University of Massachusetts at Boston

College of Advancing & Professional Studies

Critical and Creative Thinking Program

Mathematical Thinking

CrCrTh 650

Course description

This course explores several types of mathematical thinking in the context of number theory, algebra, geometry, and introductory calculus, and relates them to critical and creative thinking skills. Developmental and experiential factors in learning and teaching mathematics are considered, as well as techniques for determining a learner's mathematical abilities and learning styles. Readings, discussion, research, and problem-solving are used to provide a historical context, and to suggest connections with other disciplines. Individual and small-group projects are adapted to student interests. No formal mathematical background beyond high school algebra and geometry is required.

Fall 2017 Syllabus

Components of the syllabus:

I. Quick access to key information and links that should be bookmarked on your browser

followed by

II. Information to get started, orient yourself, and refer back to from time to time.

III. Contract: What is expected overall.

IV. Schedule of classes: What is expected each session and why -- how each session contributes to the unfolding of the course (starting with list of links to specific sessions).

V. Bibliography

POST-IT the start of each component in your printed version of this syllabus

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Office hours (http://bit.ly/pjthangout or in office):	Tuesday 1.40-3.40pm ptaylor.wikispaces.umb.edu/PTOfficeHours, or by arrangement
Class time & location	Wednesdays 6.30-9pm, 9/13-12/13; by Hangout
URL for hangouts	http://bit.ly/650hangout BOOKMARK THIS! http://bit.ly/CCTbreakout1 , http://bit.ly/CCTbreakout2 , http://bit.ly/CCTbreakout3
Blog	http://crcrth650mathematicalthinking.wordpress.com BOOKMARK THIS!
Report glitches in online materials	using this form
Syllabus	crcrth650.wikispaces.umb.edu, with a menu of useful links at the top right BOOKMARK THIS! (backup copy at http://www.faculty.umb.edu/pjt/650-17.pdf)
Public g+ community	http://bit.ly/SICWgplus , for public sharing of final products (optional)

II. Information to get started, orient yourself, and refer back to from time to time

Pointers about the preparation assumed for this course
(in lieu of formal prerequisites): CrCrTh 650 is appropriate for any student with a commitment to the personal development of themselves and others in the area of mathematical thinking. You will find it helpful to be familiar with the university's library and research services. You should be prepared to make time outside class--at least 6.5 hours/week--for undistracted work on the course and to view each assignment and each session in relation to the unfolding of learning during the course. (That is, do not expect the syllabus and online links to allow you to simply cut to the chase about what to do for the following day's class.)

In Fall 2017, the format of the course has two strands, taking up half the time of each session. The first strand is centered on 4-week "collaborative explorations" (CEs), a variant of project-based learning (PBL) that begin from a scenario or case in which the issues are real but the problems are not well defined, which leads participants to shape their own directions of inquiry and develop their skills as investigators and teachers (in the broadest sense of the word). The basic mode of a CE centers on interactions in small groups (online or face-to-face) over a delimited period of time in ways that create an experience of re-engagement with oneself as an avid learner and inquirer--as this quote from a student in a PBL course evokes:

- This course is a gift – the chance to be open – open-ended in design, open to process, open to other perspectives, open to changing your ideas, and open to sharing. Of course this means it's risky too – you won't always know when you're coming from or where you are going – you might think you aren't sufficiently grounded by the course. But you have the freedom to change that – and being on the other side of it now, I see it works out beautifully. The attention to process provides you the tools to grow and by the end you're riding the wave of your earlier work...

The CE format is designed to allow each student to
a) undertake intensive reading in the area of mathematical thinking and learn from other students through their annotated bibliography entries, presentations, and written products;
b) shape a path and final products for each CE that link closely with your personal interests; and
c) see yourselves as contributors to ongoing development of the field, especially by sharing of products with future students on the blog and (optional) with the wider public on a google+ community (and eventually perhaps a book).

The second strand will involve activities or discussion based on shared readings around key concepts or issues in the field. Each activity promotes a way to improve mathematical thinking, but allows for insights about one's thinking to emerge in its own way. Plus-Delta feedback at end of most activities fosters the formation of these insights as well as future improvements of the activity for future offerings of the course. Indeed, the instructor, whose mathematical thinking was formed in the 1960s and early 70s, is looking to students' inquiries in the CEs as well as feedback on the activities to help him clarify what are the most important ways that people's needs and capacities for mathematical thinking have shifted since then.

Course Objectives
By the end of the semester, you will have:

- a set of tools, experiences, activities, knowledge of publications, and an enhanced disposition to self-directed lifelong inquiry around:

  * your own mathematical thinking; and
  * what is needed to teach or guide others re: the above in ways that might depart markedly from your previous schooling and experience.

- a critical understanding of collaborative explorations and allied approaches to project-based learning in relation to participants re-engaging with themselves as avid learners and inquirers.

Texts and Materials
Readings for the course consist of

a) articles and book chapters specified in each activity. These can be downloaded from password-protected page;
b) work read as part of CEs, which, with planning, can be borrowed from libraries.

You will need to be set up to use interlibrary loan (either at UMB or at your local library) to get materials that interest you when needed.

Recommended (available from online retailers):

- as fieldbook of tools and processes: Taylor, P. and J. Szteiter (2012) Taking Yourself Seriously: Processes of Research and Engagement Arlington, MA, The Pumping Station
- as guides to writing: Daniel, D., C. Fauske, P. Galeno, and D. Mael. (2001). Take Charge of Your Writing: Discovering Writing Through Self-Assessment. Boston, Houghton Mifflin.
- Hacker, D. (2000) A Pocket Style Manual. Boston, Bedford/St. Martins - OR equivalent pocket manual on writing.

Technical set-up

- Make bookmarks on your browser for key links in this syllabus
- Accept invitation to join the private wordpress blog where all exchanges for the course will happen
- Get Excel or another spreadsheet program on your computer (or access to Sheets on google drive)
- Prepare for meetings on hangout (sign up for a http://plus.google.com account, get the audio & video plugins installed, and let instructor know your gmail address). Practice accessing the course hangout.
- Establish reliable, undistracted access to the internet for class sessions (with ethernet, not wifi, connection to wifi modems unless absolutely impossible)
- Know your official @umb.edu student email address and password (for access to password-protected materials)
- Set up access to online bibliographic databases
- Arrange bibliographic software for references
- Establish off-campus connection to UMass library , including get the library barcode for your student ID card from the library and interlibrary loan;
- Accept invitation to join the public google+ community associated with the SICW track of the CCT program
- Read "What is plagiarism?" and choose your citation style .
- Complete on-line tutorial (if needed) and explore the library wikipage for CCT courses.

Writing Support
For graduate students, see http://cct.wikispaces.umb.edu/writingsupport.

Accommodations
Sections 504 and the Americans with Disabilities Act of 1990 offer guidelines for curriculum modifications and adaptations for students with documented disabilities. The student must present any adaptation recommendations to the professors within a reasonable period, preferably by the end of the Drop/Add period.

Code of Conduct
The University's Student Code of Conduct

(https://www.umb.edu/life_on_campus/policies/community/code) exists to maintain and protect an environment conducive to learning. It sets clear standards of respect for members of the University community and their property, as well as laying out the procedures for addressing unacceptable conduct. Students can expect faculty members and the Office of the Dean of Students to look after the welfare of the University community and, at the same time, to take an educational approach in which students violating the Code might learn from their mistakes and understand how their behavior affects others.

Students are advised to retain a copy of this syllabus in personal files for use when applying for certification, licensure, or transfer credit.

This syllabus is subject to change, but workload expectations will not be increased after the semester starts. (Version 10 September 2017)

III. Contract: What is expected overall

- The course requirements revolve around work-in-progress presentations and written products for each Collaborative Exploration (CE) and around participation items. It is expected that you will spend at least 6.5 hours per session outside class time reading, researching, and writing.
- The draft CE products and some other written assignments are commented on, but not graded. Not grading keeps the focus on interaction around written work. You are expected to read comments carefully, consult with the peer commenter or instructor if you don't understand a comment they made, revise thoughtfully—not superficially—in response to the comments, and resubmit.
- For participation, the hope is that you come to classes prepared to engage in both parts of the sessions (the CE [see "Steps" in each CE description] and the activities [see "Preparation" in each upcoming activity]), share sources you find useful so that other students can learn from them, help others through feedback on drafts, bring the instructor into your processes of learning and inquiry early and often–starting with making sure you appreciate the thinking behind the details that this long syllabus lays out (much of which a conventional syllabus leaves hidden). If you share and internalize these hopes, you will end up meeting the 80% target in the grading scheme (see below) without having to think about the specific checklist of items.
- The course works by building from CE to CE, session to session, so late submissions detract significantly from the learning possible in class sessions. However, each student can ask for extensions on two assignments or participation items, moving the due date as far back as the last session. (No explanation is needed; simply insert the new due date on your assignment checklist.)
- You should aim for 10 of 12 writing/presentation assignments submitted by the due dates with the draft CE products revised deeply in response to comments, as well as for 16 of 20 participation items fulfilled (="80% target"). (Allowing a fraction of assignments to be skipped without penalty or explanation accommodates the contingencies of your lives.)
- Use a personal copy of the checklist wikipage to keep your own log of assignments and participation items completed and to keep track of due dates. Do not expect class-time or meetings with the instructor to be taken up reminding you. Similarly, if you get behind, you take the initiative to submit a plan to catch up or reassure the instructors that you have, in light of your other commitments, chosen to take the grading consequences of missing assignments or due dates. (Incompletes are given only in special circumstances [detailed here].)
- If you reach the target (see above)—and the goal is to work with everyone to achieve that—you get at least a B+ and a rubric is used to determine B+, A- or A (respectively, <10, >10 but < 15, 15+ points on the rubric).
- Only if you do not get to the automatic B+ level is the grade based on points for assignments and participation items = 6 for each well-prepared work-in-progress presentation, 3 for each

writing assignment or half-assignment submitted by due dates, additional 3 for each CE product revised deeply in response to comments *plus* 1.25 for each participation item fulfilled, up to a maximum of 80 points. The minimum grade for B+ is 80, for B is 72.5; for B- is 65; for C+ is 57.5; and for C is 50 points.

- The different assignments and participation items are listed below so as to be explicit about the course contract. Of course, to undertake these assignments and items you need more information—see the guidelines supplied on the Notes wikipage as well as the overall expectations conveyed in the rubric below.

Written assignments and presentation (2/3 of grade)

A. Work-in-progress presentations for each CE (well-prepared-- not informal or extemperaneous) (=3 assignments)

due week 3 of the CE

B. Product for each CE (1200 words): draft that builds on W-I-P presentation and responds to plus-delta comments, then revised again in response to comments from an instructor and a peer and posted to the blog and (optionally) to SICW's public google+ community (=3 assignments)

draft submitted by email to instructor by week 4 of the CE, then distributed by instructor for peer comments; revised version posted to the blog, due two weeks later. (Students may delete or hide their blog postings any time after the semester.)

C. Blog entries required during the process of each CE (e.g., Bibliography contributions with paragraph-length annotations, Notes on inquiries pursued) (=12 half-assignments)]

Participation Items (1/3 grade)

a. Building learning community through attendance and participation at class meetings based on preparation between meetings, including i) inquiry and reading on the CE between sessions (see also C&D above) and ii) making notes on the readings in depth sufficient to undertake the week's activity (=13 items).
b. Syllabus treasure-hunt, session 2
b1. Abiding by conventions for file naming and subject lines for email submissions, whole semester (-1 for each reminder after first)
c. Minimum of two in-office, phone, or live online conferences on your assignments and projects, by session 5 and by session 11 (= 2 items)
d. Peer commentaries on other students' draft products (= 3 items)
e. Assignment checklist, recorded throughout semester, then submitted (as scanned pdf) session 13

Rubric

For each of the following 10 qualities, * [= "fulfilled very well", 2 points], OK [= "did an OK job, but room for more development/attention", 1 point], or - [= "to be honest, this was not my strength in this course", 0 points]

- A sequence of assignments paced more or less as in syllabus (and revisions timely),
  • often revised thoroughly and with new thinking in response to comments.
- CE Projects innovative,
  • well planned and carried out with considerable initiative, and
- CE Project reports clear and well structured,
  • with supporting references and detail, and professionally presented.
- Active, prepared participation in building class as learning community, including during sessions and
  • conscientious peer commentary on other student's assignments.
- Consistent work outside sessions as evidenced in Blog entries required during process of each CE

- Framework and plan for practice in CE3 indicating deep reflection about how to move from learning to implementation/teaching in your specific situation.

Plagiarism: Using another person's ideas or material you did not write without citing the source is plagiarism and is unacceptable (seelibrary guide and Academic Honesty policies).

IV. Schedule of classes: What is expected each session and why -- how each session contributes to the unfolding of the course

1. 9/13, 2. 9/20, 3. 9/27, 4. 10/4, 5. 10/11, 6. 10/18, 7. 10/25, 8. 11/1, 9. 11/8, 10. 11/15, no class 11/22, 11. 11/29, 12. 12/6, 13. 12/13

The Sessions have two parts of 60-75 minutes, with a 10-minute break between them: 1) The CE component; 2) Activities around a key concept in the field.

Session 1
Introductions and When do people use (or need) mathematical thinking?
Preparation:
Get set up on Technical matters
View video Bennett: "Why Math Instruction Is Unnecessary.
Session Exercises:

- Warm-up: What questions would you ask Bennett, based on his TEDx video?
- Freewriting (to bring students' ideas and experience to the surface): "If asked when do people —myself included— use (or need) mathematical thinking, what comes to mind includes..."
- Share something about your thinking with a neighbor (in breakout room, returning after 3 minutes)
- Autobiographical introductions : 4 minutes to explain "How I came to be a person interested in learning more about mathematical thinking--how to do it myself and teach/foster it in others. Each introduction followed by “connections and extensions” feedback from listeners.
- Activity on When do people use (or need) mathematical thinking?

• Introduction to Collaborative Explorations.

including the steps each week and the basic rhythm of the course.
• Quick preview of syllabus and tasks to get set up.
• Take stock of the session (Critical incident questionnaire)

Follow-up:

- Syllabus quiz to get acquainted with organization of course materials.
- Look ahead to what work is due in the sessions ahead.
- Read and commence research on CE1 (below).
- Peruse connections and extensions assembled at here

CE1

Shifting: Changes in the ways people need to think mathematically edit
(A CE in which students identify a range of ways in which changes in work, technology, commerce, and social life have changed our needs and capacities for mathematical thinking, find patterns in them, and consider implications for education, including lifelong learning.)

As work, technology, commerce, and social life change, our needs and capacities for mathematical thinking shift. For example, at primary school before conversion to metric in Australia, I was assigned problems involving weights in units of tonnes, hundredweight, quarters, pounds, and ounces. After weights became measured in grams and kilos and metric tons, addition was so simple teachers found others ways to occupy students' time in class. The arrival of hand calculators helped diminish further the need for arithmetic. That said, my ability to do mental arithmetic applying multiplication tables

means that I am quicker at seeing what ball park an answer should be in and thus detecting when someone has made a mistake inputting figures into their calculator. That said, class time for me as a child centered on quiet solitary solving of multiple problems of the same kind so there was little scope for collaboration, peer-to-peer support (except in the form of cheating), or for discussion of alternative approaches.

One side in the "Math Wars" maintains that computational "skills should be memorized and practiced, using time-tested traditional methods until they become automatic" (Wikipedia, n.d. *). However, instead of arguing a position in the polarized arena of the Math Wars (e.g., did the traditional methods actually work back in their time?), let us imagine a *book* that treats the audience as capable of addressing the complexities of change in work, technology, commerce, and social life as it relates to shifts in our needs and capacities for mathematical thinking. Then let us identify patterns in past changes, with a view to helping readers think about the implications for formal education as well as for the ways each of us continues to learn in response to ongoing change over our lifetimes. (*Wikipedia, n.d. "Math wars," https://en.wikipedia.org/wiki/Math_wars (viewed 4 Sep 17))

The end goal for the CE is that the class as a whole produces thought-supporting, constructive 1200 word entries for this hypothetical book. A premise for this book is that it would be unlike other mathematical thinking texts. Indeed, it may be more like a combination of provocations and resources for people—not only teachers—who want to foster ongoing development of people's mathematical thinking. In this spirit, this CE is an experiment—it is not clear in advance what a "pattern" is or what ways you will invent to "help readers think about the implications." (Steps to undertake and when.)

Session 2: Shifting: Changes in the ways people need to think mathematically (CE1) + Spreadsheet as a tool that extends and constrains our thinking

Preparation:

- Reading as listed in Preparation section for Activity.
- Get Excel or another spreadsheet program on your computer (or access to Sheets on google drive)
- Overview of dialogue process and guidelines

Session Exercises:

- Dialogue hour to share and clarify what we are inquiring into regarding the case for CE1.
- Q&A about course requirements and wiki organization, including expectation for next week's Work-in-progress presentation
- Activity: Spreadsheet as a tool that extends and constrains our thinking (details)

Follow-up:

- Continue working on CE 1.
- First office hour could happen or be scheduled; required by session 5.
- Look ahead to what work is due in the next session. (*This follow up item is assumed and won't be stated from here on.*)

Work due by the first day of this session:

- Participation item b, Syllabus Quiz.

Session 3: Shifting: Changes in the ways people need to think mathematically (CE1) + Simple rules generate complex behaviors

Preparation:

- Prepare Work-in-progress presentation
- Reading as listed in Preparation section for Activity. (*From this point reading to prepare for activities is assumed and not listed.*)

Session Exercises:

- 5-minute Presentations on Work-in-Progress (see instructions, with short peer plus-delta comments, plus any additional tips, on each talk (using form at http://bit.ly/PlusDelta). The order of presentations is alphabetical by last name.
- Activity: Simple rules generate complex behaviors (details)

Follow-up:

- Digest feedback on Work-in-progress presentation and develop a product for the CE that stands on its own (i.e., can be understood without being narrated)

Work due by the first day of this session:

- Work-in-progress presentation (during class)
- First office hour meeting either completed or scheduled by now; required by session 5.

Session 4: Shifting: Changes in the ways people need to think mathematically (CE1) + Big data allows micro-targeting

Session Exercises:

- Dialogue Hour for Taking stock of the first Collaborative Exploration
- Activity: Big data allows micro-targeting (details)

Follow-up:

- Comment on another student's draft product (forwarded to you by instructor by email)
- Arrange asap to get via Inter-library loan or other means, readings you think might interest you, e.g., in the Entry Points given at end of CE 2.

Work due by the first day of this session:

- Draft of your CE 1 product emailed as pdf attachment to instructor with subject line "650ssignment"
- First office hour either completed or scheduled by now; required to be completed before session 5.

CE2

Constructing: Best practices for fostering mathematical thinking edit

(A CE in which students learn as much as possible about how mathematical thinking is presented and promoted by others.)

Imagine a continuation of the book in CE1: a section that aims to help readers appreciate the idea that everyone can think mathematically and to help them help others appreciate that idea. The end-product of this CE are drafts of entries to this section of the book, which might take the form of text, maps, schemas, mp3s, problem sets, or something else (adding up to at least 1200 words or its page-equivalent, in one or more entries). These entries should introduce and organize key resources from how mathematical thinking is presented and promoted by others, i.e., key concepts, issues and debates, references to research, quotes or paraphrases from those references, interactive activities and personal habits, people and organizations to take note of, appropriate stories. (Do not be concerned about whether your entries overlap with anyone else's.)

Some questions that *might* stimulate your inquiries:

- How much have well-worn sources from the 80s and 90s been superseded by more recent research and writing; how much do old sources hold up?
- Could the mathematical thinking process be thought of less as adding rule-bound (or algorithmic) practices and more as recognizing and removing obstacles that have come into place and obscured natural mathematical thinking? What authors have promoted the latter approach?
- How much does the mathematical thinking process need to involve individuals seeking or creating supportive "context," e.g., arranging sounding boards or establishing one's surroundings as a "studio" to make a space where critical thinking comes easier? What is

known about how spaces for mathematical thinking, communities and historical periods came together? What does mathematical thinking mean in different fields of work?
- To the extent that the mathematical thinking process like the creative thinking process involves the capacity to manage, seek out, even welcome risk, struggle and failure, how can we feel more comfortable and supported in allowing "failures" to happen... of letting go of positions we once held strongly to?
- What is there to support, or contradict, the idea that everyone can think mathematically? In guiding those who believe that they are not mathematical thinkers, what steps might be taken to encourage them to at least explore the possibility?
- How is improvement in mathematical thinking assessed? How are different tools and activities to foster mathematical thinking evaluated?

The *process* towards the end *products* should involve reading and digesting as much as you can in the time available, guided by some of the questions above that interest you. The assumption (is this justified?) is that your experience undertaking CE1 before having looked at how mathematical thinking is presented and promoted by others will help you to choose topics that most grab your interest and be engaged in learning about them. In any case, there is no expectation that you think like a textbook writer who has to cover every topic. Instead, you should identify a theme that can govern what your writing focuses on. Entry points for readings are given by:

- National Council on Education and the Disciplines (2003), "Why numeracy matters," https://www.maa.org/sites/default/files/pdf/QL/WhyNumeracyMatters.pdf (viewed 7 Sep 17)
- the syllabi from CCT courses in Mathematical Thinking (2013, 2006 , 1999 and http://www.cct.umb.edu/courses.html#650);
- The abundant resources of NCTM, https://www.nctm.org/
- "8 alternatives to traditional mathematics education," http://davidwees.com/content/8-alternatives-traditional-mathematics-education/ (viewed 8 Sep 17)
- CCT syntheses addressing mathematics
- Videos bookmarked in diigo group on Mathematical Thinking (but always search for critiques of ideas enthusiastically presented so as to put them in perspective)

(Steps to undertake and when.)

Session 5: Constructing: Best practices for fostering mathematical thinking (CE2)+ Inquiry-based learning: problem-posing, problem-solving, and persuasion

Preparation:

- Read CE2 and begin inquiry. Arrange *without further delay* to get via Inter-library loan or other means, readings you think might interest you in CE 2.

Session Exercises:

- Autobiographical stories , retold in relation to CE 2
- Activity: Inquiry-based learning: problem-posing, problem-solving, and persuasion (details)

Follow-up:

- Continue working on CE 2.

Work due by the first day of this session:

- First office hours meeting completed by today; Schedule 2nd meeting before session 10.

Session 6: Constructing: Best practices for fostering mathematical thinking (CE2)+ Correlation, causation, and consequences

Session Exercises:

- Dialogue hour to share and clarify what we are inquiring into regarding the case. (Reminder of dialogue guidelines).

- Activity: Correlation, causation, and consequences (details)

Follow-up:

- Continue working on CE 2.

Work due by the first day of this session:

- Final version of your product from CE 1, revised in response to comments from peers and instructors, uploaded to blog (and, optionally, to public google+ community, http://bit.ly/SICWgplus)

Session 7: Constructing: Best practices for fostering mathematical thinking (CE2)+ Traps in thinking about probability

Preparation:

- Prepare Work-in-progress presentation

Session exercises:

- Work-in-progress presentations, each followed by Plus-Delta feedback (online or on paper). The order of presentations is reverse alphabetical by last name.
- Activity: Traps in thinking about probability (details)

Follow-up:

- Digest feedback on Work-in-progress presentation and develop a product for the CE

Work due by the first day of this session:

- Work-in-progress presentation for CE 2 (during class)

Session 8: Constructing: Best practices for fostering mathematical thinking (CE2)+ Tools to extend thinking by considering feedback loops

Session exercises:

- Dialogue Hour for Taking stock of the second Collaborative Exploration
- Activity: Tools to extend thinking by considering feedback loops (details)

Follow-up:

- Comment on another student's draft product (forwarded to you by instructor by email)

Work due by the first day of this session:

- Draft of your CE 2 product emailed as pdf attachment to instructor with subject line "650ssignment"
- Two more Annotated bibliography entries posted (should have submitted total of four by now).
- Second office hours meeting either completed or scheduled by now; required to be completed by session 10.

CE3

***Framing and Practicing: Ongoing development beyond the course**, CE3 (classes 9-12)* edit

(A CE in which students, building on CEs 1 & 2, formulate specific plans for how to continue your own development as a mathematical thinker and, as a result, be able to foster the same among colleagues or students in your work/life/teaching situation.)

Books such as Julia Cameron's The Artist's Way provide readers with a program for developing one's creativity, but what is the equivalent for developing one's mathematical thinking? In any case, given that a mark of creativity is to develop one's own program, not follow someone else's, what would your program—or framework—for mathematical thinking look like? This said, all invention involves borrowing, so the challenge is really to synthesize elements from sources encountered during and before this course. These syntheses should be selected and organized in a *framework*so as to inspire and inform your efforts in extending mathematical thinking *beyond the course*. For a brief introduction to the experience of past students who prepared frameworks (called "manifestos"), not for mathematical thinking, but for critical thinking, see section 2 of Taylor (2002). For the full manifestos

from a 1999 critical thinking class, see Readings. These frameworks might make up a 3rd section of the book from CE1 and CE2.

Corresponding to your framework, what is your plan for practice to develop your ability to foster the development of others as mathematical thinkers in your work/life/teaching situation? The plan should demonstrate how and when you plan to put into practice the skills and tools from the course - in your work situation or community, and/or how you could adapt and practice using those tools for opportunities in the future. You should include a plan for evaluating the outcome so you learn from experience and practice. (Test score results are not the only measure of improvement in mathematical thinking!) For examples of Plans for Practice from different CCT courses, see Readings and google+ community (search for #601). (Steps to undertake and when.)

Session 9: Framing and Practicing: Ongoing development beyond the course (CE3) + Prediction

Preparation:

- Read CE3, review manifestos from a past course (linked to password-protected readings), and begin inquiry. Arrange now to get via Inter-library loan or other means, readings you think might interest you in CE 3.

Session Exercises:

- Autobiographical stories , retold in relation to CE 3
- Activity: Prediction (details)

Follow-up:

- Continue working on CE 3.

Work due by the first day of this session:

- Comments on another student's draft product from case 2, emailed to student and cc'd to instructors, with subject line "650ssignment".
- Second office hours meeting either completed or scheduled by now; required to be completed by session 10.

Session 10: Framing and Practicing: Ongoing development beyond the course (CE3) + Designing a critical thinking activity using spreadsheets

Session Exercises:

- Dialogue hour to share and clarify what we are inquiring into regarding the case.
- Activity: Designing a critical thinking activity using spreadsheets (details)

Follow-up:

- Continue working on CE 3.

Work due by the first day of this session:

- Final version of your product from CE 2, revised in response to comments from peers and instructors, uploaded to blog (and, optionally, to public google+ community, http://bit.ly/SICWgplus)
- Second office hours meeting completed by today.

Session 11: Framing and Practicing: Ongoing development beyond the course (CE3) + Critical assumptions at the foundations of statistical analysis

Preparation:

- Prepare Work-in-progress presentation

Session Exercises:

- Presentations on Work-in-Progress, with short peer plus-delta comments, plus any additional tips, on each talk (using form at http://bit.ly/PlusDelta). The order of presentations is

alphabetical by first name.
- Activity: Critical assumptions at the foundations of statistical analysis (details)

Follow-up:

- Digest feedback on Work-in-progress presentation and develop a product for the CE

Work due by the first day of this session:

- Work-in-progress presentation for CE 3 (during class)

Session 12: Framing and Practicing: Ongoing development beyond the course (CE3) + Visualization of inequalities

Session Exercises:

- Dialogue Hour for Taking stock of the third Collaborative Exploration
- Activity: Visualization of inequalities (details)

Follow-up:

- Comment on another student's draft product (forwarded to you by instructor by email)

Work due by the first day of this session:

- Draft of your CE 3 product emailed as pdf attachment to instructor with subject line "650ssignment"
- Two more Annotated bibliography entries posted (should have submitted total of six by now).

Session 13: Heterogeneity + Taking Stock of the Course: where have we come from and where are we headed?

Session Exercises:

- Activity: Heterogeneity (details)
- Taking stock in multiple ways with the aim of:
    - feeding into your future learning (and other work), you take stock of your process(es) over the semester;
    - feeding into instructor's future teaching (and future learning about how students learn), instructor takes stock of how you, the students, have been learning.
- Sense of Place Map
- Discussion of (shareable) insights that emerged and reactions to the exercise
- Official evaluation that starts with a self-evaluation (to be administered by online form).
- Closing circle (plus-delta).

Follow-up:

- Revise in response to instructor's comments and complete report.

Work due by the first day of this session:

- Final version of your product from CE 3, revised in response to comments from peers and instructors, uploaded to blog (and, optionally, to public google+ community, http://bit.ly/SICWgplus)
- Assignment checklist, recorded throughout semester (as scanned pdf)
- Completion contract (if needed; see policies).

V. Bibliography

(Link to password-protected pdf's of readings)

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