Dimensional analysis

Dimensions of length and time: L and T

Dimension of a velocity or speed v is written [v]:

$$[v] = \frac{[\Delta s]}{[\Delta t]} = \frac{L}{T}$$

Dimension of energy E is written [E]:

$$[E] = \left[\tfrac{1}{2} m v^2\right] = [m]\left[v^2\right] = [m][v]^2 = M\,\frac{L^2}{T^2}$$

Dimension of acceleration a is written [a]:

$$[a] = \frac{[\Delta v]}{[\Delta t]} = \frac{L}{T^2}$$

Check a formula using dimensional analysis

$$v = \tfrac{1}{2} g h^2 \quad \Rightarrow \quad [v] = [g][h]^2 = \frac{L^3}{T^2} \qquad \text{No}$$

$$v = \sqrt{gh} \quad \Rightarrow \quad [v] = \sqrt{[g][h]} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} \qquad \text{Yes}$$

Units

Unit for speed or velocity v $= \frac{unit\ for\ \Delta s}{unit\ for\ \Delta t} = \frac{mi; m; km; cm; \ldots}{s; \text{min.}; h; day; year; \ldots}$

Units for length	
km	10^3 m
cm	10^{-2} m
mm	10^{-3} m
µm	10^{-6} m
nm	10^{-9} m

Units for area	
km^2	10^6 m^2
cm^2	10^{-4} m^2
mm^2	10^{-6} m^2
μm^2	
nm^2	

Units for volume	
km^3	10^9 m^3
cm^3	10^{-6} m^3
mm^3	10^{-9} m^3
μm^3	
nm^3	

Units for time	
min	60 s
h	3600 s
day	86400s
week	
year	

Units for mass: lb; kg; g; …

Change of units (Appendix C)

1 light-year	9.46×10^{15} m
1 mi	1609 m
1 ft	0.3048 m
1 in	2.54 cm
1 lb	0.454 kg

Accuracy & Significant figures

Scientific notation

$$3\ 105\ 000 \text{ m} = 3.105 \times 10^6 \text{ m}$$

$$3\ 000 \text{ s} = 3 \times 10^3 \text{ s}$$

$$v = \frac{3.105\times10^6\, m}{3\times10^3\, s} = \frac{3.105}{3}\times10^{6-3} = 1.035\times10^3\, {}^{m}\!/_{s}$$

Accuracy:

3.1416 is more accurate than 1.14

Related to number of decimal digits

Addition & subtraction: keep same accuracy as least accurate term

$$3.1416 - 1.14 = 2.0016 = 2.00$$

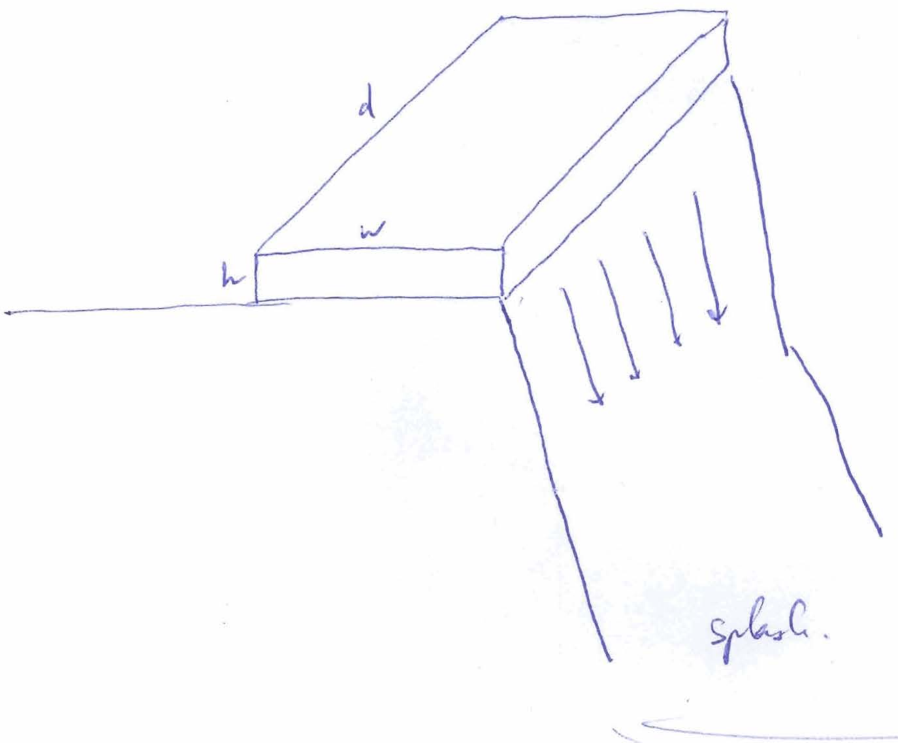
Significant figures: 6 370 000 three 6 370 001 seven

Multiplication and division: keep smallest number of s.f. except for constants, counting from the left

$$\text{Earth's circumference} = 2\pi\, R_E = 2 \times 3.1416 \times 6.37 \times 10^6 = 40.02398 \times 10^6$$

$$= 4.002398 \times 10^7 = 4.00 \times 10^7 \text{ m}$$

1.46) a) Volume of water over Niagara Falls per second.



Volume is hwd

Need $\frac{hwd}{t}$

$\left\{ \begin{array}{l} \frac{h}{t} wd \\ h\left(\frac{w}{t}\right)d \\ hw\frac{d}{t} \end{array} \right.$

$h \sim 1m$

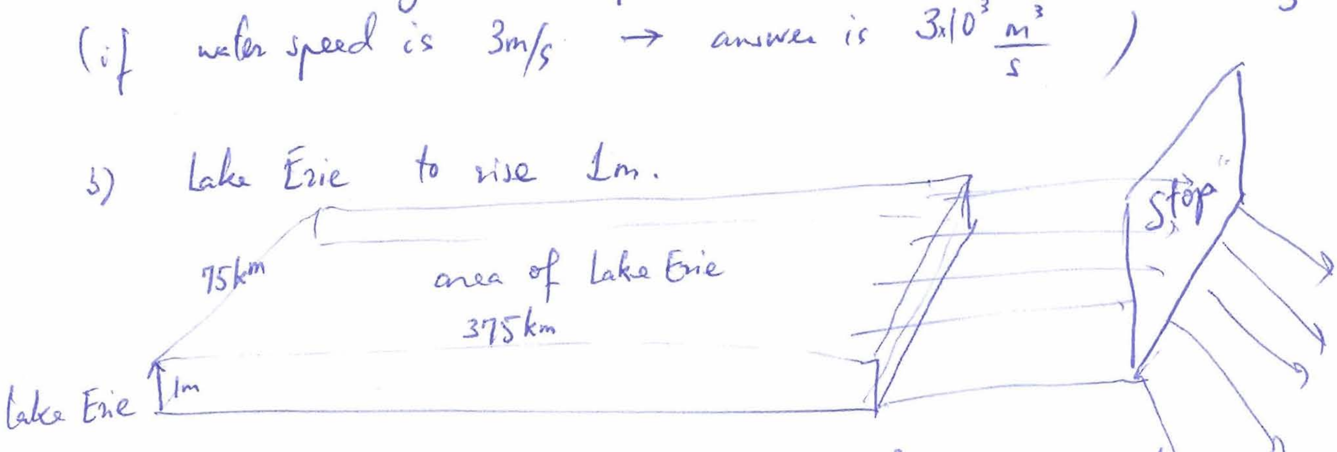
$\frac{w}{t} \sim 1 m/s$

$d \sim 1000 m$

Vol. water over Niagara Falls per second is $= 1 \times 10^3 \times 1 = 10^3 \frac{m^3}{s}$

(if water speed is 3m/s $\rightarrow$ answer is $3 \times 10^3 \frac{m^3}{s}$)

b) Lake Erie to rise 1m.



Vol water needed is $75 \times 10^3 \times 375 \times 10^3 \times 1 = 28125 \times 10^6 m^3$

at speed $3000 m^3/s$

$\rightarrow \frac{28125 \times 10^6 \ m^3}{3 \times 10^3 \ \frac{m^3}{s}} \sim 9 \times 10^6 s$ $\quad \frac{1 day}{86400 s}$

$9 \times 10^6 s \ \frac{1 day}{9 \times 10^4 s} = 100$ days.

Chapter 2: Vector description of motion

Motion in a straight line:

Speed $= \dfrac{\text{distance}}{\text{time}}$ Velocity $= \dfrac{\text{displacement}}{\text{time}}$

$$W \xrightarrow{10\,mi} E$$

$$\xleftarrow[2\,mi]{}$$

time = 1hr. (total)

speed $= \dfrac{12\,mi}{h}$ velocity $= 8\,mi/h$

average velocity : $\bar{v} = \dfrac{\Delta x}{\Delta t}$ (Δ : "change of" "increment of")

instantaneous velocity: $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ (d : "derivative")

↳ example: $x = at^3 \rightarrow v = \dfrac{dx}{dt} = \dfrac{d\,(at^3)}{dt} = 3at^2$

$x = bt^n \rightarrow v = nbt^{n-1}$

Units : $\dfrac{m}{s}$; $\dfrac{km}{h}$; $\dfrac{mi}{h}$; etc...

Change of velocity over time : acceleration

Average acceleration : $\bar{a} = \dfrac{\Delta v}{\Delta t}$

Instantaneous acceleration $a = \dfrac{dv}{dt} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$

$[a] = \dfrac{[v]}{[t]} = \dfrac{L}{T^2} \rightarrow$ Units : $\dfrac{m}{s^2}$; $\dfrac{km}{s^2}$; $\dfrac{mi}{s^2}$, etc..

Constant acceleration:

$$a = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} \rightarrow \boxed{v = v_0 + at \quad (1)}$$

For the equation (2): two intermediate steps.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} \longrightarrow x = x_0 + \bar{v} t \quad (A)$$

Average velocity is also: $\bar{v} = \frac{1}{t-0} \int_0^t v \, dt$

$$= \frac{1}{t} \int_0^t (v_0 + at) \, dt$$

$$= \frac{1}{t} \left[v_0 t + \frac{1}{2} a t^2 \right] = v_0 + \frac{1}{2} a t$$

$$= \underbrace{\frac{1}{2} v_0 + \frac{1}{2} a 0}_{\frac{1}{2}(v_0 + a0)} + \underbrace{\frac{1}{2} v_0 + \frac{1}{2} a t}_{\frac{1}{2}(v_0 + at)}$$

$$= \frac{1}{2} \left[(v_0 + a0) + (v_0 + at) \right]$$

$$= \frac{1}{2} \left[\quad v_0 \quad + \quad v \quad \right] \quad (B)$$

Plug (B) into (A):

$$x = x_0 + \frac{1}{2}(v_0 + \underset{\substack{\uparrow \\ (1)}}{v}) t$$

$$x = x_0 + \frac{1}{2}(v_0 + v_0 + at) t$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (2)}$$

$$\boxed{\frac{v^2 - v_0^2}{x - x_0} = 2a \qquad (3)}$$

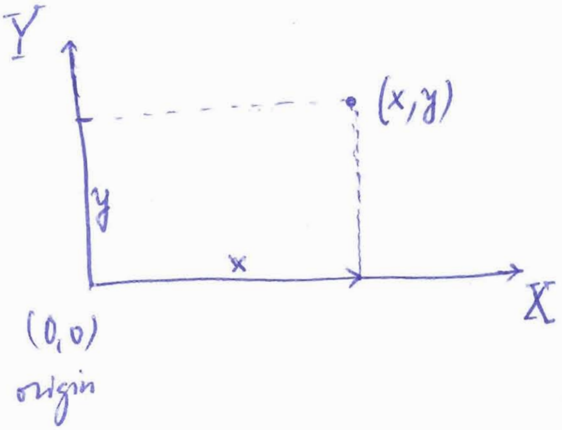
Can be derived from (1) & (2)

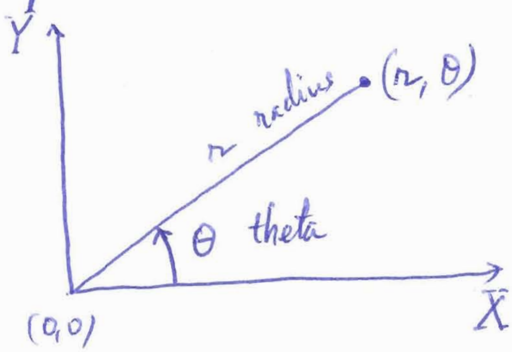
$$\boxed{v = v_0 + at \qquad (1)}$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad (2)}$$

In 2 dimensions (2D): the position is determined by 2 quantities

$$2 \text{ quantities} \begin{cases} (x, y) & \text{Cartesian components (Vector } (x, y)) \\ (r, \theta) & \text{polar components (vector } (r, \theta)) \end{cases}$$





$\boxed{\text{Notation}}$

Position vector : $\vec{r} = (x, y) = (r, \theta)$

Velocity vector : $\vec{v} = (v_x, v_y) = (v, \theta_v)$

Acceleration vector : $\vec{a} = (a_x, a_y) = (a, \theta_a)$

Relation b/w descriptions : $\vec{r} = (x, y) = (r\cos\theta, \ r\sin\theta)$

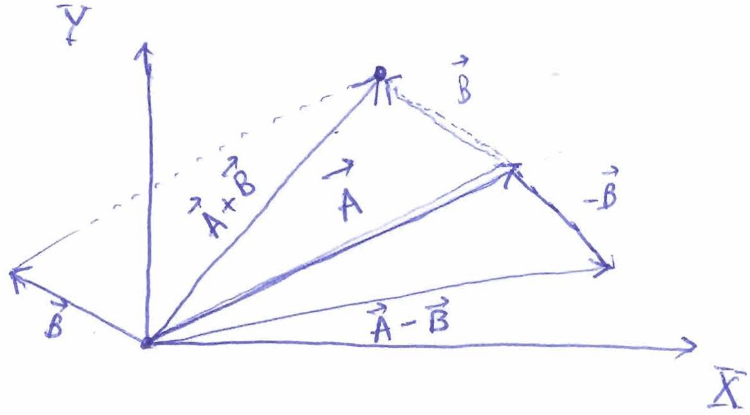
$$\vec{r} = (\underset{\substack{\downarrow \\ \text{radius or magnitude}}}{r}, \theta) = \left(\sqrt{x^2+y^2}, \ \tan^{-1}\frac{y}{x}\right)$$

<u>Unite vectors</u> : vector with magnitude $= 1$ $\begin{cases} X : & \hat{\imath} = (1, 0^\circ) \\ Y : & \hat{\jmath} = (1, 90^\circ) \end{cases}$

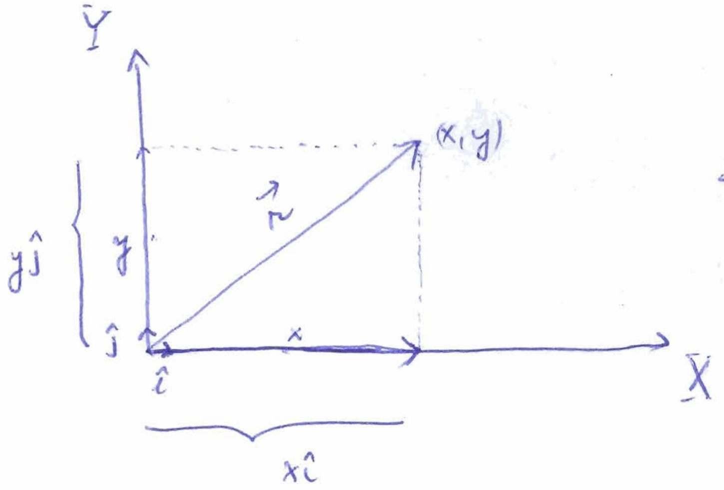
$$\vec{r} = (x, y) = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{v} = (v_x, v_y) = v_x\hat{\imath} + v_y\hat{\jmath}$$

Addition & substraction of vectors : (using parallelograms)



$\vec{r} = (x, y) = x\hat{i} + y\hat{j}$



So, $\vec{r} = x\hat{i} + y\hat{j}$ (m)
correct by vector addition

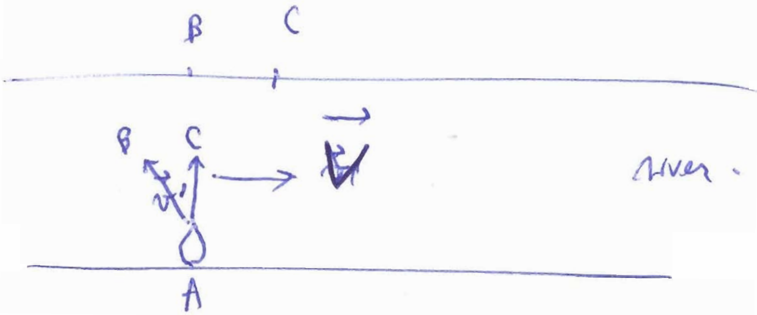
→ Kinematic equations in 2D :

Velocity Vector : $\vec{v} = v_x\hat{i} + v_y\hat{j} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j}$ $\left(\frac{m}{s}\right)$

Acceleration vector : $\vec{a} = a_x\hat{i} + a_y\hat{j} = \dfrac{d\vec{v}}{dt} = \dfrac{dv_x}{dt}\hat{i} + \dfrac{dv_y}{dt}\hat{j}$ $\left(\frac{m}{s^2}\right)$

$\vec{v} = \vec{v}_0 + \vec{a}t$ (1)

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ (2)

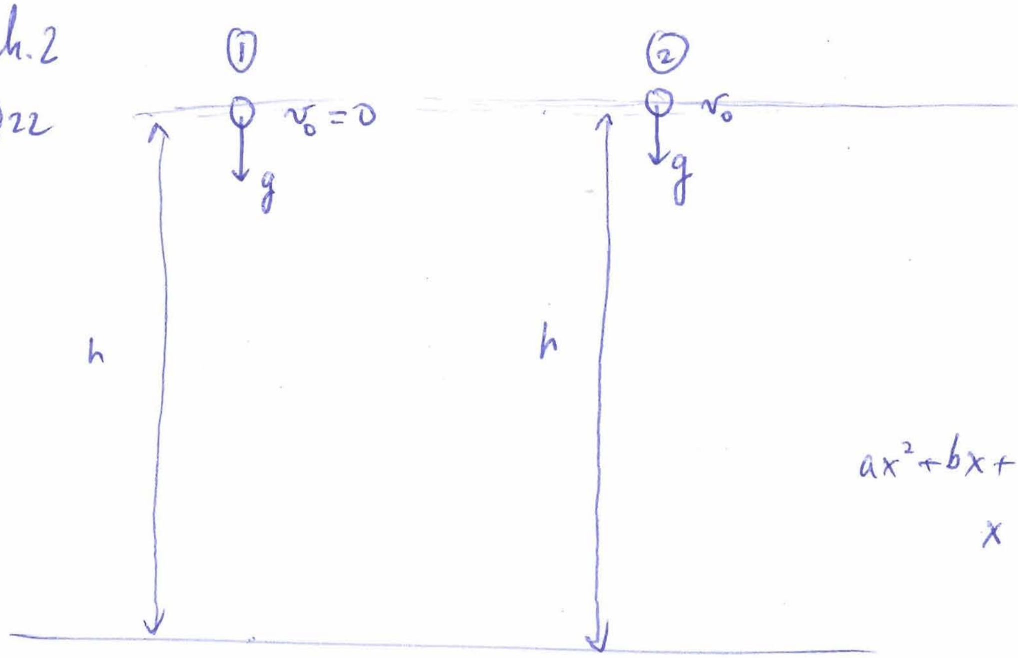


$$\vec{v} = \vec{v}' + \vec{V}$$

your speed w.r.t. ground | your speed w.r.t. water | river speed w.r.t. ground.

Ch.2

Q22



$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$h = 0 + 0t + \frac{1}{2} g t_1^2 \qquad h = 0 + v_0 t_2 + \frac{1}{2} g t_2^2 \rightarrow \frac{1}{2} g t_2^2 + v_0 t_2 - h = 0$$

$$t_1 = \sqrt{\frac{2h}{g}} \quad > \quad t_2 = \frac{-v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$$

$$v_1 = 0 + g t_1 \qquad v_2 = v_0 + g t_2$$

$$= g\sqrt{\frac{2h}{g}} \qquad = +\sqrt{v_0^2 + 2gh}$$

$$= \sqrt{\frac{2hg^{\cancel{2}}}{\cancel{g}}} = \sqrt{2hg}$$

Rock #2 hits ground at $v_2 > v_1$

$$v_2 - v_1 = \underbrace{\sqrt{v_0^2 + 2gh}}_{< v_0 + \sqrt{2gh}} - \sqrt{2gh}$$

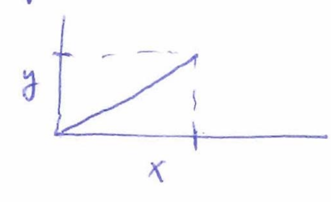
$$v_2 - v_1 \begin{cases} < v_0 \\ = v_0 \quad \times \\ > v_0 \end{cases}$$

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \qquad \sqrt{a+b} < \sqrt{a} + \sqrt{b}$$

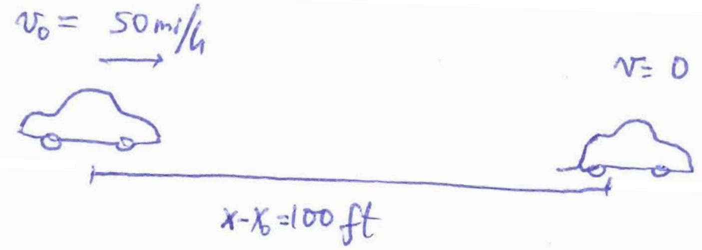
$a = x^2$

$b = y^2$

$$\underset{\text{diagonal}}{\sqrt{x^2 + y^2}} \qquad \underset{\text{side}}{x} + \underset{\text{side}}{y}$$



(2.44)



Deceleration a ?

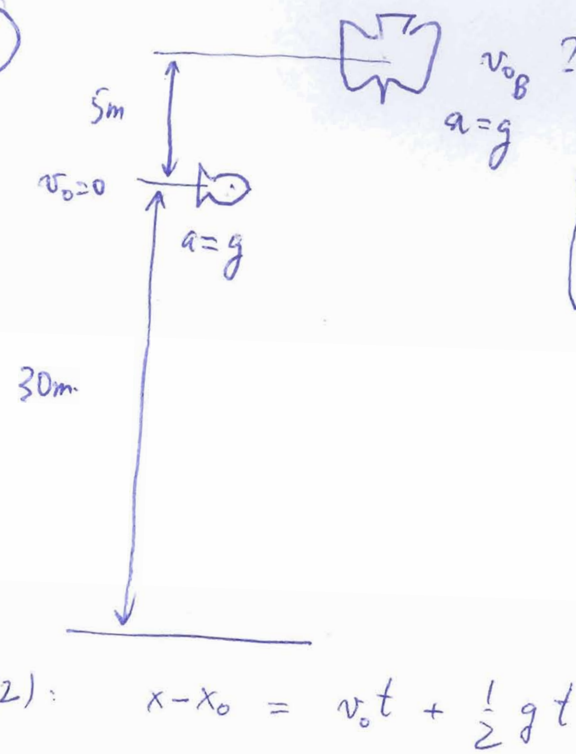
Eq 3 $\rightarrow$ $2a = \dfrac{0 - 22.35^2}{30.48} \Rightarrow \boxed{a \doteq -8.192\ m/s^2}$

$SI \begin{cases} L \rightarrow m \\ T \rightarrow s \end{cases}$

$v_0 = \dfrac{50\,\cancel{mi}}{\cancel{h}} \cdot \dfrac{1609\,m}{\cancel{mi}} \cdot \dfrac{1\cancel{h}}{3600\,s} = 22.35\ \dfrac{m}{s}$

$x - x_0 = 100\,\cancel{ft} \cdot \dfrac{0.3048\,m}{1\,\cancel{ft}} = 30.48\,m$

(2.69)



During time the fish takes to fall 30m, the bird (kingfisher) needs to cover 35m

(2): $x - x_0 = v_0 t + \frac{1}{2} g t^2$

Fish: $30m = 0 + \frac{1}{2} 9.81\, t^2$

$\rightarrow t = \sqrt{\dfrac{60}{9.81}} = 2.47\,s$

Bird: $35m = v_{0B}\, 2.47 + \underbrace{\frac{1}{2} 9.81 \cdot 2.47^2}_{30m}$

$\rightarrow v_{0B} = \dfrac{5}{2.47} = 2\ m/s$

Alternative way:

(3) $\dfrac{v^2 - v_0^2}{x - x_0} = 2g$: Fish : $\dfrac{v^2 - 0}{30} = 2g \rightarrow v = \sqrt{2 \times 9.81 \times 30}$

$= 24.25\ m/s$

(1) $v = v_0 + gt$: Fish : $24.25 = 0 + 9.81\,t$

$\rightarrow t = \dfrac{24.25}{9.81}\,s = 2.47\,s$

(2) $x - x_0 = v_{0B}\,t + \frac{1}{2} g t^2$: Bird : $35m = v_{0B}\, 2.47 + \underbrace{\frac{1}{2} 9.81 \cdot 2.47^2}_{30m}$

$\rightarrow v_{0B} = 2\ m/s$

(2-54)

a)

140m

v_0? $t = 3.6s$ $v = 53 m/s$

(1) $v = v_0 + at \rightarrow v_0 = v - at$

(2) $x - x_0 = v_0 t + \frac{1}{2} a t^2 = (v - at)t + \frac{1}{2} a t^2$

$= vt - at^2 + \frac{1}{2} a t^2 = vt - \frac{1}{2} a t^2$

$140\ m = 53 \frac{m}{s} \cdot 3.6 s - \frac{1}{2} a (3.6 s)^2 \rightarrow a = -\dfrac{(140 - 53 \times 3.6)2}{3.6^2}$

$a = 7.83\ m/s^2$

$\rightarrow$ Back in (1) $v_0 = v - at = 53 - 7.83 \times 3.6 = 24.8\ m/s$

b)

$a = 7.83 m/s^2$ $a = 7.83 m/s^2$

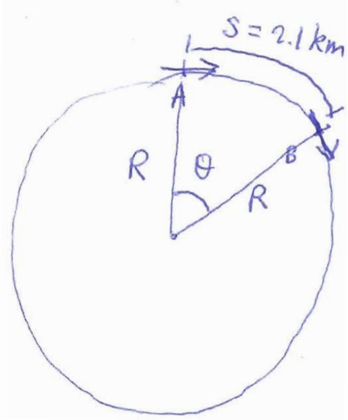
140m

$v_0' = 0 m/s$ $v_0 = 24.8 m/s$ $t = 3.6s$ $v = 53 m/s$

How far did it travel from rest to the end of the 140m distance?

(3) $\dfrac{v^2 - v_0'^2}{x - x_0'} = 2a \rightarrow x - x_0' = \dfrac{53^2}{2 \times 7.83} = 179.37m$

(1.9)



$R = 3.4\ km$

Angle $\theta = \frac{s}{R} = \frac{2.1\ km}{3.4\ km} = 0.62$ radians

$0.62 \text{ radians} \ \frac{180^{\circ}}{\pi \text{ radians}} = 35.4^{\circ}$.

(2.79)

$v_0 = 80 \frac{km}{h} \cdot \frac{10^3 m}{km} \cdot \frac{1h}{3600 s} = 22.2 \frac{m}{s}$

$a = -2.1 \frac{m}{s^2}$

$v_0' = 25 \frac{km}{h} = 6.94\ m/s$

$a' = 0$

how soon and at what relative speed will they collide? (same time & position)

$x \quad = \quad x'$

(2) $x = x_0 + v_0 t + \frac{1}{2} a t^2$

$= v_0 t + \frac{1}{2} a t^2$

$x' = x_0' + v_0' t + \frac{1}{2} a' t^2$

$= 50m + v_0' t + \frac{1}{2} a' t^2$

$\Rightarrow \quad 22.2 t - \frac{1}{2} 2.1 . t^2 = 50 + 6.94 t$

$0 = 1.05 t^2 - 15.56 t + 50$

$t = \frac{15.56 \pm \sqrt{15.56^2 - (4 \times 1.05 \times 50)}}{2.1} = \begin{cases} 10s \\ 4.7s \end{cases}$

→ They will collide 4.7s after this instant

→ Condition for no collision - $\underbrace{(v_0 - v_0')^2}_{15.56^2} - 4 \times \frac{a}{2} \times 50 < 0$

$a \quad \frac{15.56^2}{100} = 2.42\ m/s$

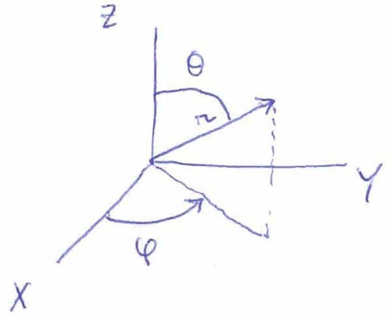
$v = v_0 + at$ $\qquad\qquad$ $v' = v_0' + a't$

$= 22.2 - 2.1 \times 4.99$ $\qquad\qquad$ $= 6.94 \ m/s$

$= 11.76 \ m/s$

At collision, relative speed is $11.76 \frac{m}{s} - 6.94 \ m/s = 4.82 \ m/s$

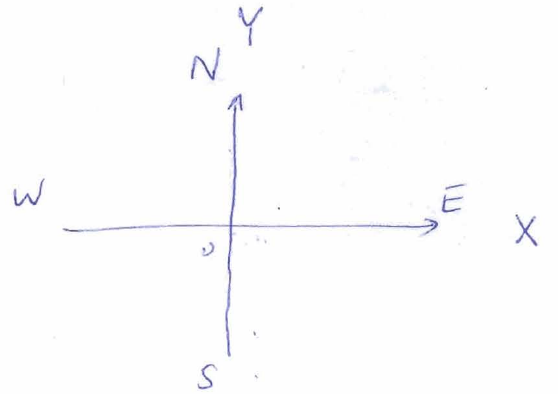
Spherical Coordinate:



(x, y, z) 3D Cartesian $\longrightarrow$ (r, φ, θ)

r: radius. φ: "phi" θ: "theta"

3.36)



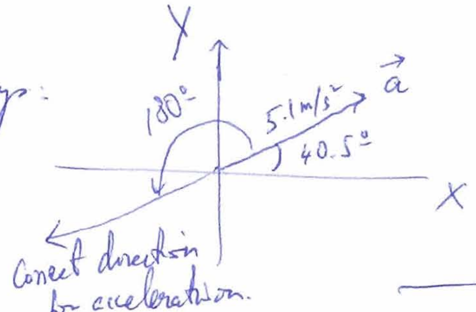
$\vec{v}_i = +2100 \frac{km}{h} \hat{i}$ $\xrightarrow{2.5 min}$ $\vec{v}_f = -1800 \frac{km}{h} \hat{j}$

Average acceleration vector $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{-500 \hat{j} - 583.3 \hat{i}}{150}$

$2100 \frac{km}{h} \cdot \frac{1h}{3600s} \cdot \frac{10^3 m}{1 km} = 583.3 \ m/s$; $1800 \frac{km}{h} \rightarrow \frac{1800}{3.6} \frac{m}{s} = 500 \ m/s$

$\vec{a} = -3.3\hat{j} - 3.9\hat{i}$ $\qquad$ $\begin{cases} a = \sqrt{(-3.9)^2 + (-3.3)^2} = \boxed{5.1 \ m/s^2} \\ \theta = \tan^{-1}\left(\frac{-3.3}{-3.9}\right) = 40.5^\circ \end{cases}$

This says:



$40.5^\circ + 180^\circ = \boxed{220.5^\circ}$