From: Tobias, Sheila (1978). Overcoming Math Anxiety. New York: W.W. Norton and Company, Inc.

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The Nature of Math Anxiety: Mapping the Terrain

A warm man never knows how a cold man feels.
—Alexander Solzhenitsyn

Symptoms of Math Anxiety

The first thing people remember about failing at math is that it felt like sudden death. Whether it happened while learning word problems in sixth grade, coping with equations in high school, or first confronting calculus and statistics in college, failure was sudden and very frightening. An idea or a new operation was not just difficult, it was impossible! And instead of asking questions or taking the lesson slowly, assuming that in a month or so they would be able to digest it, people remember the feeling, as certain as it was sudden, that they would *never* go any further in mathematics. If we assume, as we must, that the curriculum was reasonable and that the new idea was merely the next in a series of learnable concepts, that feeling of utter defeat was

simply not rational; and in fact, the autobiographies of math anxious college students and adults reveal that no matter how much the teacher reassured them, they sensed that from that moment on, as far as math was concerned, they were through.

The sameness of that sudden death experience is evident in the very metaphors people use to describe it. Whether it occurred in elementary school, high school, or college, victims felt that a curtain had been drawn, one they would never see behind; or that there was an impenetrable wall ahead; or that they were at the edge of a cliff, ready to fall off. The most extreme reaction came from a math graduate student. Beginning her dissertation research, she suddenly felt that not only could she never solve her research problem (not unusual in higher mathematics), but that she had never understood advanced math at all. She, too, felt her failure as sudden death.

Paranoia comes quickly on the heels of the anxiety attack. "Everyone knows," the victim believes, "that I don't understand this. The teacher knows. Friends know. I'd better not make it worse by asking questions. Then everyone will find out how dumb I really am." This paranoid reaction is particularly disabling because fear of exposure keeps us from constructive action. We feel guilty and ashamed, not only because our minds seem to have deserted us but because we believe that our failure to comprehend this one new idea is proof that we have been "faking math" for years.

In a fine analysis of mathophobia, Mitchell Lazarus explains why we feel like frauds. Math failure, he says, passes through a "latency stage" before becoming obvious either to our teachers or to us. It may in fact take some time for us to realize that we have been left behind. Lazarus outlines the plight of the high school student who has always relied on the memorize-what-to-do approach. "Because his grades have been satisfactory, his problem may not be apparent to anyone, including himself. But when his grades finally drop, as they must, even his teachers are unlikely to realize that his problem is not something new, but has been in the making for years."[1]

It is not hard to figure out why failure to understand mathematics can be hidden for so long. Math is usually taught in discrete bits by teachers who were themselves taught this way; students are tested, bit by bit, as they go along. Some of us never get a chance to integrate all these pieces of information, or even to realize what we are not able to do. We are aware of a lack, but though the problem has been building up for years, the first time we are asked to use our knowledge in a new way, it feels like sudden death. It is not so easy to explain, however, why we take such personal responsibility for having "cheated" our teachers and why so many of us believe that we are frauds. Would we feel the same way if we were floored by irregular verbs in French?

One thing that may contribute to a student's passivity is a common myth about mathematical ability. Most of us believe that people either have or do not have a mathematical mind. It may well be that mathematical imagination and some kind of special intuitive grasp of mathematical principles are needed for advanced research, but surely people who can do college-level work in other subjects should be able to do college-level math as well. Rates of learning may vary. Competence under time pressure may differ. Certainly low self-esteem will interfere. But is there any evidence that a student

needs to have a mathematical mind in order to succeed at *learning math*?

Leaving aside for the moment the sources of this myth, consider its effects. Since only a few people are supposed to have this mathematical mind, part of our passive reaction to difficulties in learning mathematics is that we suspect we may not be one of "them" and are waiting for our nonmathematical mind to be exposed. It is only a matter of time before our limit will be reached, so there is not much point in our being methodical or in attending to detail. We are grateful when we survive fractions, word problems, or geometry. If that certain moment of failure hasn't struck yet, then it is only temporarily postponed.

Sometimes the math teacher contributes to this myth. If the teacher claims an entirely happy history of learning mathematics, she may contribute to the idea that some people—specifically her—are gifted in mathematics and others—the students—are not. A good teacher, to allay this myth, brings in the scratch paper he used in working out the problem to share with the class the many false starts he had to make before solving it.

Parents, especially parents of girls, often expect their children to be nonmathematical. If the parents are poor at math, they had their own sudden death experience; if math was easy for them, they do not know how it feels to be slow. In either case, they will unwittingly foster the idea that a mathematical mind is something one either has or does not have.

Interestingly, the myth is peculiar to math. A teacher of history, for example, is not very likely to tell students that they write poor exams or do badly on papers because they do not have a historical mind. Although we might say that some people have a "feel" for history, the notion that one is *either* historical or nonhistorical is patently absurd. Yet, because even the experts still do not know how mathematics is learned, we tend to think of math ability as mystical and to attribute the talent for it to genetic factors. This belief, though undemonstrable, is very clearly communicated to us all.

These considerations help explain why failure to comprehend a difficult concept may seem like sudden death. We were kept alive so long only by good fortune. Since we were never truly mathematical, we had to memorize things we could not understand, and by memorizing we got through. Since we obviously do not have a mathematical mind, we will make no progress, ever. Our act is over. The curtain down.

Ambiguity, Real and Imagined

> What is a satisfactory definition? For the philosopher or the scholar, a definition is satisfactory if it applies to those things and only those things that are being defined; this is what logic demands. But in teaching, this will not do: a definition is satisfactory only if the students understand it.
>
> —H. Poincaré

Mathematics autobiographies show that for the beginning student the language of mathematics is full of ambiguity. Though mathematics is supposed to have a very precise language, more precise than our everyday use (this is why math uses symbols), it is true that mathematical terms are never wholly free of the connotations we bring to words, and these layers of meaning may get in the way. The problem is not that there is anything wrong with math; it is that we are not properly initiated into its vocabulary and rules of grammar.

Some math disabled adults will remember, after fifteen to thirty years, that the word "multiply" as used for fractions never made sense to them. "Multiply," they remember wistfully, always meant "to increase." That is the way the word was used in the Bible, in other contexts, and surely the way it worked with whole numbers. (Three times six always produced something larger than either three or six.) But with fractions (except the improper fractions), multiplication always results in something of smaller value. One-third times one-fourth equals one-twelfth, and one-twelfth is considerably smaller than either one-third or one-fourth.

Many words like "multiply" mean one thing (like "increase rapidly") when first introduced. But in the larger context (in this case all rational numbers), the apparently simple meaning becomes confusing. Since students are not warned that "multiplying" has very different effects on fractions, they find themselves searching among the meanings of the word to find out what to do. Simple logic, corresponding to the words they know and trust, seems not to apply.

A related difficulty for many math anxious people is the word "of" as applied to fractions. In general usage, "of" can imply division, as in "a portion of." Yet, with fractions, one-third *of* one-fourth requires multiplication. We can only remember this by suspending our prior associations with the word "of," or by memorizing the rule. Or, take the word "cancel" as used carelessly with fractions. We were told to "cancel numerators and denominators" of fractions. Yet nothing is being "cancelled" in the sense of being removed for all time. The same holds true for negative numbers. Once we have learned to associate the minus sign with subtraction, it takes an explicit lesson to unlearn the old meaning of minus; or, as a mathematician would put it, to learn its meaning as applied to a new kind of number.

Knowles Dougherty, a skilled teacher of mathematics, notes:

> It is no wonder that children have trouble learning arithmetic. If you ask an obedient child in first grade, "What is Zero," the child will call out loudly and with certainty, "Zero is nothing." By third grade, he had better have memorized that "Zero is a place-holder." And by fifth grade, if he believes that zero is a number that can be added, subtracted, multiplied by and divided by, he is in for trouble.[2]

People also recall having problems with shapes, never being sure for example whether the word "circle" meant the line around the circle or the space within. Students who had such difficulties felt they were just dumber than everyone else, but in fact the word "circle" needs a far more precise definition. It is in fact neither the circumference nor the area but rather "the locus of points in the plane equidistant from a center" (Fig. II-1).*

A mind that is bothered by ambiguity—actual or perceived—is not usually a weak mind, but a strong one. This point is important because mathematicians argue that it is not the subject that is fuzzy but the learner who is imprecise. This may be, but as mathematics is often taught to amateurs differences in meaning be-

*One student, learning to find the "least common denominator," took the phrase "least common" to mean "most unusual" and hunted around for the "most unusual denominator" she could find. Instead of finding the smallest common denominator, then, she found a very large one and was appropriately chastised by her teacher for misunderstanding the question.

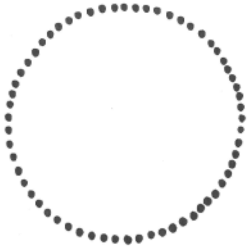

FIG. II-1

tween common language and mathematical language need to be discussed. Besides, even if mathematical language is unambiguous, there is no way into it except through our spoken language, in which words are loaded with content and associations. We cannot help but think "increase" when we hear the word "multiply" because of all the other times we have used that word. We have been coloring circles for years before we get to one we have to measure. No wonder we are unsure of what "circle" means. People who do a little better in mathematics than the rest of us are not as bothered by all this. We shall consider the possible reasons for this later on.

Meanwhile, the mathematicians withhold information. Mathematicians depend heavily upon customary notation. They have a prior association with almost every letter in the Roman and Greek alphabets, which they don't always tell us about. We think that our teachers are choosing X or a or delta (Δ) arbitrarily. Not so. Ever since Descartes, the letters at the end of the alphabet have been used to designate unknowns, the letters at the beginning of the alphabet usually to signify constants, and in math, economics, and physics generally Δ means "change" or "difference." Though these symbols appear to us to be chosen randomly, the letters are loaded with meaning for "them."

In more advanced algebra, the student's search for meaning is made even more difficult because it is almost impossible to visualize complex mathematical relationships. For me, the fateful moment struck when I was confronted by an operation I could neither visualize nor translate into meaningful words. The expression $X^{-2}=\frac{1}{X^2}$ did me in. I had dutifully learned that exponents such as ^{2} and ^{3} were shorthand notations for multiplication: a number or a letter squared or cubed was simply multiplied by itself twice or three times. Trying to translate math into words, I considered the possibility that X^{-2} *meant something* like "*X not multiplied by itself*" or "multiplied by not-itself." What words or images could convey to me what X^{-2} really meant? To all these questions—and I have asked them many times since—the answer is that $X^{-2}=\frac{1}{X^2}$ is a definition consist*ent with what has gone before. I have been shown* several demonstrations that this definition is indeed consistent with what has gone before.* But at the time

*While interviewing for this book, I have finally found out that negative two is a different kind of number from positive two and that it was naïve of me to think that it would have the same or similar effect on X. And it does work. If you divide X^3 by X^5 (remember you subtract exponents when you divide) you end up with $\frac{1}{X^2}$. See the following:

$$X^{-2} \equiv \frac{X^3}{X^5} = \frac{XXX}{XXXXX} = \frac{1}{XX} = \frac{1}{X^2}$$

I did not want a demonstration or a proof. I wanted an explanation!

I dwell on the X^{-2} example because I have often asked competent mathematicians to recall for me how they felt the first time they were told $X^{-2} = \frac{1}{X^2}$. Many remember merely believing what they were told in math class, or that they soon found the equivalency useful. Unlike me, they were satisfied with a definition and an illustration that the system works. Why some people should be more distrustful about such matters and less willing to play games of internal consistency than others is a question we shall return to later.

Willing suspension of disbelief is a phrase that comes not from mathematics or science but from literature. A reader must give the narrator an opportunity to create images and associations and to "enter" these into our mind (the way we "enter" information into a computer) in order to carry us along in the story or poem. The very student who can accept the symbolic use of language in poetry where "birds are hushed by the moon," or the disorienting treatment of time in books by Thomas Mann and James Joyce, may balk when mathematics employs familiar words in an unfamiliar way. If willingness to suspend disbelief is specific to some tasks and not to others, perhaps it is related to trust. One counsellor explains math phobia by saying, "If you don't feel safe, you won't take risks." People who don't trust math may be too wary of math to take risks.

A person's ability to accept the counter-intuitive use of time in Thomas Mann's work and not the new meaning of the negative exponent does not imply that there are two kinds of minds, the verbal and the mathematical. I do not subscribe to the simple-minded notion that we are one or the other and that ability in one area leads inevitably to disability in the other. Rather, I think that verbal people feel comfortable with language early in life, perhaps because they enjoyed success at talking and reading. When mathematics contradicts assumptions acquired in other subjects, such people need special reassurance before they will venture on.

Conflicts between mathematical language and common language may also account for students' distrust of their intuition. If several associated meanings are floating around in someone's head and the text considers only one, the learner will, at the very least, feel alone. Until someone tries to get inside the learner's head or the learner figures out a way to search among the various meanings of the word for the one that is called for, communication will break down, too. This problem is not unique to mathematics, but when people already feel insecure about math, linguistic confusion increases their sense of being out of control. And so long as teachers continue to argue, as they have to me, that words like "multiply" and "of," the negative exponents, and the "circles" or "disks" are not ambiguous at all but perfectly consistent with their definitions, then students will continue to feel that math is simply not for them.

Some mathematics texts solve the problem of ambiguity by virtually eliminating language. College-level math textbooks are even more laconic than elementary texts. One reason may be the difficulty of expressing mathematical ideas in language that is easily agreed upon. Another is the assumption that by the time students get to college they should be able to read symbols. But for some number of students (we cannot know how many since they do not take college-level math)

proofs, symbolic formulations, and examples are not enough. After I had finally learned that X^{-2} must equal $\frac{1}{X^2}$ because it was consistent with the rule that when dividing numbers with exponents we subtract the exponents, I looked up "negative exponents" in a new high school algebra text. There I found the following paragraph.

Negative and Zero Exponents

The set of numbers used as exponents in our discussion so far has been the set of positive integers. This is the only set which can be used when exponents are defined as they were in Chapter One. In this section, however, we would like to expand this set to include all integers (positive, negative and zero) as exponents. This will, of course, require further definitions. These new definitions must be consistent with the system and we will expect all of the laws of exponents as well as all previously known facts to still be true.[3]

Although this paragraph is very clear in setting the stage to explain negative exponents through definitions which are presumably forthcoming, it does not provide a lot of explanation. No wonder people who need words to make sense of things give up.

The Dropped Stitch

"The day they introduced fractions, I had the measles." Or the teacher was out for a month, the family moved, there were more snow days that year than ever before (or since). People who use events like these to account for their failure at math did, nevertheless, learn how to spell. True, math is especially cumulative. A missing link can damage understanding much as a dropped stitch ruins a knitted sleeve. But being sick or in transit or just too far behind to learn the next new idea is not reason enough for doing poorly at math forever after. It is unlikely that one missing link can abort the whole process of learning elementary arithmetic.

In fact, mathematical ideas that are rather difficult to learn at age seven or eight are much easier to comprehend one, two, or five years later if we try again. As we grow older, our facility with language improves; we have many more mathematical concepts in our minds, developed from everyday living; we can ask more and better questions. Why, then, do we let ourselves remain permanently ignorant of fractions or decimals or graphs? Something more is at work than a missed class.

It is of course comforting to have an excuse for doing poorly at math, better than having to concede that one does not have a mathematical mind. Still, the dropped stitch concept is often used by math anxious people to excuse their failure. It does not explain, however, why in later years they did not take the trouble to unravel the sweater and pick up where they left off.

Say they did try a review book. Chances are it would not be helpful. Few texts on arithmetic are written for adults.* How insulting to go back to a "Run, Sport, run!" level of elementary arithmetic, when arithmetic can be infinitely clearer and more interesting if it is discussed at an adult level.

Moreover, when most of us learned math we learned

*Deborah Hughes-Hallett is writing a book (W.W. Norton, 1978) for adults and college students that starts with arithmetic and brings the reader up to calculus, in two volumes.

dependence as well. We needed the teacher to explain, the textbook to drill us, the back of the book to tell us the right answers. Many people say that they never mastered the multiplication table, but I have encountered only one person so far who carries a multiplication table in his wallet. He may have no more skills than the others, but at least he is trying to make himself autonomous. The greatest value of using simple calculators in elementary school may, in the end, be to free pupils from dependence on something or someone beyond their control.

Adults can easily pick up those dropped stitches once they decide to do something about them. In one math counselling session for educators and psychologists, the following arithmetic bugbears were exposed:

How do you get a percentage out of a fraction like $7/16$?

Where does "pi" come from?

How do you do a problem like: Two men are painters. Each paints a room in a different time. How long does it take them to paint the room together?*

The issues were taken care of within half an hour.

This leads me to believe that people are anxious not because they dropped a stitch long ago but rather because they accepted an ideology that we must reject: *that if we haven't learned something so far it is probably because we can't.*

*See Chapter Six for a discussion of fractions and percents; see Chapter Five for a discussion of the Painting-the-Room Problem. *Pi* can be derived by drawing many-sided polygons (like squares, pentagons, hexagons, etc.) and measuring the ratio of their perimeters to their diameters. Even if you do this roughly, the ratios will approach 3.14.

Fear of Being Too Dumb or Too Smart

One of the reasons we did not ask enough questions when we were younger is that many of us were caught in a double bind between a fear of appearing too dumb in class and a fear of being too smart. Why anyone should be afraid of being too smart in math is hard to understand except for the prevailing notion that math whizzes are not normal. Boys who want to be popular can be hurt by this label. But it is even more difficult for girls to be smart in math. Matina Horner, in her survey of high-achieving college women's attitudes toward academic success, found that such women are especially nervous about

competing with men on what they think of as men's turf.[4] Since many people perceive ability in mathematics as unfeminine, fear of success may well interfere with ability to learn math.

The young woman who is frightened of seeming too smart in math must be very careful about asking questions in class because she never knows when a question is a really good one. "My nightmare," one woman remembers, "was that one day in math class I would innocently ask a question and the teacher would say, 'Now that's a fascinating issue, one that mathematicians spent years trying to figure out.' And if that happened, I would surely have had to leave town, because my social life would have been ruined." This is an extreme case, probably exaggerated, but the feeling is typical. Mathematical precocity, asking interesting questions, meant risking exposure as someone unlike the rest of the gang.

It is not even so difficult to ask questions that gave the ancients trouble. When we remember that the Greeks had no notation for multi-digit numbers and that even Newton, the inventor of the calculus, would have been hard pressed to solve some of the equations given to beginning calculus students today, we can appreciate that young woman's trauma.

At the same time, a student who is too inhibited to ask questions may never get the clarification needed to go on. We will never know how many students developed fear of math and loss of self-confidence because they could not ask questions in class. But the math anxious often refer to this kind of inhibition. In one case, a counsellor in a math clinic spent almost a semester persuading a student to ask her math teacher a question *after* class. She was a middling math student, with a B in linear algebra. She asked questions in her other courses, but could not or would not ask them in math. She did not entirely understand her inhibition, but with the aid of the counsellor, she came to believe it had something to do with a fear of appearing too smart.

There is much more to be said about women and mathematics. The subject will be discussed in detail in Chapter Three. At this point it is enough to note that some teachers and most pupils of both sexes believe that boys naturally do better in math than girls. Even bright girls believe this. When boys fail a math quiz their excuse is that they did not work hard enough. Girls who fail are three times more likely to attribute their lack of success to the belief that they "simply cannot do math."[5] Ironically, fear of being too smart may lead to such passivity in math class that eventually these girls also develop a feeling that they are dumb. It may also be that these women are not as low in self-esteem as they seem, but by failing at mathematics they resolve a conflict between the need to be competent and the need to be liked. The important thing is that until young women are encouraged to believe that they have the right to be smart in mathematics, no amount of supportive, nurturant teaching is likely to make much difference.

Distrust of Intuition

> Mathematicians use intuition, conjecture and guesswork all the time except when they are in the classroom.
>
> —Joseph Warren, Mathematician

> Thou shalt not guess.
>
> —Sign in a high school math classroom

At the Math Clinic at Wesleyan University, there is always a word problem to be solved. As soon as one is solved, another is put in its place. Everyone who walks into the clinic, whether a teacher, a math anxious person, a staff member, or just a visitor, has to give the word problem a try. Thus, we have stimulated numerous experiences with a variety of word problems and by debriefing *both* people who have solved these problems and people who have given up on them, we gain another insight into the nature of math anxiety.

One of the arithmetic word problems that was on the board for a long time is the Tire Problem:

A car goes 20,000 miles on a long trip. To save wear, the five tires are rotated regularly. How many miles will each tire have gone by the end of the trip?

Most people readily acknowledge that a car has five tires and that four are in use at any one time. Poor math students who are not anxious or blocked will poke around at the problem for a while and then come up with the idea that four-fifths of 20,000, which is 16,000 miles, is the answer. They don't always know exactly why they decided to take four-fifths of 20,000. They sometimes say it "came" to them as they were thinking about the tires on the car and the tire in the trunk. The important thing is that they *tried* it and when it resulted in 16,000 miles, they gave 16,000 a "reasonableness test." Since 16,000 seemed reasonable (that is, less than 20,000 miles but not a whole lot less), they were pretty sure they were right.

The math anxious student responds very differently.

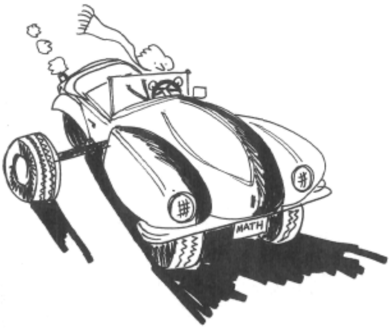

FIG. II-2

The problem is beyond her (or him). She cannot begin to fathom the *information. She* cannot even imagine how the five tires are used (See FIG. II-2.) She cannot come up with any strategy for solving it. She gives up. Later in the debriefing session, the counsellor may ask whether the fraction four-fifths occurred to her at all while she was thinking about the problem. Sometimes the answer will be yes. But if she is asked why she did not try out four-fifths of 20,000 (the only other number in the problem), the response will be—and *we have* heard this often enough to take it very seriously—"I figured that if it was in my head it had to be wrong."

The assumption that if it is in one's head it has to be wrong or, as others put it, "If it's easy for me, it can't be math," is a revealing statement about the self. Math anxious people seem to have little or no faith in their own intuition. If an idea comes into their heads or a strategy appears to them in a flash they will assume it *is wrong. They do not trust their intuition. Either they* remember the "right formula" immediately or they give up.

Mathematicians, on the other hand, trust their intuition in solving problems and readily admit that without it they would not be able to do much mathematics. The difference in attitude toward intuition, then, seems to be another tangible distinction between the math anxious and people who do well in math.

The distrust of intuition gives the math counsellor a place to begin to ask questions: Why does intuition appear to us to be untrustworthy? When has it failed us in the past? How might we improve our intuitive grasp of mathematical principles? Has anyone ever tried to "educate" our intuition, improve our repertoire of ideas by teaching us strategies for solving problems? Math anxious people usually reply that intuition was not allowed as a tool in problem solving. Only the rational, computational parts of their brain belonged in math class. If a teacher or parent used intuition at all in solving problems he rarely admitted it, and when the student on occasion did guess right in class he was punished for not being able to reconstruct his method. Yet people who trust their intuition do not see it as "irrational" or "emotional" at all. They perceive intuition as flashes of insight into the rational mind. Victims of math anxiety need to understand this, too.

The Confinement of Exact Answers

> "Computation involves going from a question to an answer. Mathematics involves going from an answer to a question."
> —Peter Hilton, Mathematician

Another source of self-distrust is that mathematics is taught as an exact science. There is pressure to get an exact right answer, and when things do not turn out right, we panic. Yet people who regularly use mathematics in their work say that it is far more useful to be able to answer the question, "What is a little more than five multiplied by a little less than three?" than to know *only* that five times three equals 15.* Many math anxious adults recall with horror the timed tests they were subjected to in elementary, junior and senior high school with the emphasis on getting a unique right answer. They liked social studies and English better because there were so many "right answers," not just one. Others were frustrated at not being able to have discussions in math class. Somewhere they or their teachers got the wrong notion that there is an inherent contradiction between rigor and debate.

This emphasis on right answers has many psychological benefits. It provides a way to do our own evaluation on the spot and to be judged fairly whether or not the teacher likes us. Emphasis on the right answer, however, may result in panic when that answer is not at hand and, even worse, lead to "premature closure" when it is. Consider the student who does get the right answer quickly and directly. If she closes the book and does not continue to reflect on the problem, she will not find other ways of solving it, and she will miss an opportunity to add to her array of problem-solving methods. In any case, getting the right answer does not necessarily imply that one has grasped the full significance of the problem. Thus, the right-answer emphasis may inhibit the learning potential of good students and poor students alike.

*A little more than five multiplied by a little less than three will produce a range between 12.5 and 16.5. Inequalities, of which this is one example, are common in more advanced math, as are equations that have more than one solution.

In altering the learning atmosphere for the math anxious the the tutor or counsellor needs to talk frankly about the difficulties of doing math. The tutor's scratch paper might be more useful to the students than a perfectly conceived solution. Doing problems afresh in class at the risk of making errors publicly can also link the tutor with the student in the process of discovery. Inviting all students to put their answers, right or wrong, before the class will relieve some of the panic that comes when students fail to get the answer the teacher wants. And, as most teachers know, looking carefully at wrong answers can give them good clues to what is going on in students' heads.

Although an answer that checks can provide immediate positive feedback, which aids in learning, the right answer may come to signify authoritarianism (on the part of the teacher), competitiveness (with other students), and painful evaluation. None of these unpleasant experiences is usually intended, any more than the premature closure or panic, but for some students who are insecure about mathematics the right-answer emphasis breeds hostility as well as anxiety. Worst of all, the "right answer" isn't always the right one at all. It is only "right" in the context of the amount of mathematics one has learned so far. First graders, who are working only with whole numbers, are told they are "right" if they answer that five (apples) cannot be divided between two (friends). But later, when they work with fractions, they will find out that five *can* be equally divided by giving each friend two and one-half apples. In fact both answers are right. You cannot divide five one-dollar bills equally between two people without getting change.

The search for the right answer soon evolves into the search for the right formula. Some students cannot even put their minds to a complex problem or play with it for a while because they assume they are expected to know something they have forgotten.

Take this problem for, example,

Amy Lowell goes out to buy cigars. She has 25 coins in her pocket, \$7.15 in all. She has seven more dimes than nickels and she has quarters, too. How many dimes, nickels, and quarters does she have?

Most people who have done well in high school algebra will begin to call the number of nickels X, the number of dimes $X + 7$, and the number of quarters \$7.15 minus $(5X + 10X + 70)$ without realizing that Amy Lowell must have miscounted her change, because even if all 25 coins in her pocket were quarters (the largest coin she has), her change would total \$6.25, not \$7.15.*

This is a tricky problem, which is fair, as opposed to a trick problem which is not. But it also shows how searching for the right formula can cause us to miss an

*I am indebted to Jean Smith for this example.

obviously impossible situation. The right formula may become a substitute for thinking, just as the right answer may replace consideration of other possibilities. Somehow students of math should learn that the power of mathematics lies not only in exactness but in the processing of information.

Self-Defeating Self-Talk

One way to show people what is going on in their heads is to have them keep a "math diary," a running commentary of their thoughts, both mathematical and emotional, as they do their homework or go about their daily lives. Sometimes a tape recorder can be used to get at the same thing. The goal is twofold: to show the student and the instructor the recurring mathematical errors that are getting in the way and to make the student hear his own "self-talk." "Self-talk" is what we say to ourselves when we are in trouble. Do we egg ourselves on with encouragement and suggestions? Or do we engage in self-defeating behaviors that only make things worse?

Inability to handle frustration contributes to math anxiety. When a math anxious person sees that a problem is not going to be easy to solve, he tends to quit right away, believing that no amount of time or rereading or reformulation of the problem will make it any clearer. Freezing and quitting may be as much the result of destructive self-talk as of unfamiliarity with the problem. If we think we have no strategy with which to begin work, we may never find one. But if we can talk ourselves into feeling comfortable and secure, we may let in a good idea.

To find out how much we are talking ourselves into failure we have to begin to listen to ourselves doing math. The tape recorder, the math diary, the self-monitoring that some people can do silently are all techniques for tuning in to ourselves. Most of us who handle frustration very poorly in math handle it very well in other subjects. It is useful to watch ourselves doing other things. What do we do there to keep going? How can these strategies be applied to math?

At the very minimum this kind of tuning in may identify the particular issue giving trouble. It is not very helpful to know that "math makes me feel nervous and uncomfortable" or that "numbers make me feel uneasy and confused," as some people say. But it may be quite useful to realize that one kind of problem is more threatening than another. One excerpt from a math diary is a case in point:

> Here I go again. I am always ready to give up when the equation looks as though it's too complicated to come out right. But the other week, an equation that started out looking like this one did turn out to be right, so I shouldn't be so depressed about it.

This is constructive self-talk. By keeping a diary or talking into a tape recorder we can begin to recognize our own pattern of resistance and with luck we may soon learn to control it. This particular person is beginning to understand how and why she jumps to negative conclusions about her work. She is learning to sort out the factual mistakes she makes from the logical and even the psychological errors. Soon she will be able to recognize the mistakes she makes *only* because she is anxious. Note that she has been encouraged to think and to talk about her feelings while doing mathematics. She

is not ashamed or guilty about the most irrational of thoughts, not frightened to observe even the onset of depression in herself; she seems confident that her mind will not desert her.

The diary or tape recorder technique has only been tried so far with college-age students and adults. So far as we can tell, it is effective only when used in combination with other nonthreatening teaching devices, such as acceptance of discussion of feelings in class, psychological support outside of class, and an instructor willing to demystify mathematics. The goal in such a situation is not to get the right answer. The goal is to achieve mastery and above all autonomy in doing math. In the end, we can only learn when we feel in control.

References

This chapter is based primarily on interviews with and observations of math anxious students and adults. These people are not typical of those who are math incompetent. Most of them are very bright and enjoy school success in other subjects, but they avoid or openly fear mathematics.

[1]Mitchell Lazarus, "Mathophobia: Some Personal Speculations," *The Principal*, January/February, 1974, p. 18.

[2]Knowles Dougherty. Personal communication to the author.

[3]J. Louis Nanney and John L. Cable, *Elementary Algebra: A Skills Approach*, Boston, Allyn and Bacon, Inc., 1974, p. 215.

[4]Matina Horner, "Fear of Success," *Psychology Today*, November, 1969, p. 38ff.

[5]Sanford Dornbusch, *as quoted in* John Ernest, "Mathematics and Sex," *American Mathematics Monthly*, Vol. 83, No. 8, October, 1976, p. 599.