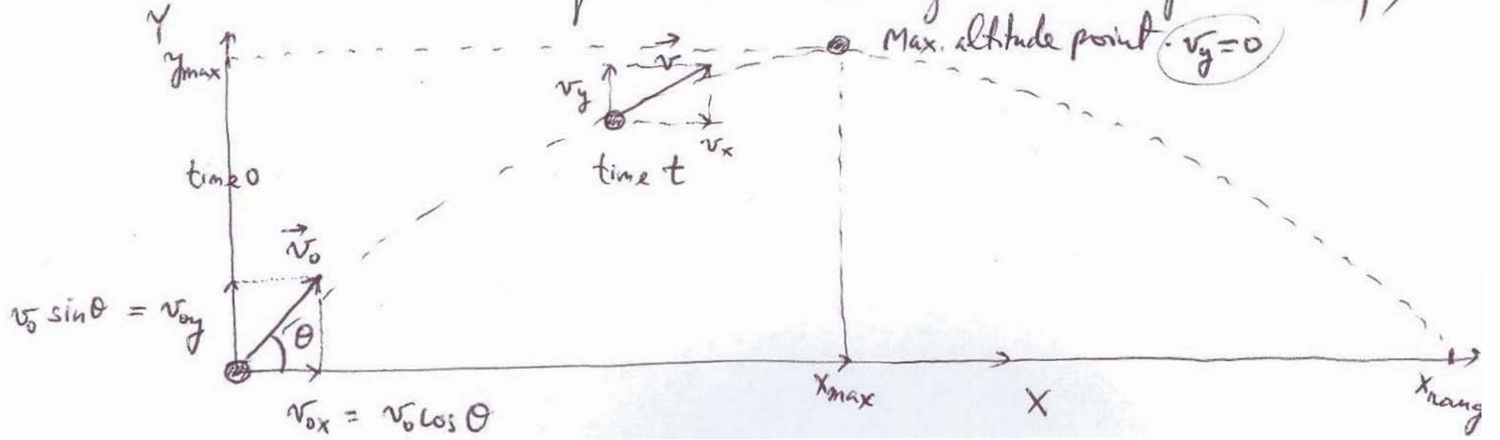
Ch 4 Motion in More than One Dimension

Shown experiments in movies to confirm x- & y- components of motion are independent.

Kinematic equations: (constant acceleration)

$$\begin{cases} (\mathrm{I}) & \vec{v} = \vec{v}_0 + \vec{a}t \\ (\mathrm{II}) & \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \end{cases}$$

<u>Projectile motion</u>: (ball & car going on horizontal track at constant speed. Then ball got tossed up vertically)



$$(\mathrm{I}) \begin{cases} v_x = v_{0x} = v_0 \cos\theta \\ v_y = v_{0y} - gt = v_0 \sin\theta - gt \end{cases}$$

$$(\mathrm{II}) \begin{cases} x = x_0 + v_0(\cos\theta)t \\ y = y_0 + v_0(\sin\theta)t - \frac{1}{2}gt^2 \end{cases}$$

gravity is along y-direction only and opposite to the motion (before ball reaches highest point.)

$(x_0, y_0) = (0, 0)$ (Place origin of coordinates on initial position)

$$(\mathrm{II}) \begin{cases} t = \dfrac{x}{v_0 \cos\theta} \\ \boxed{y = \dfrac{\cancel{v_0} \sin\theta}{\cancel{v_0} \cos\theta} x - \frac{1}{2} g \dfrac{x^2}{v_0^2 \cos^2\theta} = x\tan\theta - \frac{g}{2}\dfrac{x^2}{v_0^2\cos^2\theta}} \end{cases}$$

Trajectory is a parabola (inverted)

Since $x_{range} = 2x_{max}$ (symmetric motion w.r.t. highest point: conservation of energy)

$\rightarrow x_{max} = (v_0 \cos\theta)\, t_{max}$ (t_{max}: time ball takes to reach $v_y = 0$)

$$(I) \begin{cases} v_y = v_0 \sin\theta - g t \rightarrow v_y = 0 \rightarrow \boxed{t_{max} = \dfrac{v_0 \sin\theta}{g}} \\ v_x = v_0 \cos\theta \end{cases}$$

$$\rightarrow \boxed{x_{max} = v_0 \cos\theta \, \frac{v_0 \sin\theta}{g} = \frac{v_0^2}{g} \frac{\sin 2\theta}{2}}.$$

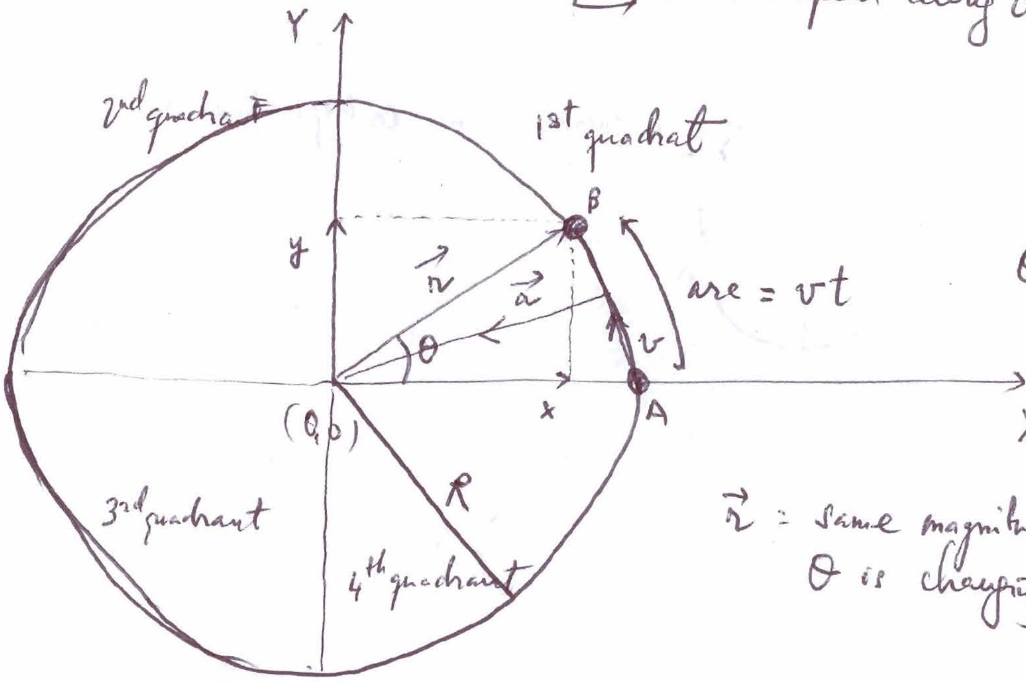
$(2 \cos\theta \sin\theta = \sin 2\theta)$

$$\boxed{y_{max} = v_0 \sin\theta \, t_{max} - \frac{1}{2} g \, t_{max}^2 = \frac{(v_0 \sin\theta)^2}{g} - \frac{1}{2} \cancel{g} \frac{(v_0 \sin\theta)^2}{g^{\cancel{2}}} = \frac{v_0^2 \sin^2\theta}{2g}}$$

$$\rightarrow (x_{max}, y_{max}) = \left(\frac{v_0^2}{2g} \sin 2\theta , \frac{v_0^2}{2g} \sin^2\theta \right)$$

$$\rightarrow (x_{range}, y_{range}) = \left(\frac{v_0^2}{g} \sin 2\theta , 0 \right)$$

Circular motion: (Uniform) $\longrightarrow$ constant speed along circular trajectory



$$\theta = \frac{arc}{R} = \frac{v.t}{R}$$

$\vec{r}$: same magnitude R along trajectory, θ is changing.

$$\vec{r} = x\hat{i} + y\hat{j} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j} = R\left[\cos\left(\frac{vt}{R}\right)\hat{i} + \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$\vec{v} = \frac{d\vec{r}}{dt} = R\left[-\frac{v}{R}\sin\left(\frac{vt}{R}\right)\hat{i} + \frac{v}{R}\cos\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$= v\left[-\sin\left(\frac{vt}{R}\right)\hat{i} + \cos\left(\frac{vt}{R}\right)\hat{j}\right]$$

uniform circ. motion

"Centripetal acceleration"

$$\vec{a} = \frac{d\vec{v}}{dt} = v\left[-\frac{v}{R}\cos\left(\frac{vt}{R}\right)\hat{i} - \frac{v}{R}\sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$= -\frac{v^2}{R}\left[\cos\left(\frac{vt}{R}\right)\hat{i} + \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

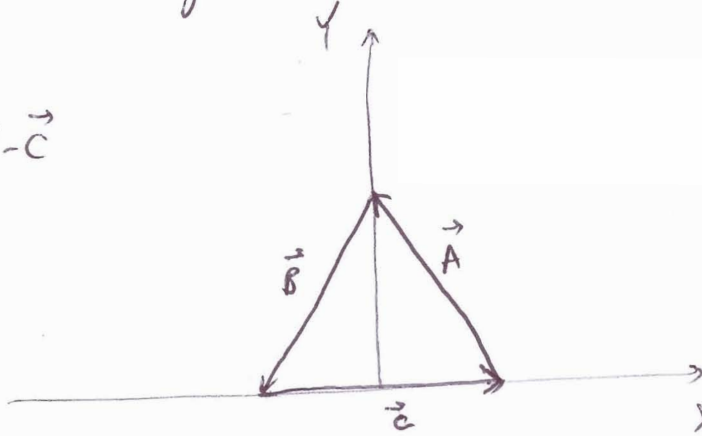
$$|\vec{a}| = \left|-\frac{v^2}{R}\right|\underbrace{\sqrt{\cos^2\left(\frac{vt}{R}\right) + \sin^2\left(\frac{vt}{R}\right)}}_{1} \rightarrow \boxed{a = \frac{v^2}{R}}$$

its direction is radially inward.

(3.9) $\vec{A}, \vec{B}, \vec{C}$ same magnitude L

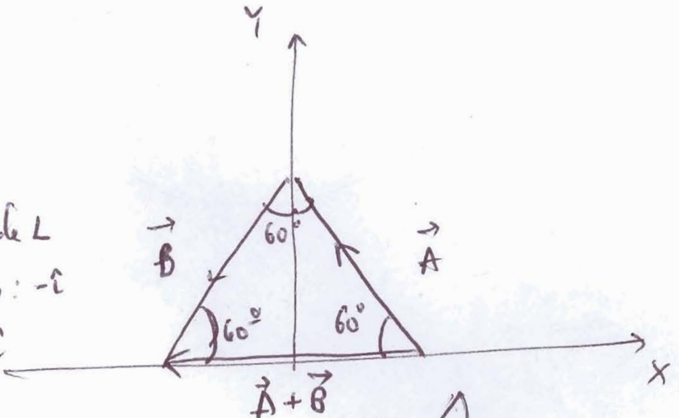
a) $\vec{A}+\vec{B}$ b) $\vec{A}-\vec{B}$

c) $\vec{A}+\vec{B}+\vec{C}$ d) $\vec{A}+\vec{B}-\vec{C}$



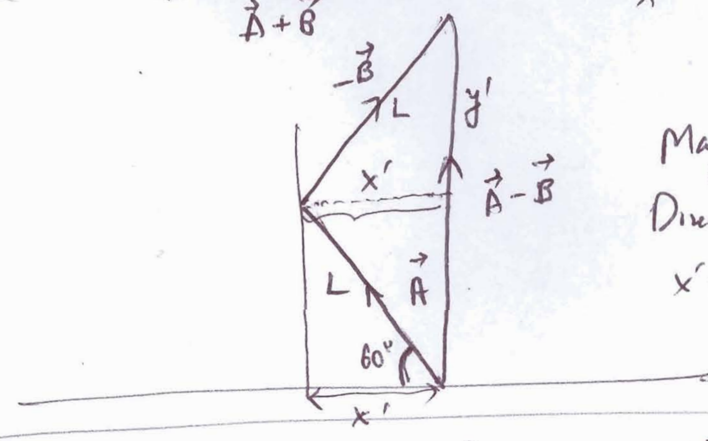
a)

Magnitude L
Direction: $-\hat{i}$
$\vec{A}+\vec{B} = -L\hat{i}$



To add 2 vectors, draw the second vector from tip of the first vector, then connect origin of 1st vector with tip of second vector.

b)



Magnitude: $2y' = 2\sqrt{L^2 - x'^2} = 2\sqrt{L^2 - \frac{L^2}{4}}$

Direction: $\hat{j}$

$= 2\sqrt{\frac{3}{4}L^2}$

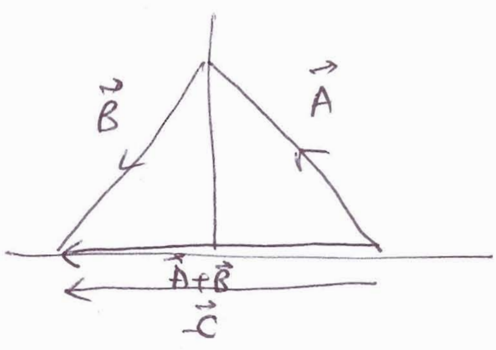
$x' = L\cos 60° = \frac{L}{2}$

$= L\sqrt{3}$

$\vec{A}-\vec{B} = L\sqrt{3}\,\hat{j}$

c) Since $\vec{A}+\vec{B} = -\vec{C}$; $\vec{A}+\vec{B}+\vec{C} = 0$

d) $\vec{A}+\vec{B}-\vec{C} = -2\vec{C}$

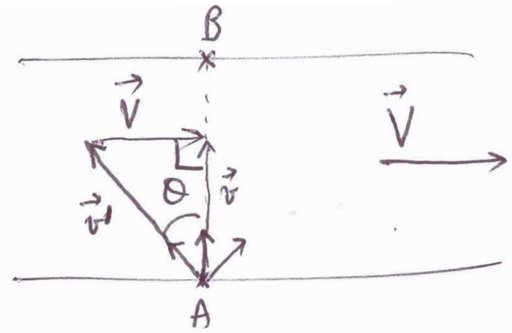


Magnitude: 2L

Direction: $-\hat{i}$

$\vec{A}+\vec{B}-\vec{C} = -2L\hat{i}$

3.57



$\vec{v} = \vec{v}' + \vec{V}$

<u>Relative motion</u>

$\vec{v}$: vel. of boat w.r.t. ground

$\vec{v}'$: vel. of boat w.r.t. water, 15 km/h

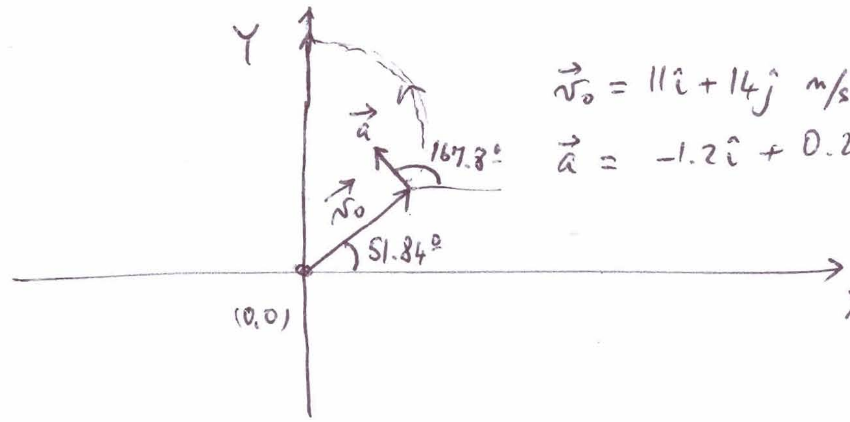
$\vec{V}$: vel. of water w.r.t. ground, 6.3 km/h

Direction of $\vec{v}'$ is such that $\vec{v}' + \vec{V} = \vec{v}$ which points along AB (for boat to get to B from A)

$$\theta = \sin^{-1} \frac{6.3 \text{ km/h}}{15 \text{ km/h}} = 24.8^\circ \simeq 25^\circ$$

3.36

4.11/



$\vec{v}_0 = 11\hat{i} + 14\hat{j}$ m/s $\begin{cases} \sqrt{11^2+14^2} = 17.8 \\ \tan^{-1}\frac{14}{11} = 51.84° \end{cases}$

$\vec{a} = -1.2\hat{i} + 0.26\hat{j}$ m/s² $\begin{cases} \sqrt{1.2^2+0.26^2} \\ \tan^{-1}\frac{0.26}{-1.2} = -12.2° \\ +180° \\ = 167.8° \end{cases}$

a) When does the particle cross the y-axis

I) Since at crossing with y-axis : x = 0

$$x - x_0 = v_{0x}\, t + \frac{1}{2} a_x t^2 \rightarrow 0 - 0 = 11t - \frac{1}{2}1.2t^2$$

$$t = \frac{11}{0.6}\,s = 18.3\,s$$

b) What is the y-coordinate?

$$y - y_0 = v_{0y}\, 18.3 + \frac{1}{2} a_y\, 18.3^2$$

$$y - 0 = 14 \times 18.3 + \frac{0.26}{2} 18.3^2 = 300\ m$$

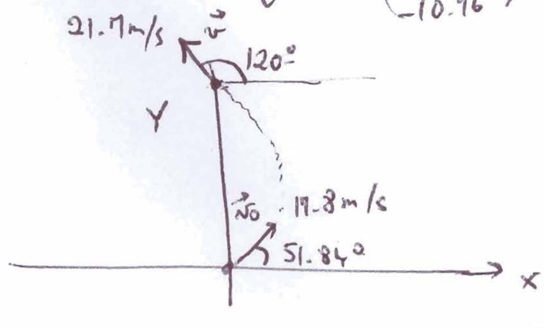
c) What is the velocity at crossing with y-axis?

I) $\vec{v} = \vec{v}_0 + \vec{a}t$

$$\vec{v} = (v_{0x} + a_x 18.3)\hat{i} + (v_{0y} + a_y \cdot 18.3)\hat{j}$$

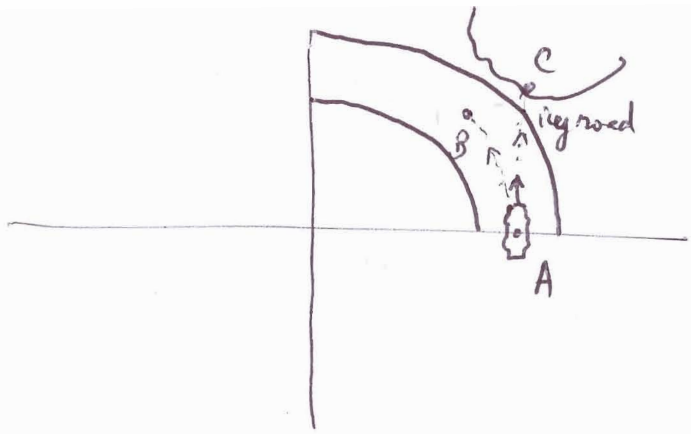
$$= -10.96\,\hat{i} + 18.8\,\hat{j} \quad m/s$$ (second quadrant)

$$\vec{v} = \begin{cases} \sqrt{10.96^2 + 18.8^2} = 21.7\ m/s \\ \tan^{-1}\left(\frac{18.8}{-10.96}\right) = -59.8° \simeq -60° + 180° = 120° \end{cases}$$



(make sure the angle corresponds to the correct quadrant)

Ch5 Force & Motion



To get from A to B we need a centripetal acceleration. What agent will provide this acceleration?
→ Friction b/w tires & road

We need a force to change motion or to change velocity (is vector including magnitude & direction). Force is also a vector, sometime more than one force affect the same motion, in this case the net force (vector addition of forces) will change the motion.

<u>1st Newton's Law</u>: body in uniform motion will stay in uniform motion, body at rest will stay at rest, unless there is a net force acting on it.

<u>2nd Newton's Law</u>: $\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$

$\vec{p}$: linear momentum $= m\vec{v}$ (mass times velocity)

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

$$\left(\frac{d(ab)}{dt} = a\frac{db}{dt} + b\frac{da}{dt} \right)$$

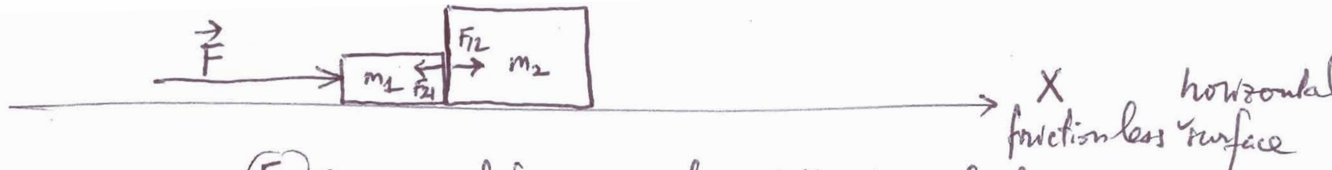
On a daily basis m is constant with time → $\vec{F}_{net} = m\dfrac{d\vec{v}}{dt} = m\vec{a}$

<u>Dimension of force</u>: $[F] = \dfrac{[p]}{[t]} = \dfrac{[m][v]}{[t]} = \dfrac{M.L}{T^2}$

Metric system (SI): unit for force is $\frac{kg\,m}{s^2}$ = N (Newton)

3rd Newton's Law : if A exerts a force on B , B exerts an equal and opposite force on A.

- No net forces along vertical direction
- No frictional force



acc. of both blocks : $a = \dfrac{F}{m_1+m_2}$ ← net force on system of the two blocks.

$a_1 = ?$

$a_2 = ?$

On m_1 : what is the net force on this block? $F - F_{21}$ (along +X)

$$F_{1net} = F - F_{21} = m_1 a_1$$

On m_2 : what is the net force on this block? F_{12}

$$F_{2net} = \underbrace{F_{12}}_{\text{3rd Law}} = \underbrace{m_2 a_2}_{\text{2nd Law}}$$

Let's solve this system of 2 equations with 2 unknowns (a_1 & a_2)

By 3rd Law of Newton : $F_{12} = F_{21}$ (action & reaction)

Since both blocks are going together $a_1 = a_2$

$$F - F_{12} = m_1 a_1$$

$$F_{12} = m_2 a_1$$

$$F = (m_1+m_2)a_1 \rightarrow a_1 = \frac{F}{m_1+m_2} = a_2 = a$$

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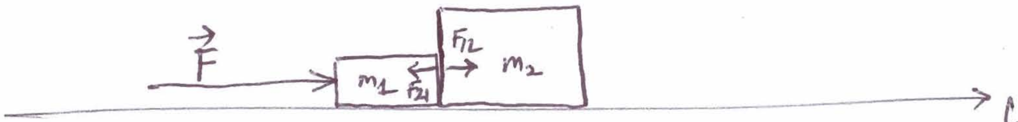

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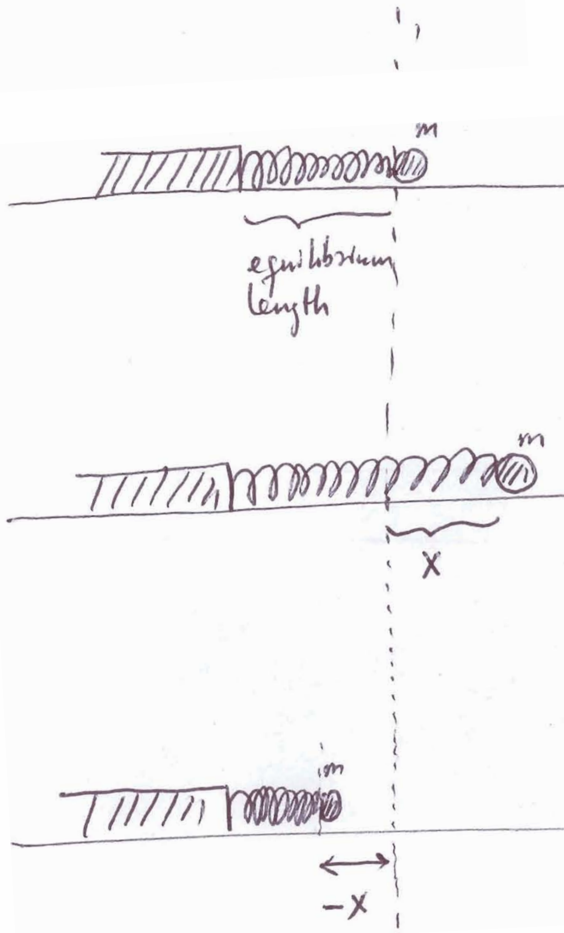
$$F = (m_1+m_2) a_1 \quad \rightarrow \quad a_1 = \frac{F}{m_1+m_2} = a_2 = a$$

<u>Measuring forces:</u>

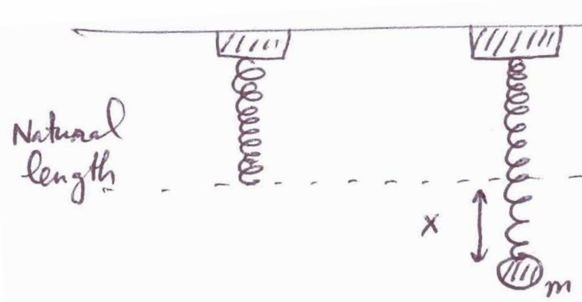
Spring : Hooke's Law : $F = -kx$ $\begin{cases} k: \text{spring constant} \\ x: \text{displacement w.r.t. the equilibrium position} \end{cases}$



$F = 0$

$F = -kx$ (force pulling on mass)

$F = -k(-x) = kx$ (force pushing on mass)



New equilibrium

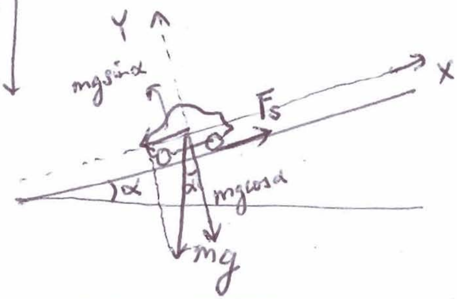
Displacement from natural length with mass m hanging

<u>Net force on mass m</u> is : $mg - kx = m\underset{0}{\underset{\shortparallel}{a}}$

$\rightarrow mg - kx = 0 \rightarrow \boxed{x = \frac{mg}{k}}$

Frictional forces: while in contact with surfaces

- Static friction: threshold force for something to start moving
- Kinetic friction: while moving



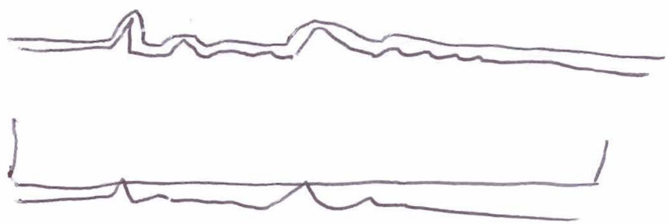
$F_s \geq mg\sin\alpha$; $\boxed{F_s = \mu_s N} = \mu_s mg\cos\alpha$

$\mu_s \cancel{m}g\cos\alpha \geq \cancel{m}g\sin\alpha \rightarrow \boxed{\mu_s \geq \tan\alpha}$

$\boxed{F_k = \mu_k N}$

$\mu_k \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \mu_s$?

hardest part when pushing a box is for it to start moving



Drag forces:

$$F_D = \frac{1}{2} C \rho_{fluid} A_{body} v_{body}^2$$

$$F_D \propto v^2$$

Sky diving: $v_0 = 0$ $\qquad v \uparrow \rightarrow \underbrace{F_D \uparrow}$ (against motion)

if there is enough altitude or enough time for F_D to catch up to mg then you will get a terminal speed (constant speed) $\quad F_D = mg$ (no further acceleration)

$$\frac{1}{2} C \rho_{fluid} A_{body} v_t^2 = mg \rightarrow v_t = \sqrt{\frac{2mg}{C \rho_{fluid} A_{body}}}$$