II.—ON THE RELATION BETWEEN THE PHILOSOPHY OF SPINOZA AND THAT OF LEIBNIZ.[1]

By Robert Latta.

It is not my intention to reopen the purely historical question regarding the actual intercourse between Spinoza and Leibniz and the particular ideas or suggestions which Leibniz may reasonably be held to have directly borrowed from Spinoza. On this point it would hardly be possible to add anything to the thorough work of Prof. Stein in his *Leibniz und Spinoza*, which seems to me to prove conclusively that Leibniz was no more a plagiarist of Spinoza than he was a plagiarist of Newton, but that he was "philosophically *homo sui generis*," strongly influenced by thinkers like Plato and Spinoza, yet in his philosophy neither Platonist nor Spinozist but always Leibnitian.[2] A few of the historical facts may, however, be mentioned as having suggestiveness in connexion with the large problem of the relation between the two systems. About a year before Spinoza's death Leibniz saw him at the Hague and had several conversations with him. At this time Leibniz was without a philosophical system of his own, dissatisfied with Cartesianism and ready to receive suggestions. He had just completed a long course of mathematical study by discovering the Infinitesimal Calculus, and on the way to Holland he wrote a paper on the principle of motion, doubtless with the view of getting Spinoza's opinion about it. This question of the laws of motion (in view of the theories of Descartes) was one of the two subjects which Leibniz mentions as having been discussed in course of the conversations at the Hague, the other subject being that of the necessity of the existence of an absolutely perfect Being.[3] In general it is clear from the evidence adduced by Stein[4] that Leibniz made a most careful study of most of Spinoza's writings and that he regarded Spinoza's as the best of modern systems with the

[1] Read before the Aristotelian Society. [2] *Leibniz u. Spinoza*, p. 134.
[3] *V. infra* pp. [4] *Leibniz u. Spinoza*, p. 236 *sqq.*

exception of his own Monadology.[1] "Spinoza would be right," he says, "if there were no Monads."[2] And it is interesting further to notice that the doctrine of Spinoza which most repelled Leibniz was his denial of final causes, and that in almost every philosophical letter written by Leibniz from 1679 onwards the idea of final cause appears.

My purpose in this paper is to consider what light may be thrown upon the two systems and their relation to one another by taking account of the general scientific thought of the time. The dominating science of the seventeenth century was Mathematics, so that for a seventeenth century writer exact scientific method was synonymous with mathematical method. The endeavour to make an exact study of external nature, which was one of the first fruits of the revulsion from Scholasticism, led inevitably to the development of Mathematics as a science of calculation or measurement. Problems which formerly had merely a speculative interest now pressed for immediate solution, and the practical necessities of physical science led gradually to the development of new mathematical methods, such as the introduction of the notion of "infinity" by Kepler, the Analytical Geometry of Descartes and the Infinitesimal Calculus of Newton and Leibniz. Both Spinoza and Leibniz were mathematicians and as mathematicians they shared the ideal of their time, that of a mathematically exact and certain system of knowledge, a comprehensive "scientific" philosophy. They were both interested in mathematical problems, but from somewhat different points of view. Spinoza was chiefly impressed with the certainty and necessity of such geometrical demonstration as that of Euclid, which proceeded from self-evident axioms and unfolded with rigorous truth the attributes of certain objects from precise definitions of them. Leibniz, on the other hand, was more interested in the progress of Mathematics than in the security of its established methods. He sought to grasp the real nature of matter and he found the current Mathematics too abstract to be sufficiently serviceable. Atomism (as in Cordemoi, Gassendi and others) had charmed him for a time, and the metaphysical problems of the Eucharist (in connexion with the question of the reunion of Christendom) impelled him from another side to the study of matter. But Atomism represented matter as too absolutely discrete while Cartesianism made it too smoothly continuous, and some advance in mathematical method was necessary in order to reconcile the discrete and the

[1] P. 252. [2] *Lettre à Bourquet* (1714), Erdmann, 720 ; Gerhardt, iii., 575.

continuous. Thus while Leibniz is at one with Spinoza in seeking not mere speculative probability but " demonstration " in philosophy, he is not to be regarded as thinking of demonstration in exactly the same way as Spinoza did.[1]

The form of Spinoza's *Ethics* makes it evident that he regarded demonstration in philosophy as a process analogous to the synthetic method in geometry, which endeavours to apply a canon of pure self-consistency to a variety of given geometrical figures. The aim of the inquiry is to ascertain the properties or qualities of the figures, and a property is shown to belong to a figure when it is proved to be consistent with the definition of that figure. Each kind of figure is treated as a distinct and separate species and their inter-relations are considered in a purely external way. The demonstrations are supposed to be pure, direct deductions from given premisses. But in reality there is a continual reference to experience, to the system of space, certain of the relations of which are expressed by the figures. The proof of each proposition requires a " construction " of some kind to be made, such as the producing of lines or the superposition of figures, and this construction is simply a reference to the unity of the system of space, in which the particular figure is an element (or combination of elements) related to others, and by which all the kinds of figures are ultimately determined. For instance, if you produce two sides of a triangle in order to prove something about its angles, you implicitly recognise that the triangle is not a self-complete system, the properties of which may be directly deduced from its definition, but that it is an element in a surface and that its internal properties are logically dependent on its external relations, or, at least, are in the most intimate connexion with them. Thus the synthetic method in geometry presupposes the system of space in its definitions and postulates, without showing how the figures described in the definitions or the right to demand these postulates follow from the nature of space itself. Now the mathematical form of Spinoza's *Ethics* is modelled upon that of Euclid's Geometry. There are numerous definitions of more or less independent things or ideas. Certain axioms are also assumed as self-evident, and from a combination of the axioms with the definitions the whole philosophy is regarded as necessarily following. The definitions are the substantial part of the

[1] Spinoza's demonstrations have, for the most part, the character of *reductio ad absurdum*. Leibniz writes of them : Ce Spinosa est plein de rêveries bien embarassées et ses prétendues démonstrations de Deo n'en ont pas seulement le semblant " (Gerhardt, ii., 133).

philosophy: the whole truth is an unfolding of what is implied in them. But the definitions of geometry are determined by space-experience; they are definitions of objects from which all characteristics except those of space have been thought away. And it is impossible to go a step beyond the definitions of geometry, to deduce anything from them, without a reference to the space which is their medium. Thus, as Tschirnhausen pointed out to Spinoza,[1] from the definition of a circle taken by itself it is impossible to deduce any of the properties of the circle except the uniformity of curvature by which it is distinguished essentially from all other curves. All the other properties of the circle can be deduced only through its being brought into relation with other things, such as radii, intersecting lines, etc. If, then, Spinoza's definitions correspond to the definitions of geometry, *i.e.*, if his method is a geometrical one, the definitions presuppose a system in which the things defined are elements, and apart from a reference to this system there can be no legitimate demonstration.

Now while it is legitimate for a special science, which does not propose to answer ultimate questions, to make postulates presupposing a system within which the objects of the science are inter-related, such a procedure is inconsistent with the purpose of an absolute philosophy. In order to expound the meaning of the universe *ordine geometrico* you must begin with a definition of the universe, just as in order to expound the meaning of a geometrical figure, you must begin with a definition of the figure. But while there are other geometrical figures by the aid of which the meaning of the figure defined may be further expounded, there is no other through which the meaning of the universe may be set forth. Either the definition must already include and express the whole of the properties of the thing defined, in which case it must say everything that is to be said, or it must express some property from which nothing further can be deduced except by the aid of other considerations, in which case it is inadequate as a definition. Spinoza, however, contends that while it is perhaps true in the case of very simple things or *entia rationis* (including geometrical figures) that the definition of the thing, apart from its relation to other things, yields only one property, this is untrue as regards real things. "For from this alone, that I define God as a Being to whose essence belongs existence, I infer several of His properties; namely, that He necessarily exists, that He is one, immutable,

[1] Ep. 82, Van Vloten and Land (71 in Bruder).

infinite," etc.[1] But the very terms of this definition imply a reference to other things. A Being whose essence involves existence is intelligible only in relation to a being whose essence does not involve existence; that which is *in se* can be thought only in relation to that which is *in alio*. And it is in virtue of this reference that the other properties of the object are deduced from the definition. Each of the properties is negatively proved by the use of such disjunctive axioms as: *Omnia quae sunt vel in se vel in alio sunt*,[2] and consequently the properties do not follow from the definition alone, but from the definition *plus* the interpretation of the terms of the definition, which is given in the axiom. That which is *in se* is that which is not *in alio*. If we go on afterwards (as seems to be the way of Spinoza) to deny the reality of that which is *in alio*, we stultify the whole procedure. To deny the reality of that which is *in alio* while we continue to assert the reality of that which is *in se*, is to alter the meaning of the axiom, to make it a disjunction, not between two kinds of things, but between the universe and nonentity. In other words, the axiom becomes tautologous: that which is *in se* is *in se*, the universe is the universe. Accordingly if the axiom has any meaning, Spinoza's definition of God implies that God is an element in a wider system, that He is *in se* in contrast with that which is really *in alio*. And yet Spinoza means by "God" the universe as one.

This is confirmed by an examination of Spinoza's own account of Definition in the *Tractatus de Intellectus Emendatione*,[3] where he gives rules for the definition of a created (*in alio*) and of an uncreated (*in se*) thing. The rules for the definition of a created thing are (1) that the definition must include the proximate cause, and (2) that the definition should be such that all the properties of the thing can be deduced from the definition, considered by itself and not in conjunction with others. This is evidently equivalent to saying that in order to know truly a created thing, we must see clearly both how it is produced and what it produces (for, according to Spinoza, the relation of cause and effect is reducible to that of substance and attribute). The thing defined must, in short, be removed out of the realm of the empirical or casual and regarded in its fixed and eternal relations. It must be perfectly conditioned, put in its own place in the ordered system of things. Again, for the defini-

[1] Ep. 83, Van Vloten (72 in Bruder).
[2] *Ethics*, i., Axiom 1; *cf.* Axiom 2: *Id quod per aliud non potest concipi, per se concipi debet.*
[3] Van Vloten, i., 29 *sqq.*; Bruder, ii., 36 *sqq.*

tion of an uncreated thing the rules are (1) that it should exclude all cause, *i.e.*, that the object should need for its explanation no other thing besides its own being; (2) given the definition there should remain no room for doubt whether the thing exists or not; (3) it should contain no substantives which can be used as adjectives, *i.e.*, the object defined should not be explained by abstractions and (4) we should be able to deduce all the properties of the thing from its definition. Now these rules are practically the same as those for the definition of a created thing. The first and second rules amount to saying that the proximate cause of the uncreated thing must be the thing itself, that it must be produced by no other thing. The fourth rule requires, as in the case of the created thing, that the idea be tested by its consequences, in other words, that the thing is real through its necessary relation to the whole system of things. The third rule is a caution against abstractions, which is equally applicable to the definition of a created thing, but is especially in point here, because in the definition of an uncreated thing proximate cause becomes *causa sui*. If it had been possible, as in the case of the created thing, to refer the uncreated thing to something else necessarily presupposed in it, there would have been less danger of abstraction. As it is, it seems to me impossible to escape abstraction in the definition of an uncreated thing. The definition of a thing can only mean a statement of the relations of that thing within some system of which it is a member or element, and this is virtually acknowledged by Spinoza in his rules for the definition of a created thing. But if this is so, every definition must be adjectival, must be made up of abstractions. In other words, it is impossible to give a true definition of an uncreated thing, if by an uncreated thing is meant the universe, the system of reality itself, which is the presupposition of all definition. Yet Spinoza bases his philosophy upon the definition of an uncreated thing and believes that he has deduced all from this definition.

Spinoza's imperfect recognition of the system which is presupposed in all demonstration appears to me to be due (in great part at least) to the way in which mathematical problems were regarded by him as by most of his contemporaries. The ancient geometers found that there were many problems which could not be solved directly by the aid of Euclid's definitions and postulates. In plane geometry Euclid postulated the straight line and the circle. But many problems (such as that of the area of a circle or the relation of its radius to its circumference) depend for their exact solution upon the discovery of a relation between the straight line and the circle. Somehow it must be possible to express the circle in terms of the straight line. But you cannot do it with a ruler and a pair of compasses: you cannot draw or construct any figure which will solve the problem. The nearest approach to a solution that can be made is to construct a polygon with so many sides that it will come very near indeed to the circle. But you can never make the sides small enough for the figure to coincide with the circle. The sides will always remain finite straight lines, while the circle is the locus of a point which is continuously changing its direction. Accordingly the Greek geometers had recourse to the method of "exhaustions". Thus they regarded the area of a circle as being equivalent to the "limit" area of a circumscribed and an inscribed polygon, having the same number of sides, when the sides are made infinitely numerous. The polygons can never actually *become* the circle, but the ultimate difference is negligible, being as little as we like to make it, and accordingly the "limit" area to which each polygon approaches may be taken as practically equivalent to the area of the circle. Now this method is one of proof *per impossibile* or *reductio ad absurdum.* The area of the circle must be either equal to, greater than, or less than the limit area of the polygons. But to suppose it greater or less would be to suppose that the polygons do not yet coincide, *i.e.*, that the area is not the limit area. Therefore the area of the circle must be equal to the limit area of the polygons. But all proof *per impossibile* is merely a negative verification. It shows that anything other than the suggested law or truth (the thing to be proved) would be inconsistent with the general principles or constitution of some system, such as the system of quantity or the system of space. But it does not show how these general principles apply to the particular case or how the particular case follows necessarily from them, is an organic element in the constitution of the system. Thus, in the instance we have considered, the proof depends upon an actual construction or picturing in space of two dimensions *plus* a general reference to the nature of quantity as being such that every element in it must be either greater than, equal to, or less than any other. Space is assumed to be quantitative, and space of two dimensions is assumed to be such that straight lines and circles can be drawn in it; but neither the relation of space to quantity nor the nature of space of two dimensions as expressing itself in the straight line and circle is thought out or made an explicit premiss in the argument.

The reasoning is grounded on a more or less blind appeal to a system or systems that are presupposed without being thoroughly thought out.

A considerable advance upon the ancient methods was made by Kepler, who introduced the notion of infinity in connexion with the solution of geometrical problems, and by Descartes, who invented the analytical geometry or geometry of co-ordinates.[1] The introduction of the idea that a finite figure or a finite area is reducible to an infinite number of elements was an explicit recognition of the inadequacy of the Enclidean postulates as principles of demonstration, and it was the beginning of a train of thought which led inevitably to the Infinitesimal Calculus; but, as Pascal pointed out in defending Cavalieri, the geometrical method which proceeds upon the principle that the infinitely little may be neglected differs only in manner of expression from the method of exhaustions used in the Greek Mathematics.[2] Both are ultimately based on *reductio ad absurdum*. On the other hand, the general effect of the changes introduced by Descartes was (1) to make the relation between the system of space and that of quantity in general more clear and definite, by finding (in the co-ordinates) units of space-relation, and (2) to substitute for the empirical reference to space that is implied in the use of a ruler and compasses a method by which figures and their properties may be shown by calculation (without drawing or construction) to follow from the nature of space as extension in three or in two dimensions. The Cartesian method in geometry is thus more positive, direct and explicit than the method of the Greeks. Eliminating the postulates of Euclid, or rather going beneath them to the grounds on which they rest and thinking out what they imply, it gives a more perfect demonstration of the propositions of Euclid and solves more complex problems than the Greeks could have attempted. Nevertheless, while the Cartesian geometry was much more positive and thorough in its method of demonstration than was the synthetic geometry, it still retained the doctrine or hypothesis of limits in a negative form. It was (considering plane geometry alone) on the right lines towards a positive solution

[1] For a full history *v.* Gerhardt, *Die Entdeckung der höhern Analysis*, p. 6 *sqq.*, and Cohen, *Das Princip der Infinitesimal-Methode*, § 35 *sqq.*

[2] So Leibniz says in a letter to Varignon that the infinitesimal calculus "donne directement et visiblement, et d'une manière propre à marquer la source de l'invention, ce que les anciens, comme Archimède, donnoient par circuit dans leur reductions ad absurdum" (Gerhardt, *Math. Schriften*, iv., 92).

of the problem of the relation between a straight line and a curve,—a problem insoluble by Euclid because he postulated them independently; but the solution had still to be worked out, the unity of which the straight line and curve are immediate differences had still to be determined. The solution was obtained in connexion with the problem of drawing a tangent to a curve. If the method of limits is followed, the tangent is the limit of a secant cutting the curve in two points, when these two points are brought infinitely near to one another, *i.e.*, when they are separated from one another by less than any assignable distance. But even in the limit case we have still two points and a line,—an infinitely little line, it is true, but yet a line. The infinitely little distance is regarded as real but as negligible. Now just about the time of Leibniz another step forward was taken.[1] In connexion with the fact that finite numbers may be resolved into infinite series, it was contended that the finite line rests upon the infinitely little, that the infinitely little is really its generating principle. Every line has length and direction. An infinitely little line has infinitely little length; but no reduction in its length can make any alteration in its direction. Accordingly the infinitely little line means really the direction, which is the essence or generating principle of the line. Given the direction, the line may be drawn to any length, great or small. The essence of every line is thus its direction, that is its quality or characteristic and not its quantity as the distance between two points. The points presuppose the line. Thus, if we regard a curve as generated by the motion of a point, the tangent to the curve at any point will simply be the direction of motion at that point. The direction of the moving point changes continuously and, in the case of a regular curve, uniformly, in accordance with a law which is characteristic of the particular curve. Accordingly, in general, the straight line and the curve are essentially varieties of direction in space, the straight line being a continuous uniform direction, while the curve is a continuously varying direction of more or less complexity. And the direction of a curve at any point must be regarded as a ratio between two infinitely small quantities, because change of direction in a plane is relative to two axes and continuous change of direction means infinitely small variation from point to point. It was the solution of problems resulting from such conceptions as these that led to the discovery of the Infinitesimal Calculus.

[1] The advance was made by Roberval (1602-1675).

By this view that the infinitely little is the basis of the finite the older doctrine of limits is transcended. According to this negative doctrine of limits, an infinitely little difference between two figures (say) is negligible. But if an infinitely little difference is negligible, it must be for some reason. Infinite littleness is a matter of degree. An infinitely small quantity is a quantity less than any that can be assigned. But such a conception has no meaning unless we are speaking of an infinitely small *thing* or unity of differences, at the very least an infinitely small element in a numerical series which is not a bare addition or subtraction of homogeneous units but has some characteristic law of increment or decrement. It is the law or principle of the series, the nature or character of the whole, which enables us to say that the infinitely little difference may be neglected. Thus, adopting a phrase from Grandi, Leibniz writes to him in 1713: "*Infinite parva concipimus non ut nihila simpliciter et absolute, sed ut nihila respectiva (ut ipse bene notas), id est ut evanescentia quidem in nihilum, retinentia tamen characterem ejus quod evanescit*".[1] Accordingly, when it can be shown that two things ultimately "run into" one another or are continuous with one another, that is to say that the ultimate difference between them is infinitely little, it is presupposed that they are differences of a unity or that their difference is one of degree and not of kind. Thus the negative doctrine of limits implicitly presupposes a system within which its various objects are related, while the positive method, of which the fullest expression is to be found in the Calculus, explicitly recognises this system and regards the various objects or elements as necessarily determined by it. The method of limits was a true method so far as it went; but it was inadequate because it did not think out its presuppositions. The advance that was made by Leibniz and his contemporaries consisted in investigating these presuppositions by inquiries (direct and indirect) into the true meaning of mathematical infinity.

We are now in a position to consider the agreement and the difference between the scientific standpoint of Spinoza and that of Leibniz. The mathematics of Spinoza are the mathematics of Descartes. Spinoza is at the negative point of view implied in the method of limits, while Leibniz is at the positive point of view implied by the method of infinitesimals. In mathematics the method of limits is logically dependent upon the method of infinitesimals; it assumes, without

[1] Gerhardt, Leibniz's *Math. Schriften*, iv., 218. So also the conception of "infinities of infinity" is a favourite one with Leibniz, who frequently argues against the possibility of an absolute quantitative infinite.

justification or explanation, what the method of infinitesimals justifies and explains. The method of limits presupposes that the discrete is ultimately reducible to the continuous, the finite to the infinite; but it does not show, as the method of infinitesimals does, how the continuous develops the discrete, how the infinite constitutes the finite. Similarly in the metaphysics of Spinoza the unity of an all-comprehensive system is presupposed throughout; but the varieties of individual existence are not shown as proceeding from this system, as its logical development. The finite presupposes the infinite, modes presuppose attributes, attributes presuppose substance; but the infinite is reached by thinking away the varieties of the finite, the attribute is that which is common to all the modes, *substantia in se* or *vere considerata* is *substantia depositis affectionibus*.[1] Thus for Spinoza "determination is negation," "the determinate denotes nothing positive, but only a privation of the existence of that nature which is conceived as determinate".[2] Geometrical figures as definite figures are unreal, because their definiteness is dependent on other figures: their reality is indeterminate extension. And in general, definite quantities of any kind, separate parts, are unreal: real quantity, "as it is in the understanding," "as it is in itself," is infinite, indivisible and single [*unica*].[3] The infinite is thus the basis of the finite, the continuous of the discrete; but the reality of the infinite and continuous is conceived in such a way as to imply the unreality, and therefore the negation, of the finite and discrete. Not merely is it maintained that the infinite and continuous are not products of the finite and discrete, but it is implied that the finite and discrete are not really (as finite and discrete) products of the infinite and continuous. Now it is interesting to find that, in thus emphasising the unity of "extended substance" and real "quantity," as against the variety of finite "bodies" and "quantities," Spinoza says that the attempt to show that "extended substance is composed of parts or bodies really distinct from one another" is as absurd "as if one were to attempt by the mere addition and aggregation of many circles to make up a square or a triangle or something else totally different in essence" or to make a line out of points.[4] But the mathe-

[1] *Eth.*, i., 5, demonst.; *cf. Eth.*, ii., 10, Schol. 2: Res singulares non possunt sine Deo esse nec concipi; et tamen Deus ad earum essentiam non pertinet.

[2] Ep. 36, Van Vloten (41 Bruder).

[3] *Ibid.*, 12, Van Vloten (29 Bruder).

[4] *Loc. cit.*

maticians of Spinoza's own day were showing that rectilineal figures are not "totally different in essence" from circles and that finite quantity is the product of an infinite series, having a definite law or characteristic. The various geometrical figures are, it is true, not products of one another nor products of discrete quantities of any kind; but they are products or expressions of the qualities or characteristics of extension. Infinite extension is not something totally different in essence from all finite figures, something to be obtained only by getting rid of all finite extension. To call it "infinite" is to insist on its qualities or relations as determining its quantities, to regard it as a system from which certain finite figures, in all their finitude, necessarily follow, or rather a system of which these finite figures are the expression. And in general "infinite" quantity, in so far as it is really anything, is a negative name for quality, and to say that the finite presupposes the infinite is to say that quantity presupposes quality. This is the truth involved in Spinoza's account of the Attributes of Substance as infinite *in their kind;*[1] but it is a truth which is inconsistent with Spinoza's other contention that Substance is *absolutely* infinite. To think of anything as infinitely great or as infinitely little is to recognise negatively that the conception under which we are thinking it is inadequate, that the thing (as conceived by us) and its other are elements or differences within a higher unity. A circle, the radius of which is infinite, is a circle which is not a circle, and when we speak of it we mean to indicate that the conception of a circle as an independent finite figure is inadequate and that the difference between a circle and a straight line is a difference determined by some higher unity, which (so far) we do not explain. In the same way, when we speak of infinite space we mean that the space of mathematics is, by itself, an inadequate conception and that the system of space must itself be an element in some more comprehensive system. And in general, to say that a thing is infinite in its kind is to say that its kind is relative to some other kind and that neither is to be fully understood except through that of which they are both differences.[2] In other words, a thing which is infinite in its kind is a thing which is to some extent indeterminate. A thing absolutely infinite will consequently be a thing absolutely indeterminate. That is to say, a thing

[1] *Eth.*, i., Def. 6; *cf.* Ep. ii. and *Korte Verhandeling*, appendix, prop. iii.

[2] This, of course, means (what Spinoza would deny) that finite *Modes*, as well as Attributes, are each infinite in its kind. Thus, according to Leibniz, every finite thing "contains infinity," *v. infra.*

absolutely infinite must be a thing of which we have no conception whatever, for if we had an inadequate conception of it, it would be *infinite in its kind*, and if we had a perfectly adequate conception of it, it would no longer be infinite in the sense of indeterminate, it would be absolutely determined. In short, the mathematical infinite is always the indeterminate, while the infinite as applied to the real universe is the self-determined.

Now the characteristic feature of the philosophy of Leibniz is that, however imperfectly, it endeavours to give a positive solution of the problem of reality. And this is closely connected with Leibniz's point of view in Mathematics. Instead of regarding the infinite as the negation of the finite, to be reached by thinking away the finite, he conceives the infinite as the reality of the finite, to be reached by thinking out the finite. Every finite thing, according to Leibniz, "contains infinity": it is in some way constituted by the infinite, made up of infinitesimals. His account of the way in which the infinite actually constitutes or determines the finite is far from being perfectly satisfactory; but he has a sure grasp of the principle that the determining infinite means quality, characteristic, relation of some kind, and that it is impossible to get behind relations, behind the world as a system, or, in other words, to reach substance *depositis affectionibus*. Thus in the letter to Grandi already quoted (p. 342) Leibniz writes: *Infinitudo vera non cadit nisi in infinitum virtutis omni parte carens . . . et quantitates illæ calculi nostri extraordinariæ sunt fictiones, non ideo tamen spernendæ sunt. . . cum in calculo perinde sit ac si essent veræ quantitates, habeantque fundamentum in re et veritatem quandam idealem ut radices imaginariæ.*[1] All quantity is accordingly quantity of something non-quantitative, quantity of some quality or characteristic. A finite straight line is a quantity of uniform direction, a finite curve is a quantity of direction which varies according to some law, a finite extension is a quantity of something extended. "Extension presupposes some quality, some attribute, some nature in the extended thing, which quality extends or diffuses itself along with the thing, continues itself."[2] This quality is conceived by Leibniz as potentiality, not in the sense of empty capacity (*puissance nue*), but in the sense of something which contains implicitly within itself its own

[1] Gerhardt, Leibniz's *Math. Schriften*, iv., 218; *cf.* iii., 500: *Reale infinitum fortasse est ipsum absolutum, quod non ex partibus conflatur, sed partes habentia eminenti ratione et velut gradu perfectionis comprehendit.*

[2] Leibniz, Erdmann's ed., 692 b; Gerhardt's ed., vi., 584.

realisation (entelechy or *tendance*). The infinite develops into the finite, the qualitative into the quantitative. The infinitely little line is a direction, but in the direction there is contained implicitly every finite line having that direction: in other words, the line is a development of the direction. But, as we have seen, all such development is the development of a unity, or rather of a system, into its differences; it is something permanent unfolding itself in its changes. Now this implies that reality is not a bare unity, from which the differences have been thought away, but a system of differences, a unity which implicitly contains its differences within itself. This is the principle of the law of Continuity, which governs Leibniz's mathematics [1] and which has a considerable function in his philosophy. According to the law of Continuity, a thing may (as Leibniz himself puts it) be regarded as "equivalent to a species of its opposite," [2] *e.g.*, rest may be regarded as a species of motion (an infinitely little motion), equality as a species of inequality, unconsciousness as a species of consciousness, the finite as a species of the infinite. By this, of course, is meant not that the thing is a species of which its opposite is genus, but that the relation between them is reciprocal, it being possible to regard each as a species of the other. But this implies that both are elements within some unity or system which is inseparable from them. And it is this that leads Leibniz to insist so strongly on the explicit recognition of the principle of sufficient reason as a principle of method. The principle of sufficient reason is the principle that everything has a ground or reason which is at once identical with it and different from it, in other words that nothing is self-evident, purely self-identical. Thus the principle of sufficient reason is the principle that the ultimate reality is not a unity from which the differences have been thought away, but a system of elements in relation, a unity in difference. And of this principle the law of Continuity is manifestly a particular application, for it amounts to saying that, while all the varieties of things are real, no one of them is independent of the rest, the world is a system of "compossible" things.

[1] Leibniz very frequently speaks of the law of Continuity as derived from the consideration of "the infinite" and as being the basis of the Calculus. For instance, in the *Specimen Dynamicum* (1687) he speaks of it as *principium ordinis generale, nascens ex infiniti et continui notione, accedente ad illud axioma, quod datis ordinatis etiam quæsita sunt ordinata* (Gerhardt, *Math. Schriften*, vi., 250; *cf.* Cohen, *Princip der Infinitesimal-Methode*, § 52 *sqq.*).

[2] *Math. Schriften*, iv., 93. Leibniz says *contradictoire*, but the context shows that he means "contrary," opposite.

On the one hand, there is no absolute surd, no purely contingent thing: on the other hand the surd and the contingent are not absolutely "irrational" or illusory. The surd is reducible to an infinite series, the contingent is the product of an infinity of conditions, and thus each is a form of its other.[1]

Accordingly we may, I think, put the difference between Leibniz and Spinoza in this way, that Spinoza expressly proceeds upon a method of deduction from self-evident first principles, *i.e.*, from a basis of pure identity, while this procedure is possible only because a system of identity in difference is presupposed throughout; and Leibniz, on the other hand, explicitly recognises this system as *practically* ultimate, while at the same time he professes to give a shadowy ground for the system itself (a ground of its *existence* but not of its *essence*) in the "choice" of God, which is rather a negative release into existence than a positive creation. Thus Spinoza's presupposition of a system of unity in difference as constituting the ultimate reality of things appears in his constant references to the "order and connexion" of things and ideas, to the proximate cause as giving the essence of a thing and to substance as *causa sui*, *natura naturans* and *natura naturata* (*i.e*, substance as cause and effect, ground and consequent, yet both ultimately the same), to the *conatus*, effort or tendency in things, to the "series of fixed and eternal things" (universal singulars)[2] and to many similar conceptions.[3] And, on the other hand, Leibniz shows the imperfection of his grasp of the principle which he himself insists upon, by treating the law of sufficient reason as an addition to the law of identity and by speaking of the essences of all abstractly possible worlds as being in the understanding of God, a *regio idearum* behind the actual world. In short the inconsistencies of the two philosophies

[1] *Vide* Leibniz, Erdmann, 83 b; Gerhardt, vii., 200: "The difference between necessary and contingent truths is indeed the same as that between commensurable and incommensurable numbers. For the reduction of commensurable numbers to a common measure is analogous to the demonstration of necessary truths or their reduction to identical truths. But, as in the case of surd ratios the reduction involves an infinite process and yet approaches a common measure, so that a definite but unending series is obtained, thus also contingent truths require an infinite analysis, which God alone can accomplish" (*Cf.* Cohen, *Infinitesimal-Methode*, § 43).

[2] *Vide Tractatus de Intellectus Emendatione.*

[3] *E.g.*, Spinoza uses the very terms in which Leibniz states his principle of sufficient reason: *Cujuscunque rei assignari debet causa seu ratio, tam cur existit, quam cur non existit* (*Eth.*, i., 11, demonstr. 2).

are similar, but the emphasis is on opposite sides. A comparison between Spinoza's "Attributes" and the qualities which Leibniz attributes to his Monads may serve to illustrate this. Spinoza speaks of substance as *constans infinitis attributis*,[1] which means that substance must contain every possible kind of reality. Each of these attributes "expresses eternal and infinite essence," *i.e.*, each expresses the whole and in its own way expresses it completely. There is no degree in their expression of the whole (as, for example, there is degree in the perfection with which the Monads express the whole). And an attribute is defined as *id quod intellectus de substantia percipit tanquam ejusdem essentiam constituens*.[2] The human understanding, because of its finitude, perceives only two of these attributes, and we are thus left to infer that an infinite understanding must perceive the infinite attributes. But the infinite attributes do not limit one another. One idea limits another and one body limits another; but thought does not limit extension nor extension thought. Accordingly the infinite attributes must mean simply the totality of abstract possibilities for an infinite intellect. That is to say, they are very much the same as Leibniz's infinity of "possible" ideas or essences in the understanding of God. Ultimately, then, there is no connexion between the attributes. They do not form part of one system; otherwise they would limit one another. In Leibniz's language they would not merely be "possible" but "compossible". Yet they are held to be parallel expressions of substance, and this parallelism seems to imply that they do belong to the same system, that they are differences within its unity. On the other hand, when Leibniz attributes to every substance two fundamental qualities, "perception" and "appetition," he is defining substance as system within system. Perception is simply a name for the relation of one term or element to every other element in the system, while appetition is a name for the development of the system from within itself. Ultimately it is implied in Leibniz's view that appetition means simply change of perception, variety of relationship. But the perception and appetition are attributed by Leibniz, not to one substance or to one ultimate system of things, but to each of an infinite number of substances, which are indeed regarded as related to one another, but which are so externally related, so independent in their own being, that each lives its own life as if there existed nothing but God and itself. Thus the notion of system is

[1] *Eth.*, i., 11, and def. 6.

[2] *Ibid.*, i., def. 4.

explicitly recognised by Leibniz, without being thoroughly thought out. His "system" is not all-inclusive. The world is not the one system of reality, but "the best of all possible worlds". The elements of which it is composed are essentially "possibles," in their own nature completely independent. Thus the world is the system of the "compossible," resting on the chaos of the "possible".

The results of this general argument cannot be worked out within the limits of this paper, but I may take up one or two special points. (1) In the first place, as most closely connected with the general line of thought we have been following, let us consider the views of Descartes, Spinoza and Leibniz regarding extension and motion. According to Descartes, extension and motion are absolutely *given.* Extension is a created substance, in the sense that its existence presupposes nothing else except the *concours ordinaire* of God. Motion is also a direct creation of "God Himself, who in the beginning created matter along with motion and rest and now, by His *concours ordinaire* alone, preserves in the whole the same amount of motion and rest that He then placed in it".[1] From the combination of these two absolutely given elements—given in separation from one another—Descartes in his *Principia,* part iii., tries to show that the whole material world in its endless variety comes into being. Ultimately, then, all matter is space of three dimensions *plus* motion. Spinoza, excluding the idea of creation, reduces the independence of extension, treating it not as substance but as an attribute of substance, *i.e.,* as something which on the one hand is not relative to anything else except understanding, while on the other hand, being relative to understanding, it is not substance itself. This attribute of extension, however, is not what we call space of three dimensions, for it is one and indivisible.[2] In short, extension, for Spinoza, is that which is presupposed in extended things, that which remains when all the limits (the finitude) of extended things are thought away. And thus, of course, Spinoza rejects the view of Descartes that the essence of matter or corporeal substance can be an extension that is divisible. Divisible extension is extension conceived "abstractly or superficially, as by means of the senses we have it in the imagination".[3]

[1] *Principia,* ii., 36.

[2] Extended substance, according to Spinoza, can have no parts; for if it had parts, each of them would be a substance and would be finite, which is a contradiction of the nature of substance as that which is infinite inasmuch as the conception of it requires the conception of nothing else; *cf.* Ep. 12, Van Vloten (29 Bruder).

[3] *Loc. cit.*

Again motion, according to Spinoza, is an infinite mode, that is to say, it is an immediate modification of the attribute of extension, "following from the absolute nature of that attribute".[1] But he makes no attempt to show how motion "follows from the absolute nature" of extension. All that he can really mean is that motion presupposes extension. Motion is the stepping-stone between finite bodies and the infinite attribute. The differences of finite bodies all presuppose (or are reducible to terms of) the motion of particles, this motion of particles as a totality presupposes (when we think away the finite element in it, the parts or particles) an infinite motion, which similarly presupposes extension, which in turn presupposes substance. Each stage is obtained from that which preceded it by the removing of certain determinations, until we reach the "absolutely indeterminate".[2] Now the characteristic feature both of Descartes's and of Spinoza's view is the negative form in which the relation between extension and motion is regarded. According to Descartes, motion comes to extension entirely *ab extra:* according to Spinoza, motion, being a mode, presupposes extension, but extension, being an attribute, must be conceived through itself alone and is therefore independent of motion. Hence, when Descartes takes it as the fundamental principle of his laws of motion that the quantity of motion and rest in the universe (or in any isolated system of bodies) is fixed and unchangeable, he leaves out of account the direction of motion, because that is a quality not of motion *per se* but of motion in space. Further it is interesting in this connexion to recall the fact that Leibniz on his journey to Holland to visit Spinoza wrote a paper on the principle of motion, and that one of the few things he tells us about his interviews with Spinoza is that "Spinoza did not quite clearly see the defects of Descartes's laws of motion: he was surprised when I began to show him that they were inconsistent with the equality of cause and effect".[3] Now Leibniz's objection to Descartes's laws of motion is that they are too abstract. Motion, of course, mathematically considered, must be an abstraction; but motion regarded as something given quite independently of extension is motion considered more abstractly than is necessary. In fact motion and extension mutually presuppose one another: they are both abstractions from one reality. This might be illustrated by the fact that

[1] *Eth.*, i., 21; *cf.* Ep. 64, Van Vloten (66 Bruder).

[2] Ep. 36, Van Vloten (41 Bruder).

[3] Foucher de Careil, *Réfutation inédite de Spinoza*, p. lxiv.

(as we have already seen) the figures or determinations of extension are reducible to directions of motion (leaving out of account mass, or moving body, and velocity). All real motion, then, has direction; it is given, not independently, but in relation to extension. And consequently the motion whose quantity in the universe is fixed must be motion having direction: the direction is conserved as well as the quantity of abstract motion. But the direction of a motion is not something actual in the sense that it can be seen or pictured as a whole. It is a quality, a potentiality or partly hidden tendency in the motion, an infinitesimal, out of which the finite motion develops. This potentiality or tendency, which is presupposed by all actual motion when we take into consideration its direction, is what Leibniz means by Force. And thus for Leibniz Force, as qualitative, as a potency passing into actuality, an identity in difference, is the substance or reality from which actual visible or picturable motion and extension are abstractions.[1] An infinitely little line is a direction of motion and an infinitely little motion (or direction of motion) is a force. Thus the positive interpretation of the infinitely little means a passing from superficial ideas of sense and imagination to deeper and more comprehensive notions of thought, from the abstract to the concrete. But the attitude of sense or imagination is not absolutely cut off from the attitude of thought or understanding. Comprehension by the understanding is a thinking out of what appears imperfectly in sense.

(2) This leads naturally to a brief consideration of the difference between Spinoza's theory of knowledge and that of Leibniz. Spinoza draws a sharp line between *opinio* or *imaginatio*, on the one hand, and *ratio* and *scientia intuitiva*, on the other. *Opinio* or *imaginatio* is the cause of falsity, while the knowledge given by *ratio* and *scientia intuitiva* is necessarily true.[2] Thus in the *Tractatus de Intellectus Emendatione* we find Spinoza insisting mainly on the distinction

[1] Thus Spinoza and Leibniz are both opposed to Descartes's theory that extension is the essence of corporeal substance, on the ground that divisible extension presupposes something *omni parte carens*. But this indivisible basis of extension is conceived by Spinoza negatively, as being entirely without parts in any sense, as being *one* in opposition to *many*, while Leibniz conceives it positively, as something which has degrees or varieties and thus as one *in* many. The difference is so considerable and so closely connected with Leibniz's mathematics that I think it ought to weigh heavily against the suggestion of Stein (p. 64 *sqq.*) that Leibniz was probably influenced by Spinoza in his criticism of Descartes's view of "extended substance".

[2] *Eth.*, ii., 40, 41.

between the empirical order of events, which is the work of imagination, and the real order of existence, as it is known by reason. Mere perception or the history of events which has no higher principle of order than memory, mere sequence in short, is dismissed absolutely as illusion. But, on the other hand, *veritas norma sui et falsi est.*[1] Falsity presupposes truth, imagination presupposes understanding. But there is no positive relation between them. Without understanding and truth there can be no imagination and falsity; but without imagination and falsity there might be understanding and truth. Leibniz, on the other hand, makes the difference between sense or imagination and understanding one of degree. The difference between them is ultimately an infinitely little one, or rather they are elements in a continuous series of perceptions, differing from one another by infinitely little degrees of clearness and distinctness. And, just as every finite number may be resolved into an infinite series, so every finite perception is made up of an infinity of *petites perceptions*, which are relatively obscure and confused. Every perception thus "contains" or "involves infinity," and the notion of perception is stretched out so as to include every kind of relation, whether conscious or unconscious. Accordingly the relation between sense (or imagination) and understanding comes to be reciprocal. Each presupposes the other. Understanding is the evolution of sense, while sense is the involution of understanding. To this extent the positive view of Leibniz transcends the negative position of Spinoza. But Leibniz does not see clearly all that is involved in his method. For instance, the infinity of *petites perceptions* into which Leibniz resolves a particular sense-perception is an infinity of elements, each of which is and is not a sense-perception, each of which belongs in some way to sense but does not belong to sense-*consciousness*. Now (as we saw when dealing with the relation between the finite and the infinite) this means that the distinction between the conscious and the unconscious is not ultimate, that it is an expression of some deeper unity, that the conscious and the unconscious are inseparable elements in a system. Consequently in the *petite perception* we ought to find that which determines the distinction between the conscious and the unconscious, *i.e.*, the comprehensive unity in difference, which expresses itself in them. Such a unity would be the unity or system of reason or of self-consciousness, which reveals itself in the distinction between conscious and unconscious, subject and

[1] *Eth.*, ii., 43, Schol.

object, and which thus transcends that distinction. But we shall look in vain for any such system in the *petites perceptions* of Leibniz. It is true that he regards them as *somehow* having order in them, as containing implicitly a law of *some* sort; but in reality he conceives them, not positively but negatively, as sensations *minus* consciousness, *i.e.*, as "limits" of conscious sensations, and thus any order they may be supposed to have is not an order of their own, but the order of conscious perception read into them. There must, for example, be among conscious perceptions an order or system which is expressed in the distinctions between (say) sensations of hearing and sensations of sight. A similar order must be supposed to exist among the *petites perceptions*. But this second order is presupposed in a purely negative way. If we have a conscious perception of the sound of 100,000 waves, we must *somehow* have perception (though unconscious) of the sound of each;[1] but Leibniz makes no attempt to indicate exactly *how*. His argument here is simply the *reductio ad absurdum*, which is the characteristic argument of Spinoza. And Leibniz's failure at this point accounts for the difficulty he finds in dealing with the rational or self-conscious soul. He sees clearly that the conscious in some way presupposes the unconscious; but he has not an equally clear grasp of what is involved in the truth that the unconscious presupposes the conscious. Hence it becomes increasingly difficult for him to carry out his law of continuity when he comes to consider the higher parts of the scale of being. He cannot, for instance, conceive that a self-conscious soul should ever lose its self-consciousness and permanently become merely conscious or unconscious. And thus he hesitates between the hypothesis that rational souls have been raised from the rank of sensuous souls "by the extraordinary operation of God" and the hypothesis that "only those souls which are destined some day to attain to human nature contain in germ [*enveloppent*] the reason which will some day appear in them".[2] On the whole matter Leibniz is very inconsistent and unsatisfactory; but, whichever of his hypotheses we follow, it is evident that he did not realise

[1] *Nouveaux Essais, Introduction* (Erdmann, 197; Gerhardt, v., 47). One might ask—why a separate *petite perception* for each *wave* and not for every possible *element* in each wave? The single wave is quite an arbitrary standard for the unit of perception: there is nothing to show why it should be chosen.

[2] *Théodicée*, § 397; *cf.* § 91, and *Lettres à Arnauld* (1686-7), Gerhardt, ii., 75 and 99; also *Lettre à des Maizeaux* (1711), Erdmann, 676; Gerhardt, vii., 534.

the true consequence of his own principles, *viz.*, that self-consciousness, as the more concrete principle, is necessarily implied or presupposed in the continuity of the conscious and the unconscious, that it is the system in which they are elements. Such a conclusion would, of course, have destroyed the monadology by making the universe a single all-comprehensive Monad. Accordingly Leibniz at this point falls back upon the method of Descartes and Spinoza, practically (though not avowedly) treating the self-conscious soul as discontinuous with the conscious and the unconscious, as having some new quality that is a sheer addition to the qualities of these lower souls.

(3) This beginning of a rift in continuity widens into an open self-contradiction when we come to Leibniz's account of God, the highest in the scale of being. The contradiction consists in regarding God as at once the highest Monad and the being in whose understanding the essences of all possible systems are and who by His choice makes the best possible system real. God is thus both within and without the system of monads. In so far as He is merely an element in the system, He is less than God: in so far as He is outside of the system, the continuity is broken. Leibniz's own suggestion regarding the proof of the existence of God would, if thought out, have revealed the contradiction. He says that the Cartesian ontological proof of the existence of God is incomplete. It ought, he says, to run: *if the most perfect Being is possible* (*i.e.*, if the idea of a most perfect Being is not self-contradictory), it follows that the most perfect Being exists. And he argues that, for instance, there is no swiftest possible motion, because the idea of it can be shown to be self-contradictory. But Leibniz failed to observe that, if the most perfect Being is regarded as one of a series, the idea of it is self-contradictory. For either it contains all the perfections (*i.e.* in Leibniz's sense, the positive reality) of the other members of the series or it does not. If it does, it is no longer to be regarded as *one* member of the series; if it does not, it is no longer most perfect, for *ex hypothesi* it lacks some perfections.[1] Leibniz misses the contradiction by arguing that the idea of a most perfect Being is not self-contradictory, for all perfections are mutually compatible. This argument, however, was made by him long before he had thought out his monadology, and he tells us that in one of the interviews at the Hague he submitted it to Spinoza

[1] That is to say, we should have a "best possible" God, corresponding to the best possible world.

who, though inclined at first to oppose it, ultimately admitted it to be satisfactory.[1] The fact is interesting when we consider that the contradiction in Leibniz's account of God is the exact counterpart of the contradiction in Spinoza's view of substance. Leibniz treats God as at once an element in the system of things and a Being independent of the system, but of such a nature that the system itself seems unnecessary; while Spinoza, as we have seen, regards God or Substance as equivalent to the Universe as one, and yet his definition of God implies that He is an element in some wider system. From opposite sides Spinoza and Leibniz fall into the same pit.

In this paper I have been able to do little more than indicate a line of thought which, it seems to me, may be fruitfully developed. It is easy, on the one hand, to show that Spinoza and Leibniz are both inconsistent and, on the other hand, to maintain that they both say exactly the same thing in slightly different ways. The armoury of the more recent philosophy equips us for the one task, and a collection of parallel passages might fortify us for the other. But neither of these things profits us a whit. Turning from them, I have endeavoured to show that what is admittedly implicit in the philosophy of Spinoza is made comparatively explicit in the philosophy of Leibniz, although Leibniz does not by any means thoroughly work out the consequences of his own method. And the philosophical attitude of each is, I think, very closely connected with their views of mathematics. The negative doctrine of limits, when it is thought out, issues in the positive doctrine of infinitesimals, which it presupposes. Thus Spinoza argues vigorously against the reality of final causes as involving the introduction of the negative, the finite, the determinate into substance, while in his constant references to the order and connexion of things[2] and to the *conatus* or self-preserving tendency in each individual thing, he presupposes that determinate system of inter-related elements which his explicit argument against final causes would exclude. Leibniz, on the other hand, is concerned for nothing more than for the reality of final cause. It is the point regarding which he most sharply differs from Spinoza and in his correspondence he returns to it again and again. Nevertheless in the end he puts behind his rational or

[1] Gerhardt, vii., 261.

[2] Compare these with the passage in the appendix to part i. of the *Ethics*, where Spinoza attributes the belief that there is order in things to imagination, as distinct from understanding.

"inclining" necessity, a necessity of blind fate, behind his "compossible" system a chaos of empty "possibilities," so that the real world is practically taken as a creation out of nothing, a development of that indeterminate capacity, that *puissance nue*, which Leibniz himself most frequently derides.