\vec{r} = mg\int_{y=0}^{y=h} dy = mgh$ (steady upward motion)

Work done by gravity: $-mgh$ (received work)

dropping { work done by dropper $= 0$

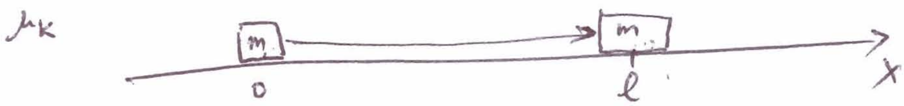
Work done by gravity: mgh

energy is conserved

↕

gravitation is a conservative force

$F_{min} = \mu_K mg$



$0 \to l$ { Work done by friction: $-\mu_K mgl$

Work done by pusher: $\mu_K mgl$

$l \to 0$ { Work done by friction: $-\mu_K mgl$

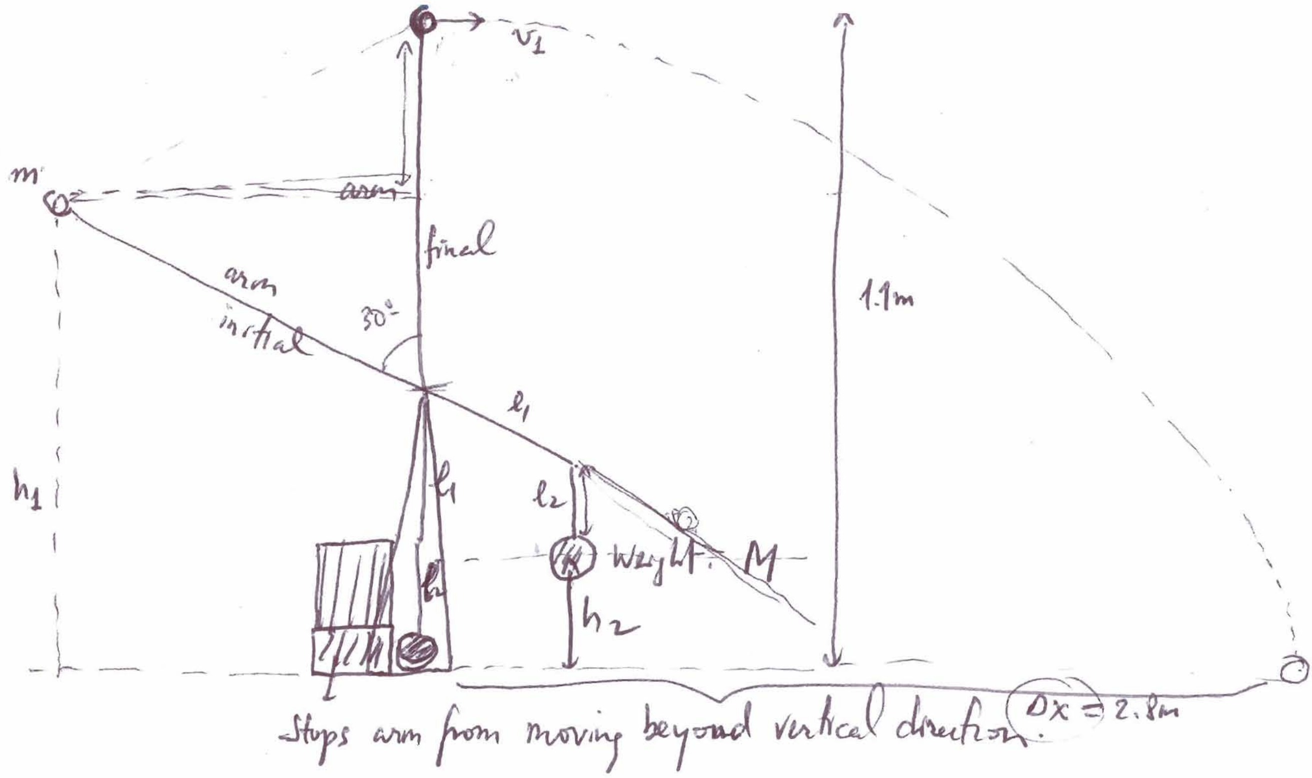
Work done by pusher: $\mu_K mgl$

energy is not conserved

or friction is non-conservative

- Conservative forces: gravitational; electric; magnetic; ...
- Non-conservative: friction

P.P. 8.1



Initial: Final

Mechanical Energy: $KE + P.E$ $\downarrow$ 0 $\quad Mgh_2 + mgh_1$ $=$ Mech. Energy: $KE + P.E.$ $\downarrow$ $\frac{1}{2}mv_1^2 + \frac{1}{2}M0^2 \quad mg1.1 + Mg.0$

Conservation of Energy:
(will get v_1 from this).

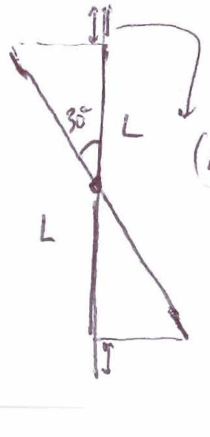
Once the ball left the spoon, it follows the projectile motion:

$\vec{v}_1$ is horizontal $\rightarrow \Delta x = v_1 t \longrightarrow v_1 = \frac{\Delta x}{t} = \frac{2.8m}{\sqrt{\frac{2.2}{9.81}}}$

$\Delta y = 1.1m = \frac{1}{2}g t^2 \rightarrow t = \sqrt{\frac{2 \times 1.1}{9.81}}$

$\boxed{v_1 = 5.9m/s}$

$$Mgh_2 = \frac{1}{2}mv_1^2 + \underset{0.1}{m}g\underbrace{(1.1 - h_1)}_{\sim 0.13} \simeq \underbrace{\frac{1}{2}0.1 \cdot 36}_{1.8} + \underbrace{0.1 \times 9.81 \times 0.13}_{0.13}$$



$$(L - L\cos 30)\frac{1}{2} = \frac{L}{2}(1 - \cos 30)$$

$$h_2 = \frac{1.8}{Mg} = \frac{1.8}{1 \times 9.81} = 18cm$$

<u>Conservation of Mechanical Energy</u> : sum of KE & $\underbrace{\text{potential energy}}_{\text{from gravitation}}$

$$\left(\frac{1}{2}mv^2 + mgh\right)_{\text{time 1}} = \left(\frac{1}{2}mv^2 + mgh\right)_{\text{time 2}}$$

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$