Predictions and Tests of Multiverse Theories *

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Abstract

Evidence for fine-tuning of physical parameters suitable for life can perhaps be explained by almost any combination of providence, coincidence or multiverse. A multiverse usually includes parts unobservable to us, but if the theory for it includes suitable measures for observations, what is observable can be explained in terms of the theory even if it contains such unobservable elements. Thus good multiverse theories can be tested against observations. For these tests and Bayesian comparisons of different theories that predict more than one observation, it is useful to define the concept of "typicality" as the likelihood given by a theory that a random result of an observation would be at least as extreme as the result of one's actual observation. Some multiverse theories can be regarded as pertaining to a single universe (e.g. a single quantum state obeying certain equations), raising the question of why those equations apply. Other multiverse theories can be regarded as pertaining to no single universe at all. These no longer raise the question of what the equations are for a single universe but rather the question of why the measure for the set of different universes is such as to make our observations not too atypical.

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1 Multiverse explanations for fine-tuning

Many of the physical parameters of the observed part of our universe, whether constants of nature or cosmological boundary conditions, seem fine-tuned for life and us [1, 2, 3, 4]. There are three common explanations for this. One is that there is a Fine-Tuner who providentially selected the physical parameters so that we can be here. Another is that it is just a coincidence that the parameters turned out to have the right values for us to be here. A third is that our observed universe is only a small part of a much vaster universe or multiverse or megaverse or holocosm (my own neologism for the whole), and that the physical parameters are not the same everywhere but take values permitting us in our part.

These three explanations are not necessarily mutually exclusive. For example, combining a Fine-Tuner with coincidence but without a multiverse, perhaps the universe was providentially created by a God who had a preference for a particularly elegant single universe which only coincidentally gave values for the physical parameters that allowed us to exist. Or, for a Fine-Tuner with a multiverse but without coincidences, perhaps God providentially created a multiverse for the purpose of definitely creating us somewhere within it. Or, for coincidence and a multiverse without a Fine-tuner, if the universe weren't providentially created, it might be a multiverse that has some parts suitable for us just coincidentally. Or, it might even be that all three explanations are mutually true, say if God providentially created a multiverse for reasons other than having us within it, and yet it was a coincidence that this multiverse did contain us.

On the other hand, it seems conceivable (in the sense that I do not see any obvious logical contradiction) that the universe is determined by some sort of blind necessity that requires both our own existence and a single world with a single set of physical parameters. In this case, the universe is not providential (in the sense of being foreseen by any God) but nor is our existence coincidental.

Thus, logically, I don't see that we can prove that any combination of the three explanations is either correct or incorrect. However, it does seem a bit implausible that none of these explanations is at least partially correct, and it also seems rather implausible that the large number of fine-tunings that have been noticed are mere coincidences.

I should perhaps at this point put my metaphysical cards on the table and say that – as an evangelical Christian – I do believe the universe was providentially created by God, and that – as quantum cosmologist with a sympathy toward the Everett 'many-worlds' version of quantum theory – I also strongly suspect that the universe is a multiverse, with different parts having different values of the physical parameters. It seems plausible to me that – in a quantum theory with no arbitrary collapses of the wavefunction – God might prefer an elegant physical theory (perhaps string/M theory with no adjustable dimensionless parameters) that would lead to a multiverse that nevertheless has been created providentially by God with the purpose of having life and us somewhere within it.

Although personally I have less confidence in string/M theory than in either providence or the multiverse, nevertheless string/M theory is very attractive. It does seem to be the best current candidate for a dynamical theory of the universe (i.e. for its evolution, if not its state) and it does strongly appear to suggest a multiverse. Since string/M theory has no adjustable dimensionless constants, if it predicted just a single set of parameters, it would seem very surprising if these parameters came out right for our existence. Thus if string/M theory – or some alternative with no adjustable dimensionless constants – were correct, it would seem much more plausible that it would lead to a multiverse, with different parts of the universe having different physical parameters.

Indeed, string theorists [5, 6, 7, 8, 9, 10, 11] have argued that string/M theory leads to an immense multiverse or landscape of different values of physical parameters and 'constants of nature'. It is not yet known whether the range of values can include the physical parameters that allow life, such as those within our part of the universe, but that does seem at least plausible with the enormous range suggested in the string landscape or stringscape.

One objection that is often raised against the multiverse is that it is unobservable. Of course, this depends on how the multiverse is defined. One definition would be the existence of different parts, where some physical parameters are different, but this just shifts the arbitrariness to the choice of this set of physical parameters. Obviously if some quantity which varies with position (like energy density) were included in the set of physical parameters, then even what we can see could be considered a multiverse. But if we just include the so-called 'constants of nature', like the fine structure constant and various other coupling constants and the mass ratios of the various elementary particles, then what we can observe directly seems to consist of a single universe. Indeed it would be rather natural – if *ad hoc* – to define a multiverse with respect to the physical parameters that have no observable variation within the part we can directly see. In this case, the multiverse becomes unobservable, and it becomes an open question whether parts of the universe we cannot see have different values of these constants. Many would argue that it is a purely metaphysical concept that has no place in science.

However, in science we need not restrict our entities to be observable – we just want the simplest theory, whether using observable or unobservable entities, to explain and predict what is observable. One cannot test scientifically a theory that makes predictions about what is unobservable, but one can test a theory that makes use of unobservable entities to explain and predict the observable ones. Therefore, if we find a multiverse theory that is simpler and more explanatory and predictive of what is observed than the best single-universe theory, then the multiverse theory should be preferred. The success of such a multiverse theory itself would then give credence to the existence of the unobservable multiverse.

Another objection that is often raised against multiverse theories is that naïvely they can 'explain' anything and predict nothing, so that they cannot be tested and considered scientific. The idea is that if a multiverse gives all possible physical parameters or other conditions somewhere within the multiverse, then the parameters and conditions we observe will exist somewhere. Hence what we observe is 'explained' at least somewhere. On the other hand, if every possibility exists, then we cannot predict any non-trivial restriction on what might be observed. If a theory makes no non-trivial predictions, then it cannot be tested against observations, and it can hardly be considered scientific.

2 Testable multiverse explanations

Sufficiently sophisticated multiverse theories can provide predictions as well as explanations, and hence can be tested against observations scientifically. Unlike single-universe theories, in each of which one can in principle predict uniquely the physical parameters, in multiverse theories one usually can make only statistical predictions for ranges of parameters, but this can still be much better than making no prediction at all. However, to make such statistical predictions, the multiverse theory needs to include a *measure* for the different observations that can be made. If it allows all possible observations without putting any measure on them, then one can make no predictions.

Since we have strong evidence that we live in a quantum universe, it would be natural to seek a quantum multiverse theory. If this just includes some quantum states, unitary evolution, path integrals, operators, some operator algebra and the like, one has the bare quantum theory eloquently described by Sidney Coleman [12], which by itself does not give any measures or probabilities. The Copenhagen version of quantum theory does give these, but at the apparent cost of the collapses of the wave-function at times undetermined by the theory and to states that are random.

Here I shall take essentially an Everett 'many-worlds' view that in actuality there is no collapse of the wavefunction. However, to get testability of the quantum theory, I shall assume that there is one aspect of the Copenhagen version that should be added to the bare quantum theory: measures for observations that are expectation values of certain corresponding 'awareness operators'.

In Copenhagen theory, these operators are projection operators, and the measures are the probabilities for the results of the collapse of the wave-function. Here

I shall not necessarily require the operators to be projection operators, though – to give the positivity properties of measures – I shall assume they are at least positive operators. Also, I shall not assume that anything really random occurs, such as wave-function collapse, but that there are simply measures for all the different observations that might occur. In testing the theory against one's observation, one can regard that observation as being selected at random (with the theory-given measure) from the set of all possible observations, but ontologically one can assume that all possible observations with non-zero measure really do occur, so that there is never a real physical random choice between them.

For the quantum theory to be fundamental, one would need to specify which observations have measures and what the corresponding operators are whose expectation values give those measures. In my opinion, the most fundamental aspect of a true observation is a conscious perception or awareness of the observation. Therefore, I have developed the framework of "Sensible Quantum Mechanics" (SQM) [13] or "Mindless Sensationalism" [14] for giving the measures for sets of conscious perceptions as expectation values of corresponding positive operators that I call "awareness operators". This is only a framework (analogous to the bare quantum theory without the detailed form of the unitary evolution or operator algebra), rather than a detailed theory, since I have no detailed proposal for the sets of possible conscious perceptions or for the corresponding positive operators. Presumably, for human conscious perceptions, these operators are related to states of human brains, so understanding them better would involve brain physics. However, I do not see how they could be deduced purely from an external examination of a brain, since we cannot then know what is being consciously experienced by the brain.

To avoid the complications of brain physics, one might use the observed correlation between external stimuli and conscious experiences to replace the unknown awareness operators acting on brain-states with surrogate operators acting on the correlated external stimuli. Of course, this would not work well for illusory or hallucinatory conscious perceptions, for which the fundamental awareness operators would presumably still work if they were known. However, one might prefer to focus on conscious perceptions that are correlated with external stimuli and hence better fit what is usually meant by observations.

If the awareness operator for a conscious perception is correlated with a single set of external stimuli at a single time, it could be approximately replaced with a single projection operator onto some external system. Alternatively, if it is correlated with a sequence of measurement processes, then it could be approximately replaced with a product of projection operators or a sum of such products, a class operator of the decoherent histories approach to quantum theory [15, 16, 17].

Therefore, though I would not regard either the projection operators of Copenhagen quantum theory or the class operators of decoherent histories quantum theory as truly fundamental in the same way that I believe awareness operators are, it might be true that in certain circumstances these are reasonable approximations to the fundamental awareness operators. Then one can take their expectation values in the quantum state of the universe as giving the measure for the corresponding conscious perception.

One example of this replacement would be to calculate the measure for conscious perceptions of a certain value of the Hubble constant. In principle, in SQM this would be the expectation value of a certain awareness operator that presumably acts on suitable brain-states in which the observer is consciously aware of that particular Hubble constant value. But the expectation value of this operator might also be well approximated by that of some suitable operator acting on the logarithm of the expansion rate of the part of the universe that is observed. Because the latter operator does not involve brain physics, it might be easier to study scientifically and so could be used as a good surrogate for the actual awareness operator.

However, it would presumably not be a good approximation to use the latter operator if its expectation value depended significantly on parts of the universe where there are no conscious observers: if one wants to use it to mimic the expectation value for the perceptions of conscious observations, one must include a selection effect which restricts to parts of the universe where there are conscious observers.

To include this selection effect in operators that are external to brains (or whatever directly has the conscious perceptions), so that their expectation values can be good approximations for that of the fundamental awareness operators, is a difficult task, since we do not know the physical requirements for conscious observers. For example, there is nothing within our current understanding of physics that would tell us whether or not some powerful computer is conscious, unless one makes assumptions about what is necessary for consciousness. Also, I know of nothing within our current understanding of physics that would enable us to predict that I am currently conscious of some of my visual sensations but not of my heartbeat, since presumably information about both is being processed by my brain and would be incorporated in a purely physical analysis.

Nevertheless, to get some very crude guess for a selection effect for conscious observers, one might make the untested hypothesis that typical observers are like us in requiring suitable complex chemical reactions and perhaps a liquid compound like water. Then one could use the existence of liquid water as a very crude selection effect for observers and attach it onto other projection or class operators used to approximate some conscious perception depending on the external stimuli that are described by the projection or class operators.

Thus one might use projection or class operators to ask the following two questions: Does liquid water exist in part of the universe? Is that part of the universe expanding at a suitable logarithmic rate? If the answer to these questions is yes with some measure, then one might expect that there would be a roughly corresponding expectation value for the awareness operator for conscious perceptions of that value of the Hubble constant. This is an extremely crude approximation to what I postulate would objectively exist as the expectation value of the true corresponding awareness operator, but since these awareness operators are as yet largely unknown, the crude approximation may be useful during our present ignorance.

One problem with calculating the measure for sets of conscious perceptions as expectation values of corresponding 'awareness operators' is that naïvely one might get infinite values. By itself this would not necessarily be a problem, since only ratios of measures are testable as conditional probabilities. However, when the measures themselves are infinite, it is usually ambiguous how to take their ratios.

The problem arises if the awareness operators are sums of positive operators that are each localized within finite spacetime regions (as one would expect if the operators correspond to finite conscious beings). Assume that one such operator in the sum has support within one of N spacetime regions of equal volume within the total spacetime. Then by translational or diffeomorphism invariance, one would expect the sum of the operators for a particular awareness operator to include a sum over the corresponding operators in each of the N regions. (There would also be a sum over operators that overlap different regions, but we need not consider those for this argument.) This is essentially just the assumption that, if a suitable brain-state for some conscious perception can occur in one of the N spacetime regions, then it can also occur (depending on the quantum state) in any of the other $N-1$ regions. Also where it occurs in some coordinate system should not affect the content of the conscious perception produced by the corresponding brain-state.

If the conditions for observers with the corresponding conscious perception occur within all N spacetime regions, so that the expectation value of the operator within each region has a positive expectation value bounded from below by a positive number ϵ, then the total awareness operator (a sum of at least the individual positive operators within each of the N regions) will have an expectation value at least as large as $N\epsilon$. This is infinite if the number N of such spacetime regions is infinite. Essentially the argument is that, if the measure for a conscious perception has a strictly positive expectation value for each spacetime volume in some region, then for an infinite volume of spacetime where this is true the measure will be infinite. One can regard this as arising from the infinite number of conscious observers that arise in an infinite volume of spacetime with conditions suitable for life and conscious observers.

Since inflation tends to produce a universe that is arbitrarily large (with an infinitely large expectation value for the spatial volume at any fixed time after inflation and hence presumably infinitely many conscious observers), it tends to produce an infinite measure for almost all non-zero sets of conscious observations. There has been a lot of discussion in the literature [18, 19, 20, 21] of how to get well-defined ratios of these infinite measures (or of related quantities, since the discussion is not usually in terms of measures for conscious perceptions) but I think it is fair to say that there is as yet no universally-accepted solution.

This is a serious problem that needs to be solved before we can hope to make rigorous testable predictions for an inflationary multiverse. A vague hope is that somehow the dimensionality of the part of the Hilbert space (or quantum state space if it is bigger than the Hilbert space) where conscious observers are supported is finite, so that – for all finite quantum states – the expectation values of all finite positive operators (including the awareness operators) would be finite, thus giving finite measures for all conscious perceptions. But what would limit conscious observers to a finite dimensional part of the presumably infinite-dimensional quantum state space eludes me.

3 Testing multi-observation theories with typicality

If we can find a theory that gives finite measures for sets of observations (perhaps conscious perceptions) or which can be approximated as the expectation values of other positive operators, how can we test it? If the theory predicts a unique observation (at least unique under some condition, such as observing a clock reading to have some value), then one can simply check whether one's observation fits the prediction. This would typically be the case in a classical model of the universe with a single observer who reads a clock that gives monotonically increasing readings (so that there is only a single observation for each clock reading).

Although a classical solipsist might believe this is true for his universe, for most of us the evidence is compelling that there are many observers and hence presumably many different observations even at one value of some classical time. Quantum theory further suggests that there are many possible observations – even for a single observer at a single time.

There is a debate as to whether the observations given by quantum theory are actual or are merely unrealized possibilities. The Copenhagen view seems to imply that – for each value of the time and for each observer – there is only one observation that is actualized (say by collapse of the wave-function), so that all the other possibilities are unrealized. This seems to come from a naïvely WYSIWYG[1] view of the universe, so to me it is much simpler to suppose that all possible observations predicted by the quantum theory are actualized, with no ugly collapse of the wave-function to give a single actualized observation for each observer at each time. We are already used to the idea of many different times (which are effectively just different branches of the quantum state, at least in the Wheeler-DeWitt approach to quantum gravity) and – except for solipsists – to the idea of many different observers, so why should we not accept the simple prediction from quantum theory of many observations at the same time by the same observer?

In any case, whether in a classical universe or a quantum universe without collapse of the wave-function, each time an observation occurs, there are many observations even at the same time and so one needs to be able to test this. To do this for a theory that gives measures for all sets of observations, I would propose using the concept of "typicality" [13], which is a suitable likelihood that one may use to test or compare theories or to calculate their posterior probabilities in a Bayesian analysis after assigning their prior probabilities.

The basic idea is to choose a set of possible observations that each give a single real parameter, such as the Hubble constant or the value of one of the constants of nature. Then we use the measure for sets of observations to get the measure for all ranges of this single real parameter. For simplicity, we normalize the total measure in the set of observations being considered to be unity.

Now we want to test one's observation against the theory by calculating the typicality for that observation within the set. For simplicity, I shall call the observation being tested the 'actual' observation, even though the theory would say that all possible observations with non-zero measure are realized as actual observations. To do this, one calculates the total 'left' and 'right' measures for all possible observations in the set under consideration, i.e. the total measure to the left or right of and including the 'actual' observation when they are ordered on the x-axis by the value of the real parameter under consideration. These two measures will add up to 1 plus the measure of the 'actual' observation, which is counted in both of the measures.

Next take the smaller of these two measures (the total measure on the more extreme side if the 'actual' observation is not in the middle of the total measure) as the 'extreme' measure of the 'actual' observation. We then use the normalized measure of the set of observations to calculate the probability that a random observation within the set would give an 'extreme' measure as small as that of the 'actual' observation. This probability is what I call the 'typicality' of the actual observation of the real parameter within the chosen set of possible observations. The typicality is thus the probability that a random observation in the set is at least as extreme

[1]What You See Is What You Get (If you needed this footnote, WYSIWYG is not WYSIWYG.)

as the actual observation. It depends not only on the actual observation but also on the theory predicting the measure for the sets of observations. This is what is needed to calculate the conditional probability of subsets of observations within the set under consideration.

In the case in which the real parameter takes a continuum of values and there is zero measure for an observation to have precisely any particular value, the left plus right measures add up to unity. Then the extreme measure (the smaller of the left and right measures) will will take continuous values from 0 to 1/2 with a uniform probability distribution, so the typicality is twice the extreme measure. In this simple case, the typicality is a random variable with a uniform probability distribution ranging from 0 (if the actual parameter is at the extreme left or right) to 1 (if the actual value is in the middle of its measure-weighted range, with both left and right measures being 1/2).

If the real parameter takes on discrete values, then the situation is more complicated. For example, suppose that the real parameter is k, with possible values $k = -1$ (with measure 0.2), $k = 0$ (with measure 0.35) and $k = +1$ (with measure 0.45). Then $k = -1$ has a left measure of 0.2 and a right measure of $0.2 + 0.35 + 0.45 = 1$ for an extreme measure of 0.2; $k = 0$ has a left measure of $0.2 + 0.35 = 0.55$ and a right measure of $0.35 + 0.45 = 0.8$ for an extreme measure of 0.55; and $k = +1$ has a left measure of $0.2 + 0.35 + 0.45 = 1$ and a right measure of 0.45 for an extreme measure of 0.45. Thus the probability of an extreme measure of 0.2 is 0.2 (the probability of $k = -1$); the probability of an extreme measure of 0.45 is 0.45 (the probability of $k = +1$); and the probability of an extreme measure of 0.55 is 0.35 (the probability of $k = 0$). The typicality of $k = -1$ is the probability that the extreme measure will be at least as small as 0.2, which is 0.2; the typicality of $k = 0$ is the probability that the extreme measure will be at least as small as 0.55, which is $0.2 + 0.45 + 0.35 = 1$, and the typicality of $k = +1$ is the probability that the extreme measure will be at least as small as 0.45, which is $0.2 + 0.45 = 0.65$.

Note that only for the most extreme parameter value or values (for which the extreme measure is the smallest possible within the set) is the typicality the same as the normalized measure of the observation giving that value itself. For less extreme parameter values, the typicality is greater than the measure of the observations giving that parameter value. On the other hand, the least extreme parameter value or values (the middle one, for which the 'extreme' measure is the greatest possible within the set) has a typicality of unity. Thus the typicality always attains its upper limit of unity for some member of the set, but the lowest value it attains is the measure of the most extreme observation (which would be zero if the observed parameter formed a continuum with zero measure for any particular value of the parameter).

The typicality is thus a likelihood, given a theory for the measures of sets of values of a real parameter, for a parameter chosen randomly with the probability measure given by the theory, to be at least as extreme as the 'actual' observed parameter. The typicality has the advantage over the probability measure for the actual observed parameter of being a probability that has values up to unity for some possible observation. This differs from the probability measure for the parameter itself, which may have a very small upper limit (e.g. if there is an enormous number of possible discrete values for the parameter) or even a zero upper limit (e.g. if the parameter ranges over a continuum and has a smooth probability density, with no delta functions at any particular values of the parameters).

If one uses the probability measure itself as a likelihood, one cannot directly do a Bayesian analysis with an observation of a continuous parameter having a smooth probability density, since the resulting likelihood will be zero for all possible observed values of the parameter. One might try to use the probability density instead of the probability itself, but this depends on the coordinatization of the parameter and so gives ambiguous results. For example, one would get a different likelihood for an observed value of the Hubble constant H by using its probability density than one would for H^2.

Another approach that is often used for results that have a large number of possible values is to bin them and then use the total probability for the bin in which the actual observation lies as the likelihood. But again this depends on the bins and so gives ambiguous results. The ambiguity of both the probability density and the binning are avoided if one uses the typicality as I have defined it here.

Admittedly, if there are $N > 1$ parameters being observed, then there are ambiguities even with the typicalities. First, with more than one parameter, one gets more than one typicality. Second, if there are N independent parameters, one can construct N independent combinations of them in arbitrarily many different ways. Both of these problems are related to the issue of how one chooses to test a theory, which has no unique answer.

Once one has made a choice of what set of observations to include and what parameter to determine the typicality for, how do we use the typicality to test a theory? It can be used – like any other likelihood – in the following manner: Let H_n be an hypothesis that gives measures to observations in the set, so that an actual observation O has typicality $T_n(O)$ according to this hypothesis. At the simplest level, one can say that, if $T_n(O)$ is low, then H_n is ruled out at the corresponding level. For example, if $T_n(O) < 0.01$, then one can say that H_n is ruled out at the 99% confidence level.

A better approach would be to assign initial or prior probabilities $P_i(H_n)$ to different hypotheses H_n, labeled by different values of n. Then the typicalities

$T_n(O)$ for these different hypotheses would be used as weights to adjust the $P_i(H_n)$ to final or posterior probabilities $P_f(H_n)$ that are given by Bayes' formula:

$$P_f(H_n) = \frac{T_n(O)P_i(H_n)}{\sum_m T_m(O)P_i(H_m)} \,. \tag{1}$$

Apart from the ambiguity of choosing the set of possible observations and the parameter to be observed and the physics problem of calculating the typicalities $T_m(O)$ assigned by each theory H_m, there is now the new ambiguity of assigning prior probabilities $P_i(H_m)$ to the theories themselves. This appears to be a purely subjective matter, though – in the spirit of Ockham's razor – scientists would generally assign higher prior probabilities to simpler theories. Of course, there are arbitrarily many ways to do that. However, if one just considered an infinite countable set of theories that one could order in increasing order of complexity, from the simplest H_1 to the next simplest H_2 and so on, then one simple assignment of prior probabilities would be

$$P_i(H_m) = 2^{-m} \,. \tag{2}$$

The idea of restricting attention to a countable set of theories seems plausible, since humans could really consider only a finite set of theories, but it could be inappropriate if the ultimate theory of the universe contained an infinite amount of information, even if merely in the form of a single real coupling constant or some other parameter whose digits are not compressible (i.e. generated by a finite amount of input information). Note that it is considered to be a merit of string/M theory that there is not even the possibility of having infinite amounts of information in any dimensionless coupling constants, at least in the dynamical equations of the theory, although it is apparently not yet ruled out that the quantum state might have an infinite amount of information. This might apply to the expectation value of the dilaton, although most theorists would also prefer to avoid this possibility.

4 Testing the single-universe and multiverse hypotheses

Tegmark [22] has classified multiverse hypotheses into Levels 1, 2, 3 and 4. Level 1 is regions beyond our cosmic horizon, with the same 'constants of nature' as our own region. Level 2 is other post-inflation bubbles, perhaps with different 'constants of nature'. Level 3 are the Everett many worlds of quantum theory, with the same features as Level 2. Level 4 is other mathematical structures, with different fundamental equations of physics as well as different constants of nature.

Levels 1-3 can all come from a single universe if we define a universe to be some quantum state in some quantum state space (e.g. some C*-algebra state). In this case, the quantum state-space may be regarded as a set of quantum operators and their algebra, and the quantum state as an assignment of an expectation value to each quantum operator. To get measures for observations in the form of conscious perceptions, one must add to this bare quantum theory an assignment of a particular positive operator for each set of conscious perceptions. The resulting 'awareness operators' then form a positive-operator-valued set obeying the appropriate sum rules when one forms unions of disjoint sets of conscious perceptions, so that the resulting expectation values have the properties of a measure on sets of conscious perceptions [13, 14].

Different hypotheses H_m that each specify a single SQM universe would give different quantum state spaces, different operator algebras, different quantum states, different sets of conscious perceptions and/or different sets of awareness operators corresponding to the sets of conscious perceptions. (A quantum state is here defined, in the C*-algebra sense, as the quantum expectation values for all possible quantum operators in the set.) By the SQM rule that the measure for each set of conscious perceptions is the expectation value given by the quantum state for the corresponding awareness operator, a definite SQM theory H_n would give a definite measure for each set of possible conscious perceptions. This would be a theory of a single SQM universe, though that universe could be a multiverse in the senses of Levels 1-3.

Then by the procedure outlined above, from one's actual observation O, a sufficient intelligence should be able to calculate for each H_m the typicality $T_m(O)$ of that observation. If one has a set of such theories with prior probabilities $P_i(H_m)$, then one can use a Bayesian analysis to calculate the posterior probability $P_f(H_n)$ for any specific theory H_n and thereby test the theory at a statistical or probabilistic level.

But what if there is more than one universe? Tegmark [22, 23] has raised the possibility of a multiverse containing different mathematical structures, and it certainly seems logically conceivable that reality may consist of more than one universe in the sense of Levels 1 to 3. Tegmark discusses a Level 4 multiverse which, as he describes it, includes all mathematical structures. This seems to me logically inconsistent and inconceivable. My argument against Level 4 is that different mathematical structures can be contradictory, and contradictory ones cannot co-exist. For example, one structure could assert that spacetime exists somewhere and another that it does not exist at all. However, these two structures cannot both describe reality.

Now one could say that different mathematical structures describe different existing universes, so that they each apply to separate parts of reality and cannot be contradictory. But this set of existing universes, and the different mathematical structures with their indexed statements about each of them, then forms a bigger mathematical structure. At the ultimate level, there can be only one world and, if mathematical structures are broad enough to include all possible worlds or at least our own, there must be one unique mathematical structure that describes ultimate reality. So I think it is logical nonsense to talk of Level 4 in the sense of the co-existence of all mathematical structures. However, one might want to consider how to test levels of the multiverse between Levels 1-3 and 4.

One way to extend an SQM universe to a multiverse might be to allow more than one quantum state on the same quantum state-space, while keeping the other parts of the structure – such as the awareness operators – the same. Then if a weight is assigned to each of these different quantum states, one can get the measure for each set of conscious perceptions as the weighted sum of the measures for each quantum state. But this is equivalent to defining a new single quantum state in a new single-universe theory that is the weighted sum of these different quantum states in the original description. That is, the new single quantum state would be defined to give as the expectation value of each operator the weighted sum of the expectation values that the different quantum states would give. (If the quantum state can be described by a density matrix, then the new density matrix would be the weighted sum of the old ones.)

Since the measure for a set of conscious perceptions in an SQM universe is the expectation value given by the quantum state of the awareness operator corresponding to the set of conscious perceptions, one would get the same measure by using the new quantum state as by taking the weighted sum of the measures in the old description in which there are different quantum states.

Another way to get a broader multiverse would be to keep the same quantum state-space, quantum operators, operator algebra and set of possible conscious perceptions, but to include different sets of awareness operators in different SQM universes. But again, if one weights the resulting measures for each universe to get a total measure for this multiverse, this would be equivalent to forming a single new set of awareness operators that are each the weighted sum of the corresponding awareness operators in each of the different universes.

Yet another way to extend the multiverse would be to include universes with separate quantum state-spaces, each with its own quantum state and awareness operators. If each of these universes has a weight, then one can again get the total measure for each set of conscious perception by taking the weighted sum of the measure for that set in each universe. This would be equivalent to defining a total quantum state-space whose quantum operators were generated by the tensor sum of the operators in each of the original sets of operators that correspond to the original separate quantum state-spaces. One could take operators from different original sets as commuting to define the quantum algebra of the new set.

The new single quantum state could then be defined by giving – on any sum of operators from the separate sets of operators – the weighted sum of expectation values that the old quantum states gave. For products of operators from different sets, one could just take the new expectation value to be the product of the weighted old expectation values for the separate operators in each set. The new awareness operators could be defined as the sum of the original awareness operators. Since this would involve only sums from the different sets of operators and not products, the expectation values of the new awareness operators would all be linear in the weights for the original separate universes in the new single quantum state and hence would give in that new single quantum state the same measure as the weighted sum of the original measures.

Each of these three simple-minded ways to attempt to extend the multiverse produces nothing new, at least for the measures of sets of conscious perceptions. Thus a single SQM universe is a fairly broad concept, encompassing a wide variety of ways of generating measures for conscious perceptions. In fact, one could argue that any assignment of measures for conscious perceptions could come from a single SQM universe, since one could just define awareness operators for all sets of conscious perceptions and embed these into a larger set of quantum operators with some algebra. One could then just choose the quantum state to give the desired expectation values for all of the awareness operators.

In principle, one could even choose the algebra of operators to be entirely commuting, so that the resulting quantum theory would be entirely classical, though still possibly giving the Everett many worlds rather than just a single classical world. Thus even a universe that gives exactly the same measures for conscious perceptions as ours, and hence the same typicalities for all observations, could in principle be entirely classical in the sense of being commutative. We cannot prove that the universe is quantum just from our observations.

However, surely such a classical description of our conscious perceptions would involve a more complicated SQM universe than one in which there are non-commuting operators (and presumably even non-commuting awareness operators). Thus it is on the ground of simplicity and Ockham's razor that we assign higher probabilities to non-commuting quantum theories that explain our observations, even though the likelihoods for our observations can be precisely the same in a classical theory. In a similar way it might turn out that, although a multiple-SQM-universe theory could be reduced to a single-SQM-universe theory in one of the ways outlined above, the description could be simpler in terms of the former or even in terms of universes that are not SQM.

If we do have a true multiverse of different universes, each of which gives a measure for each set of conscious perceptions, then to get a measure covering the whole of reality, we would need a measure for each of the individual multiverses. For suppose each universe is described by an hypothesis H_n that assigns a measure $\mu_n(S)$ for each set S of conscious perceptions. When we were considering single universes, we considered different H_m just as theoretical alternative possibilities and discussed assigning subjective prior probabilities $P_i(H_m)$ to them. But when we are considering true multiverses, we need an objective weight $w(H_n)$ for each universe, since each universe with non-zero weight is being considered to actually exist. Therefore the total measure for each set of conscious perceptions from this extended multiverse would be $\mu(S) = \sum_n w(H_n)\mu_n(S)$.

Extending the multiverse to multiple SQM universes (or to any ensemble in which there is a prediction for the measure for all sets of conscious perceptions from each universe) replaces our uncertainty about which H_n is correct with the uncertainty about which $w(H_n)$ is correct. It would replace Tegmark's question [22] "Why these equations?" with "Why this measure?" We cannot evade some form of this question by invoking ever higher levels of the multiverse, even though this may provide a simpler description of a world.

In the sense that an SQM universe is a single universe, it may still encompass Level 1-3 multiverses. At the true multiverse level, we need not just a single theory H_n for a single universe but also a meta-theory I for the measure or weight $w(H_n)$ of the single universes within the set of actually existing multiverses. However, since we do not yet know what the correct meta-theory is, just as we do not yet know what the correct theory H_n is for our single universe, we may wish to consider various theoretically possible meta-theories, I_M, labeled by some index M in the same way that n labeled the single universe described by the theory H_n. Then meta-theory I_M says that single universes exist with measures $w_{M,n} \equiv w_M(H_n)$ and so a set of conscious perceptions S would have measure $\mu_M(S) = \sum_n w_{M,n}\mu_n(S)$. From the measure for conscious perceptions, one can follow the procedure outlined in the previous section to get the typicality $T_M(O)$ of an observation O in meta-theory I_M.

For example, if the single universes described by H_n are labeled by the positive integers n in order of increasing complexity, and if the meta-theories I_M are labeled by the positive integers M, one might imagine the following choice for the weights $w_{M,n}$ of the meta-theory I_M to give the universe H_n:

$$w_{2m-1,n} = \frac{1}{m}\left(\frac{m}{m+1}\right)^n , \quad w_{2m,n} = \delta_{mn} . \tag{3}$$

Then for odd M, one gets a geometric distribution of weights over all single universes described by the theories H_n, with the mean of n being $m+1$. However, for even M, one gets a non-zero (unit) weight only for the unique single universe described by the theory H_m. Thus the odd members of this countable sequence of meta-theories do indeed give multiverse theories with various weights, but the even members give single-universe theories.

Just as in a Bayesian analysis for single-universe theories we needed subjective prior probabilities $P_i(H_m)$ for the possible single-universe theories H_m, so now for a Bayesian analysis of multiverse meta-theories I_M, we need subjective prior probabilities $P_i(I_M)$. Again, although these subjective prior probabilities are really arbitrary, we may wish to invoke Ockham's razor for the meta-theories and assign the simpler ones the greater prior probabilities. For example, if we can re-order the I_M in increasing order of complexity by another natural number $N(M)$, one might use the simple subjective prior probability assignment

$$P_i(I_M) = 2^{-N(M)}\,. \tag{4}$$

This would imply that the simplest meta-theory ($N = 1$) is assigned 50% prior probability of being correct, the next simplest ($N = 2$) 25% etc.

For a more *ad hoc* choice, one could take the meta-theory weights given by the hybrid model of eqn (3) for both single and multiple universes and arbitrarily set

$$P_i(I_{2m-1}) = P_i(I_{2m}) = 2^{-m-1}\,. \tag{5}$$

This gives a total prior probability of 1/2 for single-universe (even M) theories and 1/2 for multiple-universe (odd M) theories. This might be viewed as a compromise assignment if one is *a priori* ambivalent about whether a single-universe or multiple-universe theory should be used.

5 Conclusions

Even though multiverse theories usually involve unobservable elements, they may give testable predictions for observable elements if they include a well-defined measure for observations. One can then analyze them by Bayesian means, using the theory-dependent typicality of the result of observations as a likelihood for the theory, though there is still an inherent ambiguity in assigning prior probabilities to the theories.

One can try to avoid specifying the equations or other properties of an individual universe by assuming that there is an ensemble of different universes, but this replaces the question of the equations with the question of the measure for the different universes in the ensemble. There is no apparent way to avoid having non-trivial content to a testable theory fully describing all of reality.

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